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Analog Integrated Filters

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Part-I: Overview

Classification of filters

– Analog filters





• Sampled-data



Digital filters



Digital vs. Analog Implementations

Programmability.
 Flexibility.
 Short design cycles.
 Good immunity to noise.
 Good immunity to manufacturing process tolerances.

Their performance is degraded in very high frequency applications.



The analog part provides the I/O interface (amplification, filtering, etc.) to the core of the chip which is digital.

The approximation problem

• Frequency response of ideal filters



• Practical specifications for a LP filter



• Characteristic transfer functions.



• Chebyshev vs Butterworth responses.

Higher attenuation in stopband.
Steeper roll-off near the cut-off frequency.

More complex filter realizations.
Eess linear phase characteristics.

The choice of frequency response that should be implemented, requires trade-off between the above conflicting requirements.

• Determination of the order of a Butterworth filter.





☺A doubly terminated LC ladder has the advantage of low sensitivity to component tolerances.

Cladder filters are mainly used in very high frequency applications.

③Inductors are heavy and bulky and thus they are difficult to adapt to IC realization.

Active RC filters

• Miller integrator



$$H(s) = \frac{U_o(s)}{U_{in}(s)} = -\frac{1}{RCs}$$

Frequency response

$$|H(s)| = \frac{\omega_o}{\omega}, \qquad \omega_o = \frac{1}{RC} \qquad \omega_o$$
: unity-gain frequency



 \square The cutoff frequency is determined by the integrator's time constant.

• Second-order filters (Biquads)

They are constructing using two-integrator loop.



Tow-Thomas Biquad



Design of high-order RC filters

- Using second-order sections in cascade connection or with multiple-loop feedback.
- Simulating the corresponding LC ladder prototypes.
 - Functional simulation of LC ladders.
 - Topological simulation of LC ladders.

Cascade connection of biquads

Design procedure for maximizing the Dynamic Range

•Pole-zero pairing: each pole is assigned to the closest zero.

- •Section ordering: in the order of increasing values of Q. In addition, LP or BP are employed as first section and HP or BP are employed as last section.
- •Gain assignement: the gain constants are computed in such a way that the maxima of output voltages of all sections would be made equal.

 \odot Easy to design and tune.

 \odot Higher sensitivity, in comparison to other filter configurations.

• Functional simulation of LC ladders

✓ LC ladder prototype



Voltage/current equations

$$-V_{1} = -\frac{1}{sC_{1}} \left(\frac{V_{in} - V_{1}}{R_{s}} - I_{2} \right)$$
$$-I_{2} = -\frac{1}{sL_{2}} (V_{1} - V_{3})$$
$$V_{3} = -\frac{1}{sC_{3}} \left(\frac{V_{3}}{R_{L}} - I_{2} \right)$$

✓ Signal Flow Graph (SFG)

$$-V_{1} = -\frac{1}{sRC_{1}} \left(\frac{R}{R_{s}} V_{in} - \frac{R}{R_{s}} V_{1} - I_{2}R \right)$$
$$-I_{2}R = -\frac{1}{sL_{2}/R} (V_{1} - V_{3})$$
$$V_{3} = -\frac{1}{sRC_{3}} \left(\frac{R}{R_{L}} V_{3} - I_{2}R \right)$$







• Topological simulation of LC ladders

• General impedance converter-GIC



Simulation of a grounded inductance



Antoniou circuit



f(s) = RCs



MOSFET-C filters

 \odot In low-frequency applications ($\omega c=10krad/sec$) in case of active RC filters, a resistor with value $5M\Omega$ is required, in case that C=20pF.



- © Resistors are replaced by MOS transistors operated in linear region. Their values are electronically adjustable by the gate voltage.
- ☺ The developed active RC design methods can be directly used in MOSFET-C filters.
- O They suffer from the nonlinearities introduced by MOS transistor.
- S Reduced dynamic range in comparison to active RC filters
- Fully balanced structures are used, in order to reduce the distortion introduced by MOS transistor.

Transconductance-C (G_m-C) filters

- ☺ The opamp frequency limitations (finite GBW) degrade the performance of active RC, MOSFET-C and SC filters.
 - **Basic CMOS transconductance stage**



- \bigcirc G_m is electronically adjustable.
- \odot The stage has simpler structure, in comparison to the op-amp.





✓ SFG



✓ Configuration of filter



Basic building blocks for topological simulation of LC ladders

✓ Inductance simulation



Current-Conveyor (CCII) filters

Definitions for CCII



$$b_x = b_y$$

 $i_z = i_x$

 $R_y \to \infty$ $R_x \to 0$ $R_z \to \infty$

• CCII lossless integrator



$$H(s) = \frac{I_o(s)}{I_i(s)} = \frac{1}{RCs}$$

Current Amplifier filters





$$i_{out} = A \cdot i_{in}$$

• Lossy integrator



^(C) The time-constant of integrator can be electronically adjusted

Switched-Capacitor (SC) filters

- The time-constants in active RC circuits could not be accurately implemented on chip.
- ☺ Large resistor values are needed in case of low-frequency applications.

SC integrator



 \odot The time-constant of integrator is defined by capacitors ration and thus the achieved accuracy is 0.1%.

^(C) Tuning is achieved using that clock frequency.

 \bigcirc For a resistor value of 10*M* Ω , and using 100kHz clock, a capacitor 1*pF* is needed.

- O The previous direct replacement of resistors by switched capacitors is valid only in case that the clock frequency is much higher than the signal frequency.
- O The frequency limitations of op-amps degrade the performance of SC filters.

[☉]SC filters were designed using s-z transformations.



S The circuit is sensitive to the effect of parasitic capacitances.



 \odot SC integrator insensitive to the effect of parasitic capacitances.



$$H_{1/1}(z) = \frac{V_0(z)}{V_{in}(z)} = -\frac{C_1}{C_2} \frac{1}{1-z^{-1}}$$

• Steps for designing SC filters

✓ LC ladder prototype





✓ Continuous-time SFG

✓ Discrete-time SFG using s-z transformation



The above SFG was derived using the LDI transformation

$$s = \frac{1}{T} \frac{1 - z^{-1}}{z^{-1/2}}, \qquad \Omega = \frac{2}{T} \sin\left(\frac{\omega}{2}\right)$$





Switched-Current (SI) filters

Basic memory element



•In time-slot 1 the voltage at gate capacitance is equal to:

$$=V_T + \sqrt{\frac{J + i_{in}}{\frac{\mu_o C_{ox} W}{2L}}}$$

 v_{GS}

•In time-slot 2 transistor M₁ sustains its drain current, and thus:

$$H_{1/2} = \frac{i_o(z)}{i_{in}(z)} = -Az^{-1/2}$$

© Capacitors are not further needed and as a result SI filter are fully compatible with standard digital CMOS process. 35



$$H_{1/2} = \frac{i_o(z)}{i_{in}(z)} = -z^{-1/2}$$

O This cell is insensitive to the effect of MOS transistor parameters mismatch.

 $\ensuremath{\mathfrak{S}}$ Scaling of output current is not available.

Full period delay cell


SI lossless integrator



Transfer function

$$H_{1/1}(z) = A \frac{z^{-1}}{1 - z^{-1}}$$

Frequency response

$$H_{1/1}(e^{j\omega}) = \frac{A}{j\omega} \frac{\frac{\omega}{2}}{\sin\left(\frac{\omega}{2}\right)} e^{-\frac{j\omega}{2}}$$

O The time-constant of integrator is defined by the MOS transistors aspect ratio.

• <u>SI lossy integrator</u>



Transfer function

Frequency response

$$H_{1/1}(z) = \frac{\frac{A}{1+B}z^{-1}}{1-\frac{1}{1+B}z^{-1}}$$

$$H_{1/1}(e^{j\omega}) = \frac{\frac{A}{B}}{1+j\frac{\omega}{\frac{2}{T}\frac{B}{2+B}}} \qquad a_o = \frac{A}{B} \quad \text{low frequency gain}$$

$$\omega_o = \frac{2}{T}\frac{B}{2+B} \quad \text{cut-off frequency}$$

• Effects of non-idealities in SI filters

 \otimes MOS transistor parameters mismatch



Parameter	DC Offset	Small-Signal Gain Error	Hamonic Distortion
$\Delta K = K_0 - K_I$	$2(K_0 - K_1)_{T}$	$2(K_0 - K_1)$	
	$K_0 + K_1$	$\overline{K_0 + K_1}$	
$\Delta W = W_0 - W_1$	$2(W_0 - W_1)_{T}$	$2(W_0 - W_1)$	
	W_0+W_1	$W_0 + W_1$	
$\Delta L = L_0 - L_1$	$2(L_0-L_1)$	$2(L_0-L_1)$	
	$L_0 + L_1$	$L_0 + L_1$	
	$\frac{2(V_{r0} - V_{r1})}{V_{con} - V_{r0}}J$	$\frac{(V_{10} - V_{11})}{V_{000} - V_{10}}$	$THD \approx \frac{V_{10} - V_{11}}{8(V_{col} - V_{10})} \left(\frac{i_{\text{max}}}{J}\right)$
			· /

☺ Finite input-output conductance ratio of MOS transistor.



✓ Regulated-cascode structures are used



\otimes Clock-feedthrough effect in MOS switches.



•The error that is caused is similar to that of V_T mismatch.

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Part-II: Companding filters

INTRODUCTION



They open the door to elegantly realizing a linear system with inherently non-linear building blocks, and may achieve the advantageous potential of companding (compress-expand) signal processing.

•Linear and companding signal processing



• Waveforms at the internal nodes of a typical log-domain system.



© Companding filters have potential for low-voltage operation, as the signal swings within filter are typically less than 100mV.

Log-Domain Filters

Methods for designing Log-Domain filters

- ✓ Using the well-known state-space synthesis approach in linear domain, and an appropriate mapping of the resulted equations.
 - O A large number of equations is needed in case of high order filters.
- ✓ Using the well-known Signal Flow Graph (SFG) synthesis approach in linear domain, and a set of complementary operators.
 - [©]This method is more simpler than the ESS.
- ✓ By following the concept of Wave Active Filters.
 - \odot The design procedure of high-order filters is quite facilitated.
 - ③ Modular filter structures are derived.

Log-Domain filters synthesis using the SFG approach

Representation of a Log-Domain filter using complementary operators





Log

💋 Exp

BJT Translinear principle



✓ For any closed loop comprising any number of pairs of clockwise and counterclockwise forward-biased identical junctions, the product of currents for the elements in one direction is equal to the corresponding product in the opposite direction.

$$\prod_{CW} I_C = \prod_{CCW} I_C$$

•As an extension to the above principle, when a voltage source is introduced into the loop, then: $\underline{\nu_s}$

$$\prod_{CW} I_C = e^{\frac{\sigma_s}{V_T}} \cdot \prod_{CCW} I_C$$
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Non-linear transconductance



Implementation of complementary operators



$$EXP(\hat{\upsilon}) = I_o \cdot e^{\frac{\hat{\upsilon}}{V_T}} - I_o$$
$$LOG(i) = V_T \cdot ln\left(\frac{i + I_o}{I_o}\right)$$

Log-Domain integrators



© Electronic tuneability using a DC current source.

O The time constant is depended from temperature.

Lossless integrator-subractor





^(C) Damping is achieved using a DC current source.

High-order Log-Domain filters

Operational simulation of all-pole LC ladders

✓ LC ladder prototype



✓ Linear domain Signal Flow Graph (SFG)



✓ **Transposition to Log-Domain using the following rules:**

•A LOG block is placed at the output of each integrator.

- •An EXP block is placed at each input of integrators (before scaling).
- •A LOG block is placed at the input of the system.
- •An EXP block is placed at the output of the system.



✓ Implementation using Log-Domain cells



Operational simulation LC ladders with finite zeros

LC ladder prototype $\underbrace{\sum_{i_{s} \in C_{1}} i_{2} + i_{2}}_{v_{s} \in C_{1}} \underbrace{\sum_{i_{s} \in C_{2}} i_{2}}_{C_{2}} \underbrace{\sum_{i_{s} \in C_{3}} i_{i_{s}}}_{C_{1}} \underbrace{\sum_{i_{s} \in C_{2}} i_{i_{s}}}_{C_{2}} \underbrace{\sum_{i_{s} \in C_{3}} i_{i_{s}}}_{C_{1}} \underbrace{\sum_{i_{s} \in C_{3}} i_{i_{s}}}_{C_{2}} \underbrace{\sum_{i_{s} \in C_{3}} i_{i_{s}}}_{C_{2}} \underbrace{\sum_{i_{s} \in C_{3}} i_{i_{s}}}_{C_{2}} \underbrace{\sum_{i_{s} \in C_{3}} i_{i_{s}}}}_{C_{2}} \underbrace{\sum_{i_{s} \in C_{3}} i_{i_{s}}}}_{C_{2}} \underbrace{\sum_{i_{s} \in C_{3}} i_{i_{s}}}_{C_{2}} \underbrace{\sum_{i_{s} \in C_{3}} i_{i_{s}}}}_{C_{2}} \underbrace{\sum_{i_{s} \in C_{3}} i_{i_{s}}}}_{C_{2}}$

O Differentiation is required:

$$\upsilon_{I} = \frac{1}{sRC_{I,eq}} (\upsilon_{S}' - \upsilon_{2}' + sRC_{2}\upsilon_{3})$$

③ Alternative form without differentiation:

$$\upsilon_{I} = \frac{1}{sRC_{I,eq}} (\upsilon'_{S} - \upsilon'_{2}) + \frac{C_{2}}{C_{I,eq}} \upsilon_{3}$$

✓ Log-Domain Signal Flow Graph (SFG)



✓ Implementation using Log-Domain cells

Log-Domain summing block



$$EXP(\hat{\upsilon}_{out}) = EXP(\hat{\upsilon}_1) + K \cdot EXP(\hat{\upsilon}_2)$$

Log-Domain elliptic filter



Frequency response of the filter *a* modulation index factor 0.8



The frequency response of a Log-Domain filter is evaluated using large-signal transient simulations and FFT analysis.

 \odot The AC analysis of HSPICE is valid only for small modulation index factor !!! ⁶³

Non-linear analysis



A two-tone test was performed, in order to measure the third-order intermodulation distortion factor.

☺ The IMD3 was -52dB @ modulation index equal to 1.

Noise analysis



☺ The achieved Dynamic Range was 35dB.

Log-Domain Wave filters

Wave variables



incident wave
$$A_k = i_k + \frac{\upsilon_k}{R}$$
 $(k = 1, 2)$

reflected wave
$$B_k = i_k - \frac{v_k}{R}$$
 $(k = 1, 2)$

Scattering parameters description



✓ Transposition of linear domain SFG to the Log-Domain SFG



Lossy integration



Summation





Subtraction





Inversion







✓Log-domain wave equivalents of 1st-order blocks

✓ Log-domain wave equivalents of 2nd-order blocks



Log-Domain wave filter





- \odot A quick design procedure is offered.
- ③ Modular filter structures are derived.
- \odot The circuit complexity is increased in comparison to the leapfrog filters.
Effects of non-idealities

Effect of non-zero R_E



 $I_{OUT} = I_0 e^{\frac{\hat{\nu}_{IN} - \hat{\nu}_{OUT}}{V_T + \frac{R_B}{\beta}I_0}}$

⊗ The cut-off frequency is shifted, while the filter shape remains unchanged.

• Input-output relationship for a lossy integrator



☺ The effect of non-zero R_E is compensated by multiplying the value of DC current source by a factor $(W + U P_E)$

$$k_{RE} = \frac{\left(V_T + I_0 R_E\right)}{V_T} \tag{74}$$

Effect of BJT transistor finite beta.



The finite beta introduces a scalar error and an extra feedback path.
The cut-off frequency and the phase response of the filter are changed.
The above imperfections can be electronically removed.

•Wave equivalent of a floating inductance, with additional circuitry for removing the effect of finite beta.





⊗ The cut-off frequency is shifted, while the filter shape remains unchanged.

 \odot This deviation is canceled by following the same procedure as in non-zero R_E.



The values of factors are calculated using a fitting algorithm.



Square-Root-Domain filters

A method for designing Square-Root-Domain filters

✓ Representation of a Square-Root-Domain filter using complementary operators



• Definition of operators

$$SQ(\hat{\upsilon}) = \frac{K}{2} (\hat{\upsilon} - V_T)^2 - I_o$$
 $SQRT(i) = \sqrt{\frac{2(i+I_o)}{K} + V_T}$

SQ[SQRT(i)]=i



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Square-Root-Domain integrators

SQRT

 $-\hat{v}_{OUT}$



$$i_{c} = C \frac{d\hat{\upsilon}_{OUT}}{dt} = \frac{i_{IN}I_{a}}{\sqrt{I_{o}i_{OUT}}} - \frac{I_{o}I_{a}}{\sqrt{I_{o}i_{OUT}}}$$

$$SQ(\hat{\upsilon}_{OUT}) = \frac{1}{\tau} \int SQ(\hat{\upsilon}_{IN}) \cdot dt$$

The time-constant of integrator is:



 \bigcirc Electronic tuneability using a DC current source.

Lossless integrator-subtractor



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Lossy integrator



^(C) Damping is achieved using a current mirror.

Complete system of current integrator



Simplification of circuit:



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MOS Translinear principle



✓ In the closed loop, the gate-source voltages are in series, with equal number of transistors arranged clockwise and counter-clockwise.

$$\sum_{CW} \sqrt{\frac{I_D}{W/L}} = \sum_{CCW} \sqrt{\frac{I_D}{W/L}}$$

Stacked topology



③ Offers simple circuits.

S Suffers from a strong influence of the body effect.

Up-down topology



O The body effect is much smaller than that in case of stacked topology.

B Requires some extra circuitry, in comparison with the stacked configuration.

Electronically simulated loop topology



✓ The average of the gate-source voltages of transistors M_1 and M_2 is forced equal to the average of the gate-source voltages of transistors M_3 and M_4 .

^(C) The body effect is completely eliminated.

- O The achieved bandwidth is limited by the speed of additional circuitry.
- O A relative large number of elements is required in same cases.

Basic SQRD blocks

Current geometric-mean circuit



•The voltage averaging subcircuit produces a gate voltage $v_{X'Y}$, which is forced to be equal with the average of the gate voltages of MX, and MY.

MOS translinear principle:
$$\sqrt{i_{XY}} = \sqrt{i_X} + \sqrt{i_Y}$$

Output current: $i_z = 2\sqrt{i_X} \cdot i_Y$

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Current multiplier topology



 \checkmark It is constructed by two properly connected geometric-mean circuits.

Performance of the proposed multiplier.

•Comparison with a multiplier based on up-down topology.



 ○ A reduction of about 50% for the required transistor area is achieved in the proposed circuit.

^(c) The minimum supply voltage requirement for both proposed circuits is: $V_{DD\min} = 2V_{DS,SAT} + V_{GS}$

High-order Square-Root-Domain filters

Operational simulation of all-pole LC ladders

✓ LC ladder prototype



✓ Linear domain Signal Flow Graph (SFG)



✓ **Transposition to Square-Root Domain using the following rules:**

•A SQRT block is placed at the output of each integrator.

- •A SQ block is placed at each input of integrators (before scaling).
- •A SQRT block is placed at the input of the system.
- •A SQ block is placed at the output of the system.



✓ Implementation using Square-Root-Domain cells



Frequency response of the filter *a* modulation index factor 0.8



The frequency response of a Square-Root-Domain filter is evaluated using large-signal transient simulations and FFT analysis.

38 The AC analysis of HSPICE is valid only for small modulation index factor 38

✓ Electronic tuneability of Square-Root-Domain filters.



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Non-linear analysis



A two-tone test was performed, in order to measure the third-order intermodulation distortion factor.

☺ The IMD3 was -42dB @ modulation index factor equal to 1.

Noise analysis



☺ The achieved Dynamic Range was 49.7dB.

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Square-RootDomain

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