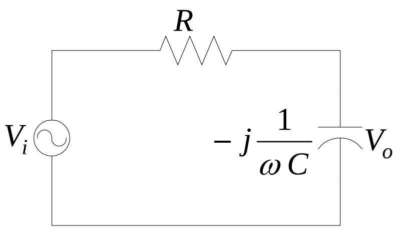
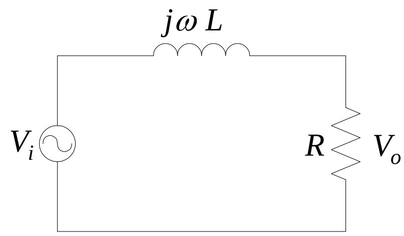
Frequency selective circuits

Low pass Filters are used to pass low-frequency sine waves and attenuate high frequency sine waves. The *cutoff* frequency ω_c is used to distinguish the passband ($\omega_c<\omega$) from the stopband ($\omega_c> \omega$). An elementary example of two passive lowpass filter is given below.





$$\frac{V_0}{V_i} = \frac{1}{1+j\omega RC} \qquad (1) \qquad \frac{V_0}{V_i} = \frac{1}{1+j\omega \frac{L}{R}}$$

$$\frac{|V_o|}{|V_i|} = \frac{1}{\left[1+(\omega RC)^2\right]^{1/2}} \qquad (2) \qquad \frac{|V_o|}{|V_i|} = \frac{1}{\left[1+\left(\omega \frac{L}{R}\right)^2\right]^{1/2}}$$

$$\theta = -\tan^{-1}\omega RC \qquad (3) \qquad \theta = -\tan^{-1}\omega \frac{L}{R}$$

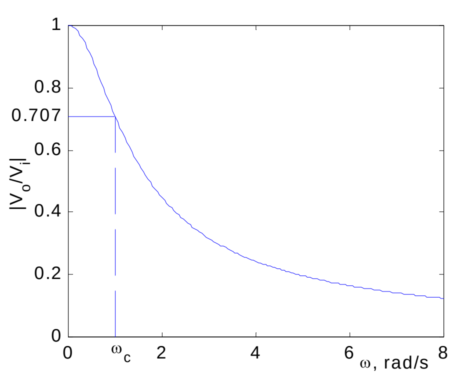
At cutoff frequency Gain $=1/\sqrt{2}$. Substituting in (2) result in

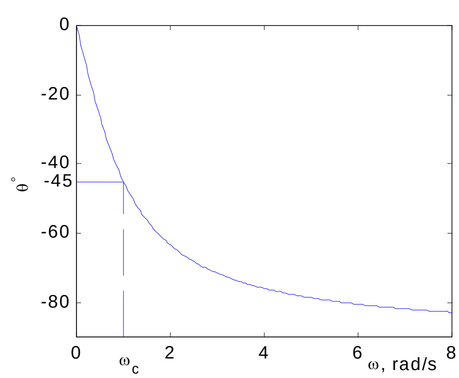
$$\omega_c = \frac{1}{RC} \qquad (4) \qquad \omega_c = \frac{R}{L}$$

From (3), we see that the phase angle at cutoff frequency is -45°

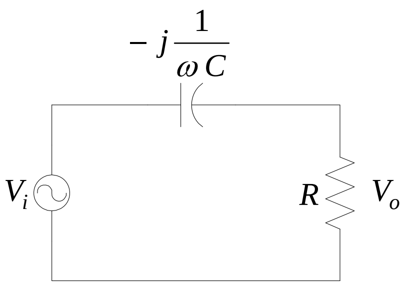
The ratio $\frac{V_o}{V_i}$ is shown by $H(j\omega)$, and is called the *frequency response transfer function.*

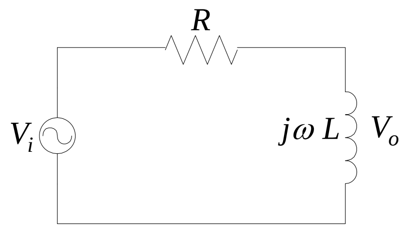
The gain versus frequency, and the phase angle versus frequency known as the *frequency response* is as shown.





High pass Filters are used to stop low-frequency sine waves and pass the high frequency sine waves. The *cutoff* frequency ω_c is used to distinguish the stopband ($\omega_c<\omega$) from the passband ($\omega_c>\omega$). An elementary example of two passive highpass filter is given below.





$$\frac{V_0}{V_i}=\frac{\omega \mathrm{RC}}{\omega RC-j1} \qquad (5) \qquad \frac{V_0}{V_i}=\frac{\omega\frac{L}{R}}{\omega\frac{L}{R}-j1}$$

$$\frac{|V_o|}{|V_i|}=\frac{\omega RC}{\left[(\omega RC)^2+1\right]^{1/2}} \qquad (6) \qquad \frac{|V_o|}{|V_i|}=\frac{\omega\frac{L}{R}}{\left[\left(\omega\frac{L}{R}\right)^2+1\right]^{1/2}}$$

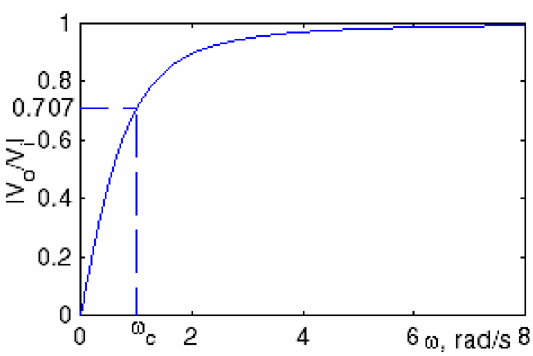
$$\theta=\tan^{-1}\frac{1}{\omega \mathrm{RC}} \qquad (7) \qquad \theta=\tan^{-1}\frac{R}{\omega L}$$

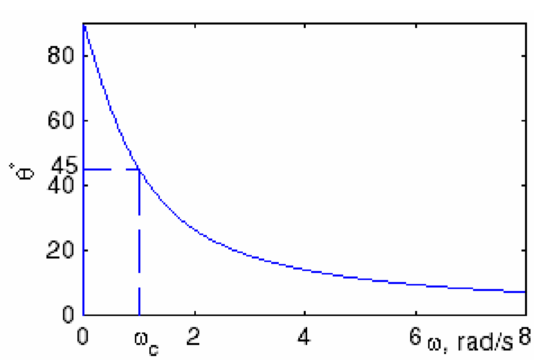
At cut off frequency Gain $=1/\sqrt{2}$. Substituting in (6) result in

$$\omega_c=\frac{1}{RC} \qquad \omega_c=\frac{R}{L}$$

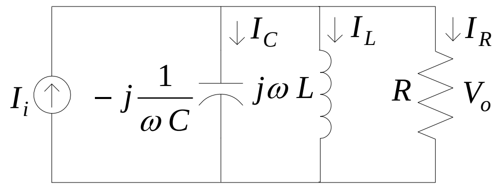
From (7), we see that the phase angle at cutoff frequency is 45°

The gain versus frequency, and the phase angle versus frequency known as the *frequency response* is as shown.



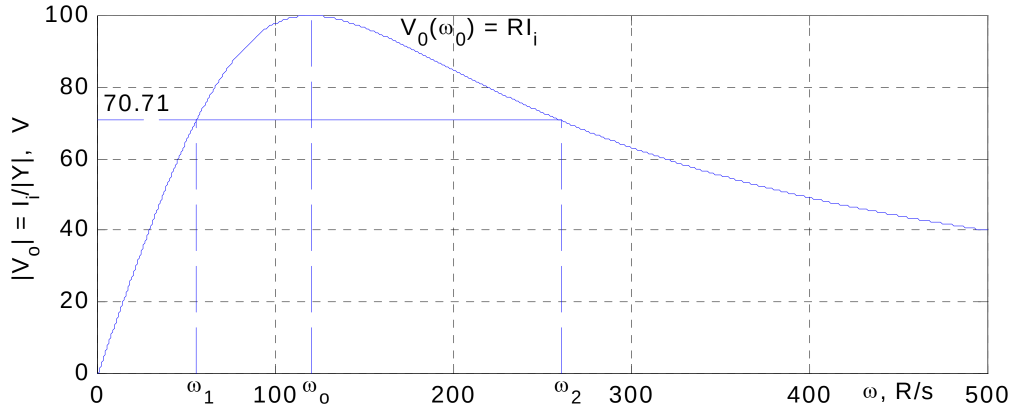


Bandpass Filters
(a) The Parallel RLC Resonance



$$V_o = \frac{I_i}{\frac{1}{R} + j(\omega C - \frac{1}{\omega L})}$$

A circuit is in resonance when the voltage and current at the input terminals are in phase. The circuit admittance is $Y = \frac{1}{R} + j(\omega C - \frac{1}{\omega \mathrm{L}})$. At resonance Y is purely conductive and $\omega C - \frac{1}{\omega \mathrm{L}} = 0$, thus $\omega_o = \frac{1}{\sqrt{LC}}$. The circuit admittance is minimum or the circuit impedance at resonance, given by $Z(\omega_o) = R$, is maximum. Thus, the output voltage at resonance is maximum and is given by $V_o(\omega_o) = RI_i$



The frequencies ω_1 and ω_2 at which the output power drops to one half of its values at the resonant frequency are called the *half-power frequencies*. At these frequencies also known as *cutoff frequencies* or *corner frequencies*, the output voltage is $|V_o(\omega_c| = 0.707\,|V_o(\omega_o)|$. This circuit which passes all the frequencies within a band of frequencies ($\omega_1 < \omega < \omega_2$) is called a *bandpass filter*. This range of frequency is known as the circuit *bandwidth*.

$$\beta = \omega_2 - \omega_1$$

The half-power frequencies are obtained from

$$|V_o(\omega_c)| = \frac{I_i}{\left[\left(\frac{1}{R}\right)^2 + \left(\omega C - \frac{1}{\omega L}\right)^2\right]^{\frac{1}{2}}} = \frac{1}{\sqrt{2}} RI_i$$

Solving for ω we obtain

$$\omega_1 = -\frac{1}{2RC} + \sqrt{\left(\frac{1}{2RC}\right)^2 + \frac{1}{LC}}, \text{ and } \omega_2 = \frac{1}{2RC} + \sqrt{\left(\frac{1}{2RC}\right)^2 + \frac{1}{LC}}$$

From the above, we obtain, $\omega_2 - \omega_1 = \dfrac{1}{RC}$, or the circuit bandwidth is

$$\beta = \frac{1}{RC}$$

The cutoff frequencies can be written in terms of ω_0 and β as follow:

$$\omega_1 = -\frac{\beta}{2} + \sqrt{\left(\frac{\beta}{2}\right)^2 + \omega_o{}^2}\text{ , and }\omega_2 = \frac{\beta}{2} + \sqrt{\left(\frac{\beta}{2}\right)^2 + \omega_o{}^2}$$

This shows that ω_o is the geometric mean of ω_1 and ω_2, i.e.,

$$\omega_o = \sqrt{\omega_1 \omega_2}$$

Notice that β is inversely proportional to *R*, i.e., smaller *R* results in a larger bandwidth. The resonant frequency ($\omega_o = \dfrac{1}{\sqrt{LC}}$) is a function of *L* and *C*. Therefore, by adjusting *L* and *C* a desired resonant frequency is obtained, whereas by adjusting *R*, the bandwidth and the height of the response curve is adjusted. The sharpness of the resonance is measured quantitatively by the *quality factor Q*. This is defined as the ratio of the resonant frequency to the bandwidth.

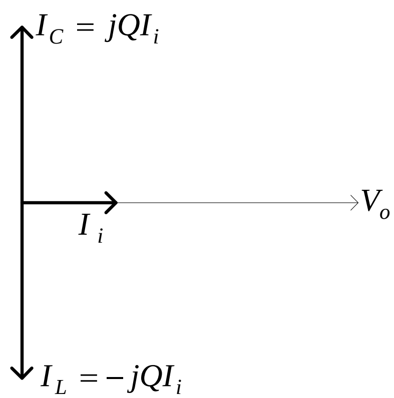
$$Q = \frac{\omega_o}{\beta}$$

Substituting for $\beta = \dfrac{1}{RC}$ and $\omega_o = \dfrac{1}{\sqrt{LC}}$ the quality factor can be expressed as

$$Q = \omega_o RC = \frac{R}{\omega_o L} = R\sqrt{\frac{C}{L}}$$

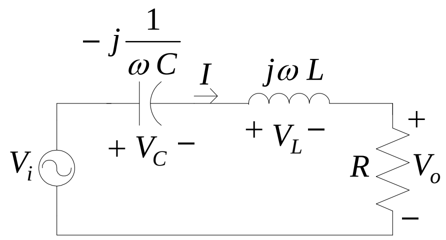
At resonance I_L and I_C are given by

$$I_L = \frac{V_o}{j\omega_o L} = -j\frac{V_o}{R/Q} = -jQI_i \quad \text{and} \quad I_C = j\omega_o C V_o = j\frac{Q}{R}V_o = jQI_i$$



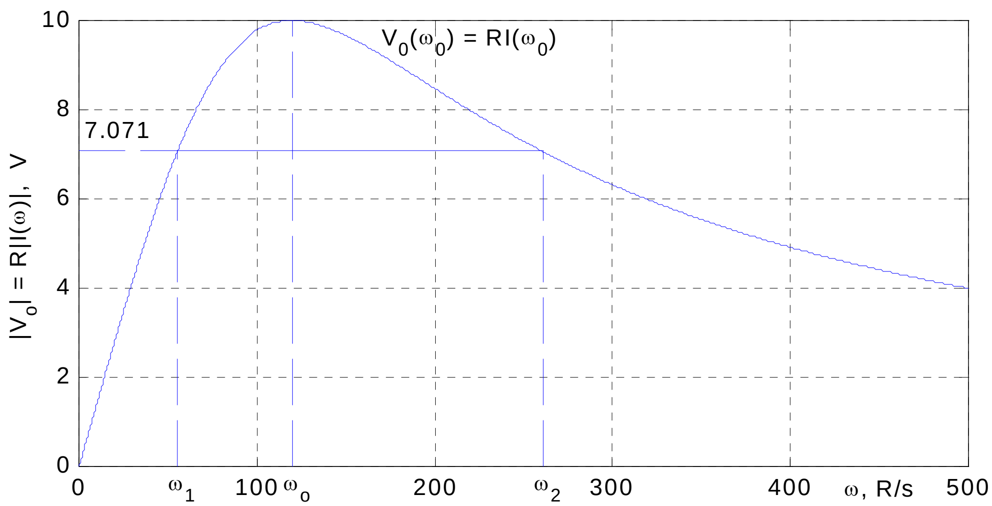
As it can be seen at resonance depending on the *Q* factor, I_L and I_C can be many times the supply current (current amplification).

(b) The Series RLC Resonance



$$I = \frac{V_i}{R + j(\omega L - \frac{1}{\omega C})}$$

A circuit is in resonance when the voltage and current at the input terminals are in phase. The circuit impedance is $Z = R + j(\omega L - \frac{1}{\omega C})$. At resonance Z is purely resistive and $\omega L - \frac{1}{\omega C} = 0$, thus $\omega_o = \frac{1}{\sqrt{LC}}$. The circuit impedance at resonance, given by $Z(\omega_o) = R$ is minimum, and the current is maximum. Thus, the output voltage at resonance is maximum and is given by $V_o(\omega_o) = RI(\omega_o)$



The frequencies ω_1 and ω_2 at which the output power drops to one half of its values at the resonant frequency are called the *half-power frequencies*. At these frequencies also known as *cutoff frequencies* or *corner frequencies*, the output voltage is $|V_o(\omega_c)| = 0.707|V_o(\omega_o)|$. This circuit which passes all the frequencies within a band of frequencies ($\omega_1 < \omega < \omega_2$) is called a *bandpass filter.* This range of frequency is known as the circuit *bandwidth*.

$$\beta = \omega_2 - \omega_1$$

The half-power frequencies are obtained from

$$|V_o(\omega_c)| = R\frac{V_i}{\left[(R)^2 + \left(\omega L - \frac{1}{\omega C}\right)^2\right]^{\frac{1}{2}}} = \frac{1}{\sqrt{2}}V_i$$

Solving for ω we obtain

$$\omega_1 = -\frac{R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \frac{1}{LC}}, \text{ and } \omega_2 = \frac{R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \frac{1}{LC}}$$

From the above, we obtain, $\omega_2 - \omega_1 = \frac{R}{L}$, or the circuit bandwidth is

$$\beta = \frac{R}{L}$$

The cutoff frequencies can be written in terms of ω_0 and β as follow:

$$\omega_1 = -\frac{\beta}{2} + \sqrt{\left(\frac{\beta}{2}\right)^2 + \omega_o{}^2}, \text{ and } \omega_2 = \frac{\beta}{2} + \sqrt{\left(\frac{\beta}{2}\right)^2 + \omega_o{}^2}$$

This shows that ω_o is the geometric mean of ω_1 and ω_2, i.e.,

$$\omega_o = \sqrt{\omega_1 \omega_2}$$

Notice that β is proportional to *R*, i.e., larger *R* results in a larger bandwidth. The resonant frequency ($\omega_o = \frac{1}{\sqrt{LC}}$) is a function of *L* and *C*. Therefore, by adjusting *L* and *C* a desired resonant frequency is obtained, whereas by adjusting *R*, the bandwidth and the height of the response curve is adjusted. The sharpness of the resonance is measured quantitatively by the *quality factor Q*. This is defined as the ratio of the resonant frequency to the bandwidth.

$$Q = \frac{\omega_o}{\beta}$$

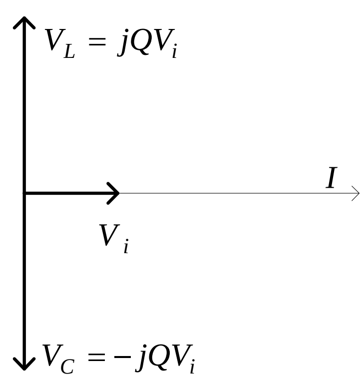
Substituting for $\beta = \frac{R}{L}$ and $\omega_o = \frac{1}{\sqrt{LC}}$ the quality factor can be expressed as

$$Q = \frac{\omega_o L}{R} = \frac{1}{\omega_o RC} = \frac{1}{R}\sqrt{\frac{L}{C}}$$

At resonance V_L and V_C are given by

$$V_L = j\omega_o L I(\omega_0) = jQV_i$$

$$V_C = -j\frac{1}{\omega_o C} I(\omega_0) = -jQV_i$$

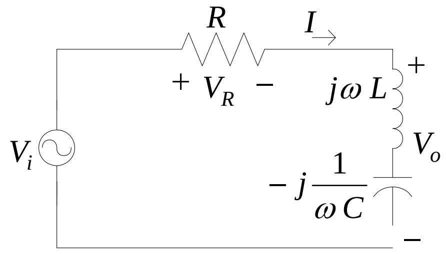


As it can be seen at resonance depending on the *Q* factor, V_L and V_C can be many times the supply voltage (voltage amplification).

For a circuit with a very high quality factor *Q*, the corner frequencies may be approximated to $\omega_1 = \omega_o - \frac{\beta}{2}$, and $\omega_1 = \omega_o + \frac{\beta}{2}$

Bandreject Filter

A bandreject filter is designed to stop all frequencies within a band of frequencies ($\omega_1 < \omega < \omega_2$). In the series RLC circuit consider the output across the series combination L and C.

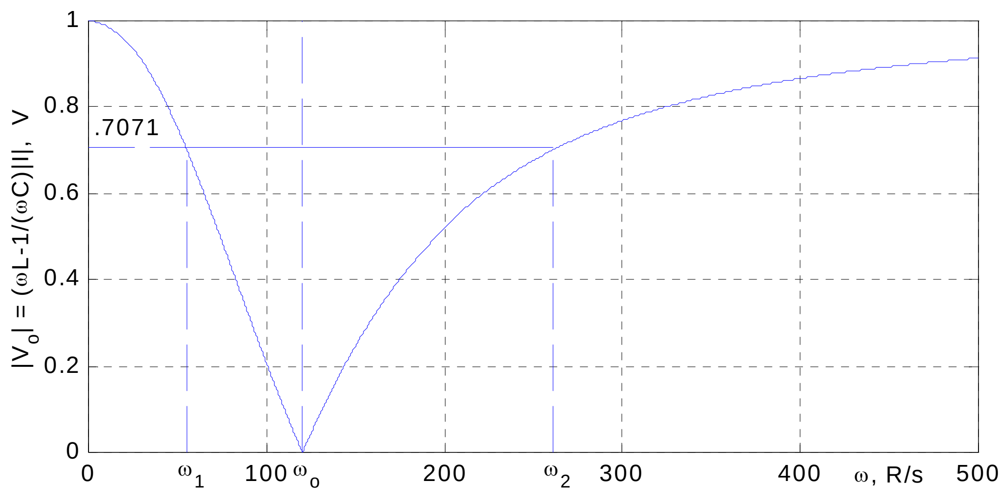


$$\frac{V_o}{V_i} = \frac{j(\omega L - \frac{1}{\omega C})}{R + j(\omega L - \frac{1}{\omega C})}$$

The voltage gain magnitude is

$$\frac{|V_o|}{|V_i|} = \frac{\omega L - \frac{1}{\omega C}}{\left[R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2\right]^{1/2}}$$

At $\omega = 0$, the inductor behaves like a short circuit, and the capacitor behaves like an open circuit, $I = 0$ and $V_o = V_i$, and the voltage gain is unity. At $\omega = \infty$, the inductor behaves like an open circuit and the capacitor behaves like a short circuit, $I = 0$ and again $V_o = V_i$, and the voltage gain is unity. At resonance Z is purely resistive and $\omega L - \frac{1}{\omega C} = 0$, thus $\omega_o = \frac{1}{\sqrt{LC}}$. Since the numerator of the voltage gain is zero, the gain drops to zero at ω_o.



The cutoff frequencies, the bandwidth, and the quality factor are the same as the series RLC bandpass filter, $\beta = \frac{R}{L}$, $\omega_1 = -\frac{\beta}{2} + \sqrt{\left(\frac{\beta}{2}\right)^2 + \omega_o^2}$, and $\omega_2 = \frac{\beta}{2} + \sqrt{\left(\frac{\beta}{2}\right)^2 + \omega_o^2}$