

Slides Aulas de Eletrônica

Material didático auxiliar

Observação importante

Os slides aqui apresentados não refletem todo o conteúdo abordado em sala de aula. Muitos exercícios, demonstrações e detalhamento da teoria, expostos na aula presencial, não estão contemplados nestes slides. Portanto, considere-o apenas como material de referência parcial a ser complementado com o auxílio de livros, apostilas, guias de laboratório e literatura correspondente. Material auxiliar adicional encontra-se disponibilizado no site da disciplina através de textos e links.

ELETRÔNICA

Filtros Elétricos
Ativos e Passivos

AOC

Análise de Circuitos

- **Domínio do tempo**

Equações diferenciais e integrais

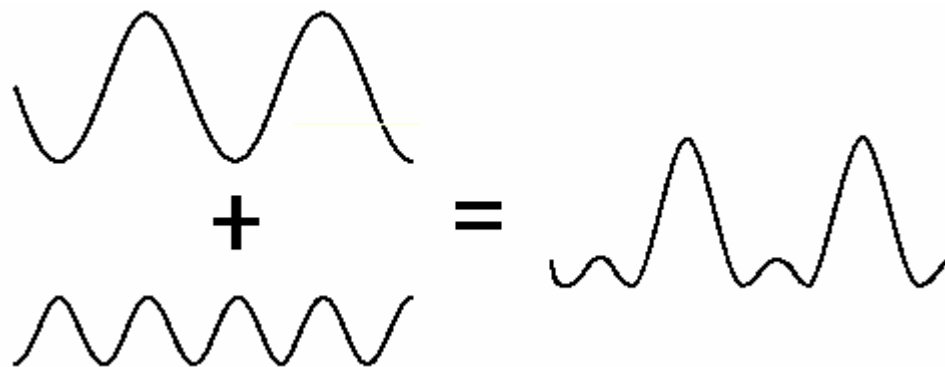
- **Domínio da frequência**

Transformada de Laplace

Transformada de Fourier

Análise Harmônica

- Fourier mostrou
 - Que é possível reduzir uma onda complexa em uma soma de ondas senoidais.
 - As únicas ondas senoidais necessárias são ondas com freqüências que são múltiplos inteiros da freqüência fundamental.



Fourier mostrou que:

“Qualquer função periódica pode ser decomposta como uma soma de senos e cossenos.”

Série de Fourier

Sendo $g(t)$ um sinal periódico (período T_0) podemos escrever:

$$g(t) = a_0 + \sum_{n=1}^{\infty} (a_n \cos n\omega_0 t + b_n \sin n\omega_0 t)$$

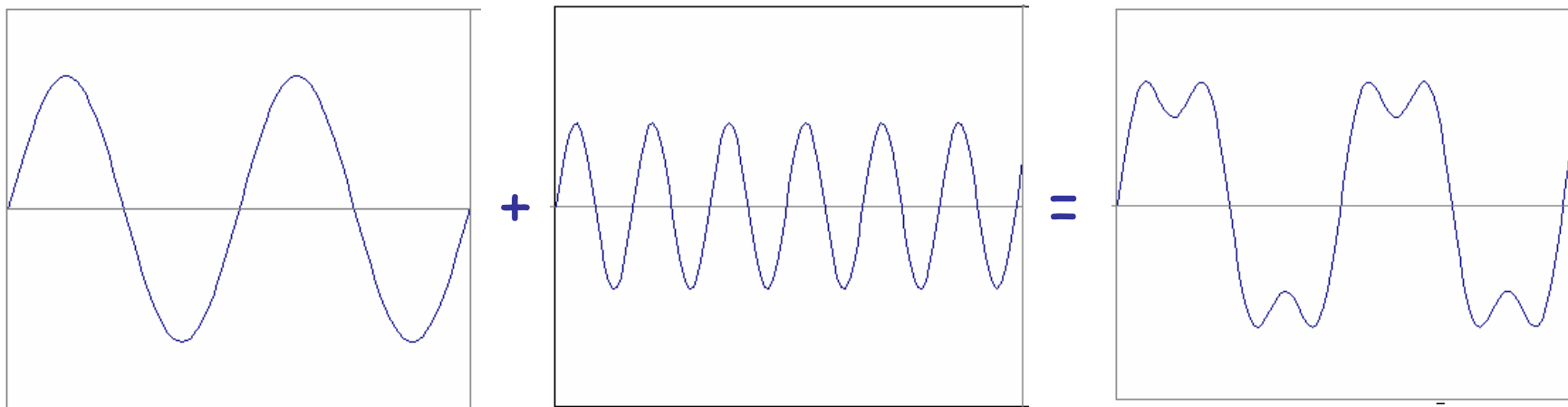
$$a_0 = \frac{1}{T_0} \int_{t_1}^{t_1+T_0} g(t) dt$$

$$a_n = \frac{2}{T_0} \int_{t_1}^{t_1+T_0} g(t) \cos n\omega_0 t dt$$

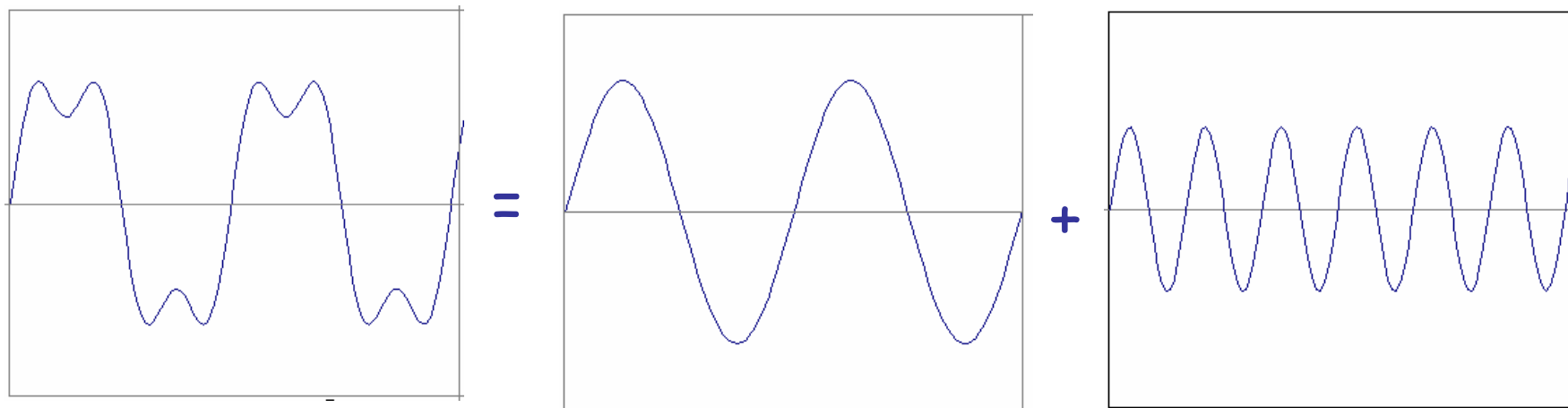
$$b_n = \frac{2}{T_0} \int_{t_1}^{t_1+T_0} g(t) \sin n\omega_0 t dt$$

$$n = 1, 2, 3, \dots$$

Síntese → *Composição*



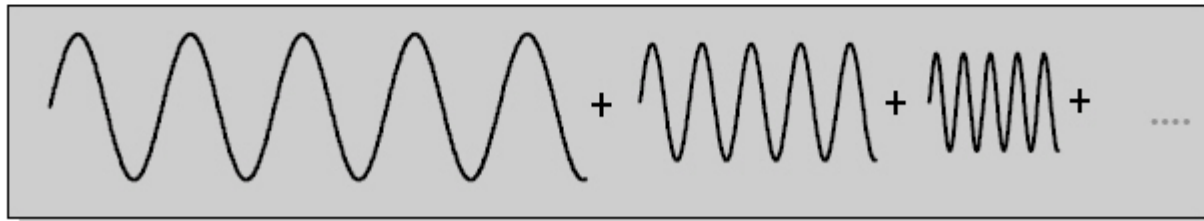
Análise → *Decomposição*



Análise de Sinais

■ Série de Fourier

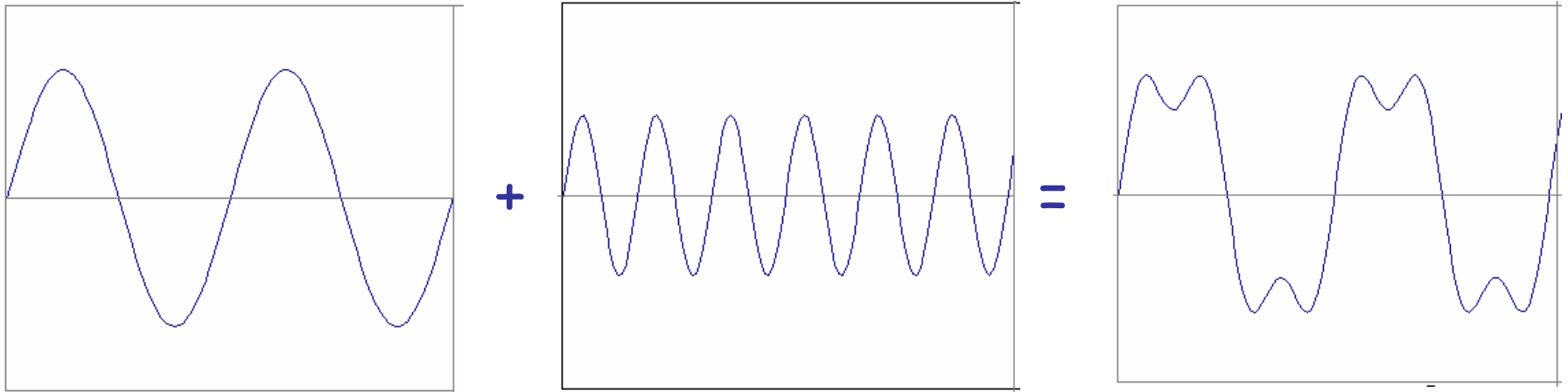
- *Qualquer sinal periódico pode ser entendido como uma soma (possivelmente infinita) de ondas senoidais de diferentes frequências e amplitudes.*



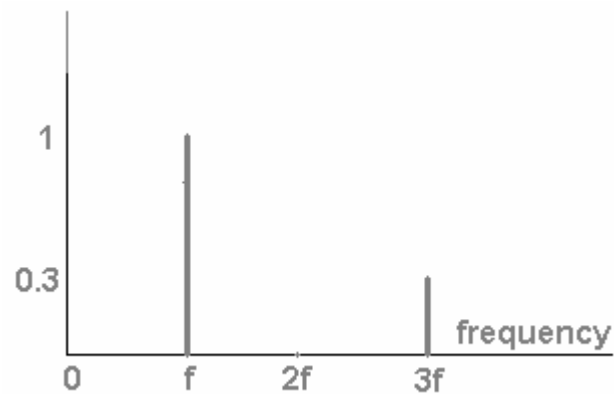
$$f(x) = a_0 + \sum_{n=1}^{n=\infty} \left(a_n \cos\left(\frac{2n\pi x}{T}\right) + b_n \sin\left(\frac{2n\pi x}{T}\right) \right)$$

Exemplo

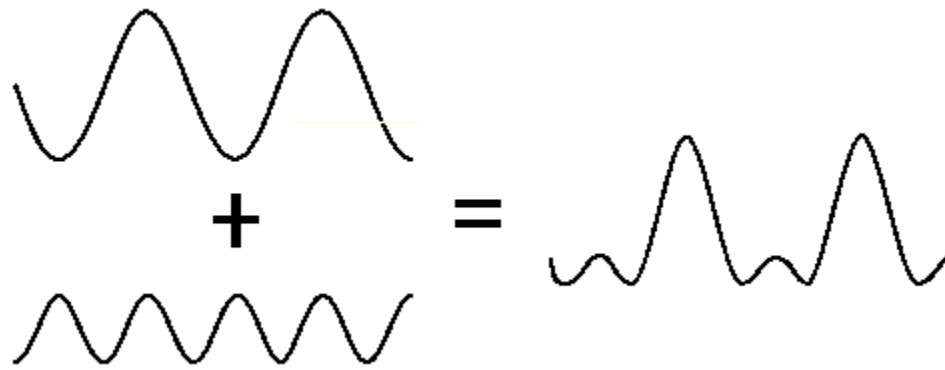
“Formas de ondas periódicas podem ser construídas a partir da soma de senóides.”



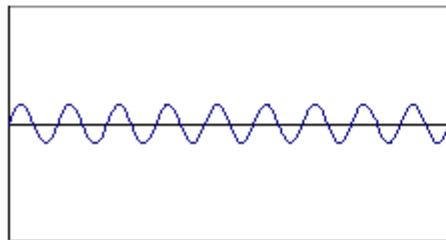
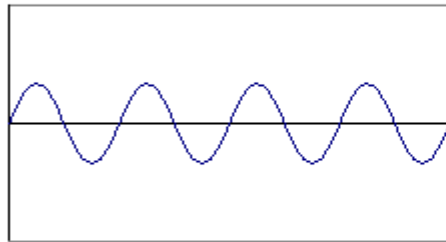
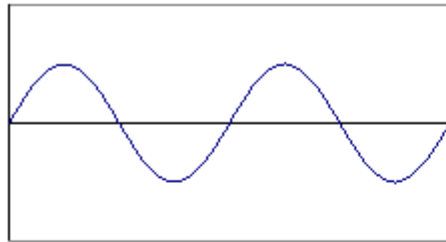
$$g(t) = \sin(2\pi f t) + (1/3)\sin(2\pi 3f t)$$



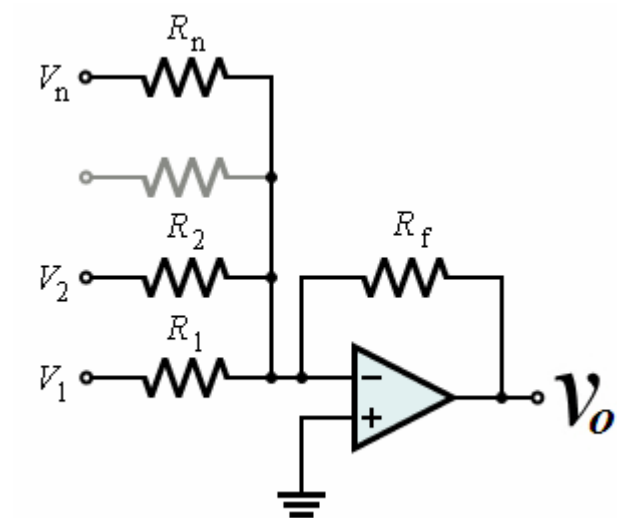
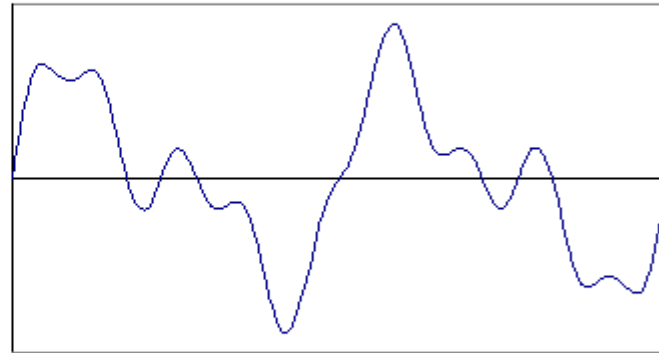
Combinando sinais de frequências distintas

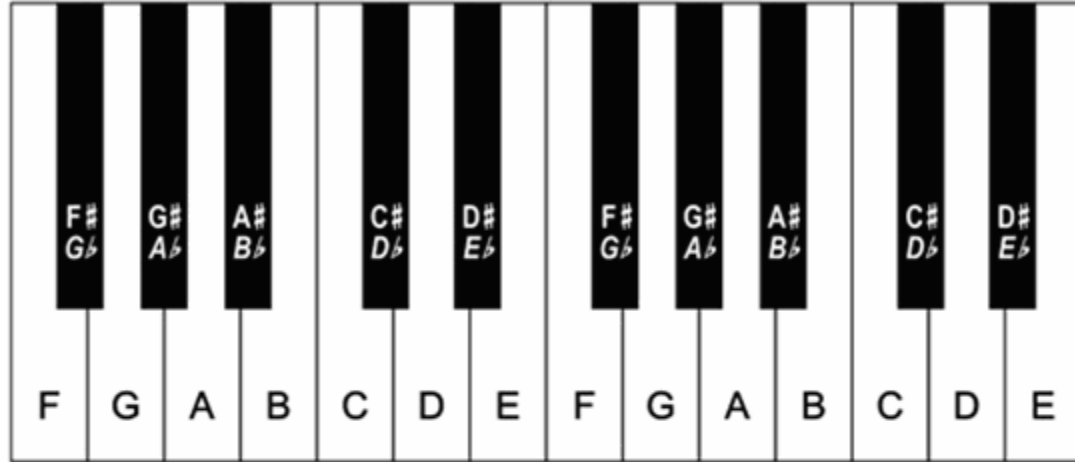


Geração de sinais complexos a partir de senóides



=






440 Hz

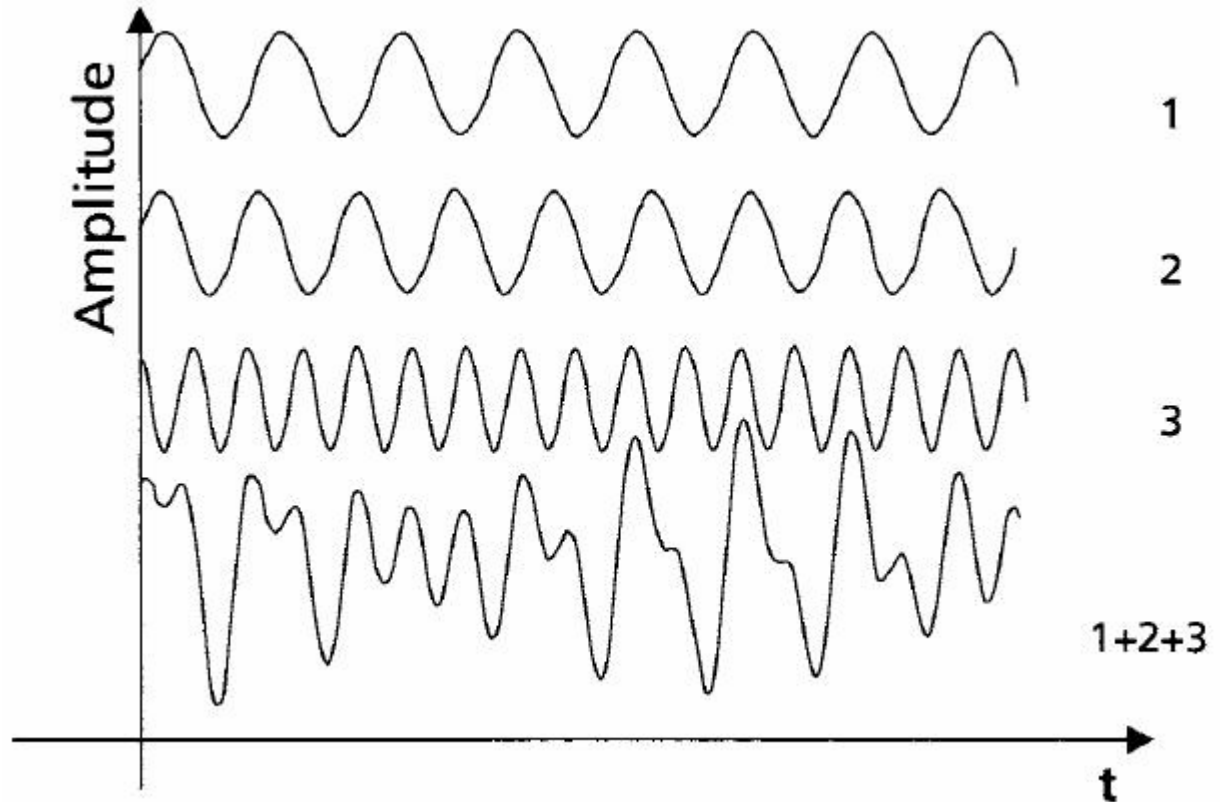






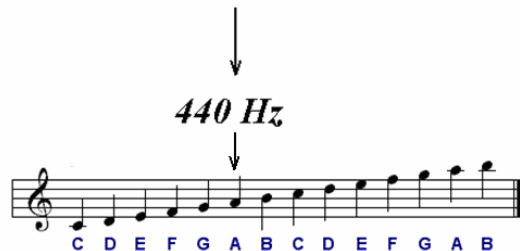
$G = \sin(2\pi \cdot 392.00t)$
 $E = \sin(2\pi \cdot 329.63t)$
 $C = \sin(2\pi \cdot 261.63t)$

Composição de Sinais



Aplicação de osciladores senoidais

Geração de Notas Musicais



Note	Frequency (Hz)
A	220 = 220 ... $2^{0/12}$
A#	233 = 220 ... $2^{1/12}$
B	247 = 220 ... $2^{2/12}$
C	262 = 220 ... $2^{3/12}$
C#	277 = 220 ... $2^{4/12}$
D	294 = 220 ... $2^{5/12}$
D#	311 = 220 ... $2^{6/12}$
E	330 = 220 ... $2^{7/12}$
F	349 = 220 ... $2^{8/12}$
F#	370 = 220 ... $2^{9/12}$
G	392 = 220 ... $2^{10/12}$
G#	415 = 220 ... $2^{11/12}$
A	440 = 220 ... $2^{12/12}$

Diapasão Musical



A = 440 Hz

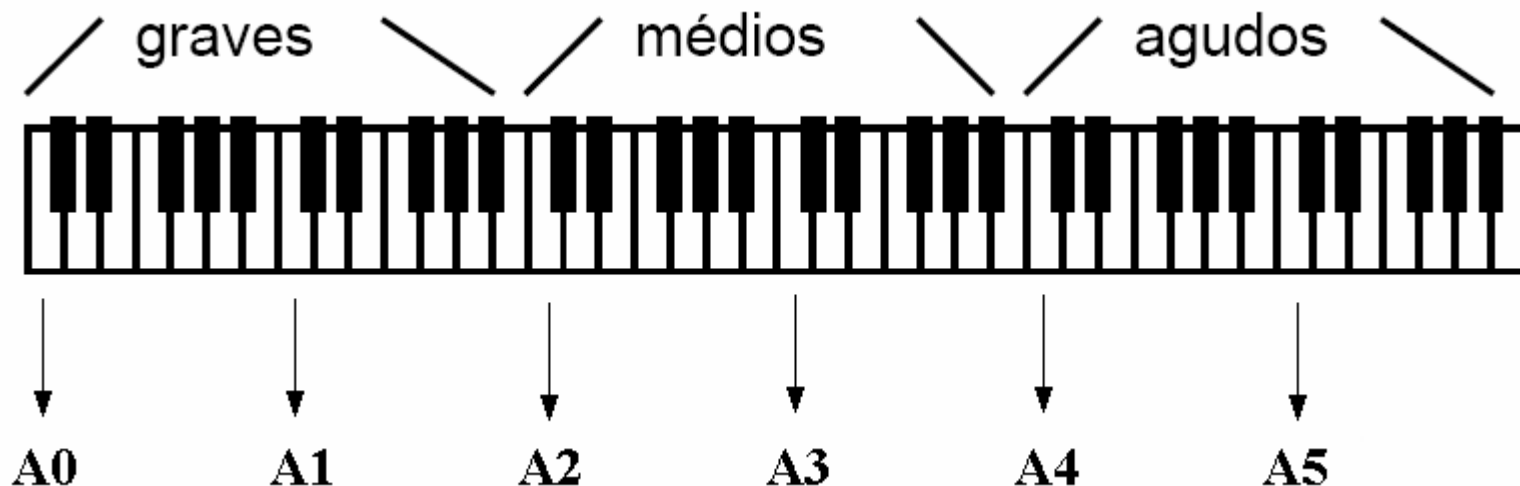
A → Nota musical LÁ

A frequência da nota musical seguinte é determinada pela relação $f = f_o \sqrt[12]{2}$

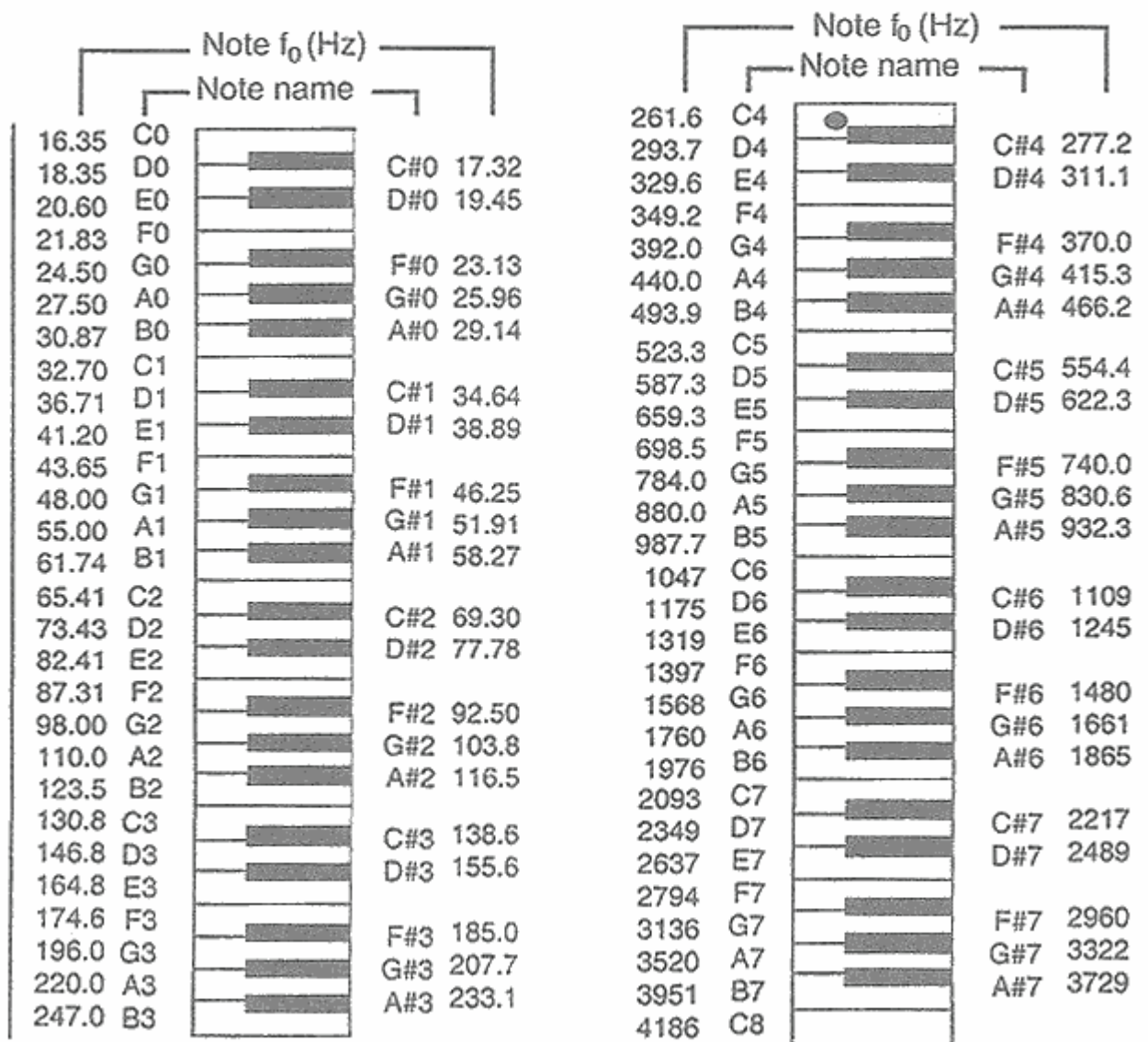
São comumente aceitos como limites da percepção humana em frequência a região do espectro entre 20 e 20.000 Hz.

Um piano produz sons com frequência entre 27,5 Hz (A0) e 4.186 Hz (C8)

A voz humana entre 80 Hz. (baixos) e 1 kHz (sopranos).



Notas musicais e frequências produzidas por um piano

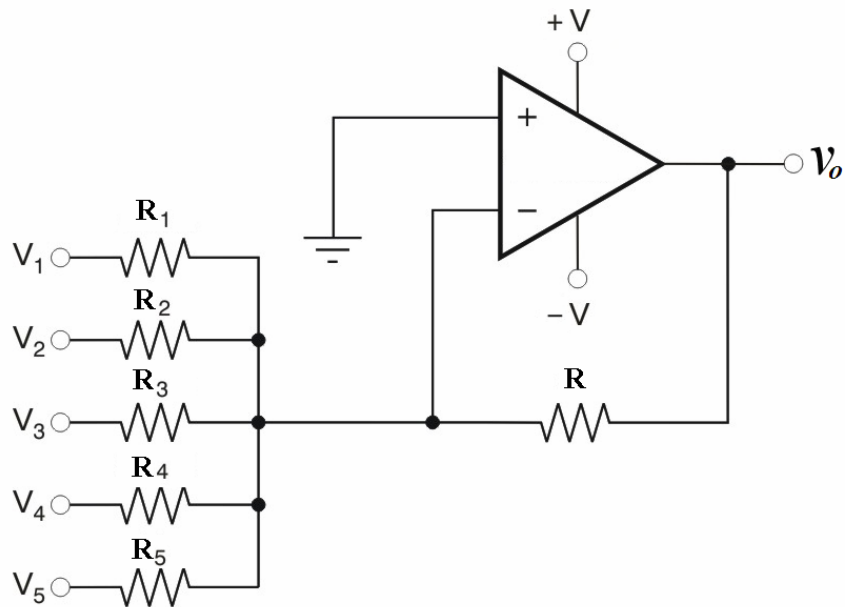


As frequências das notas musicais são determinadas pela relação $f = f_0 \sqrt[12]{2}$

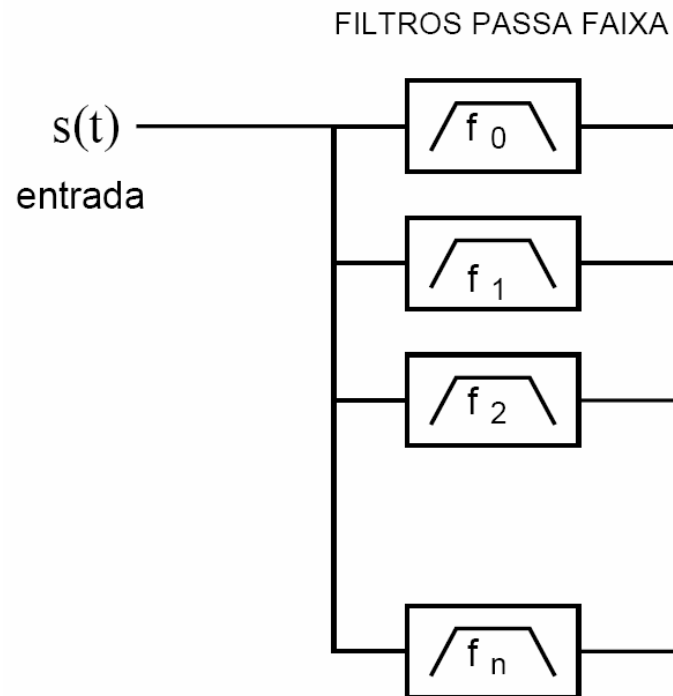


Síntese Espectral
x
Análise Espectral

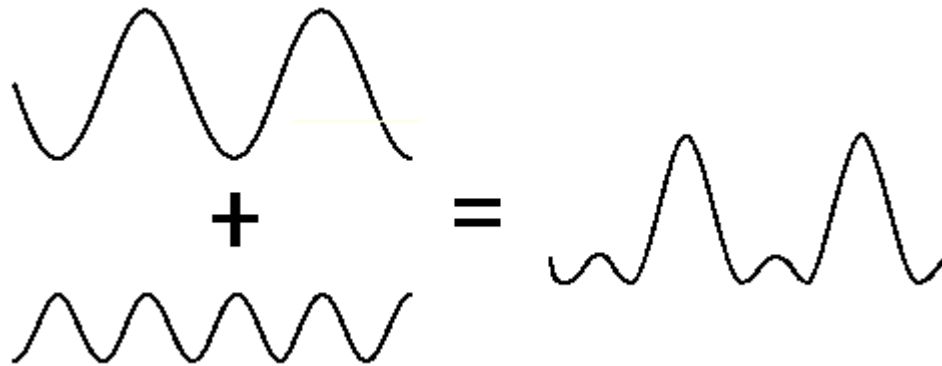
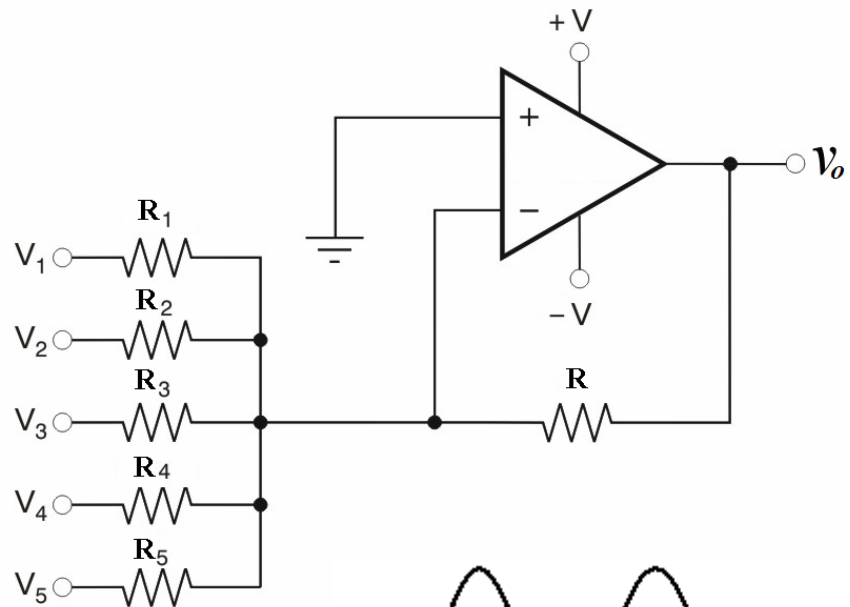
Síntese Espectral



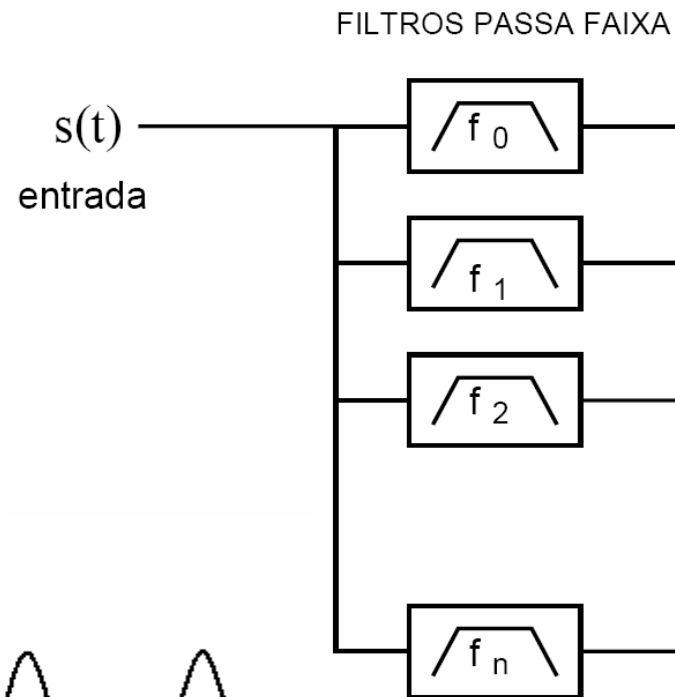
Análise Espectral



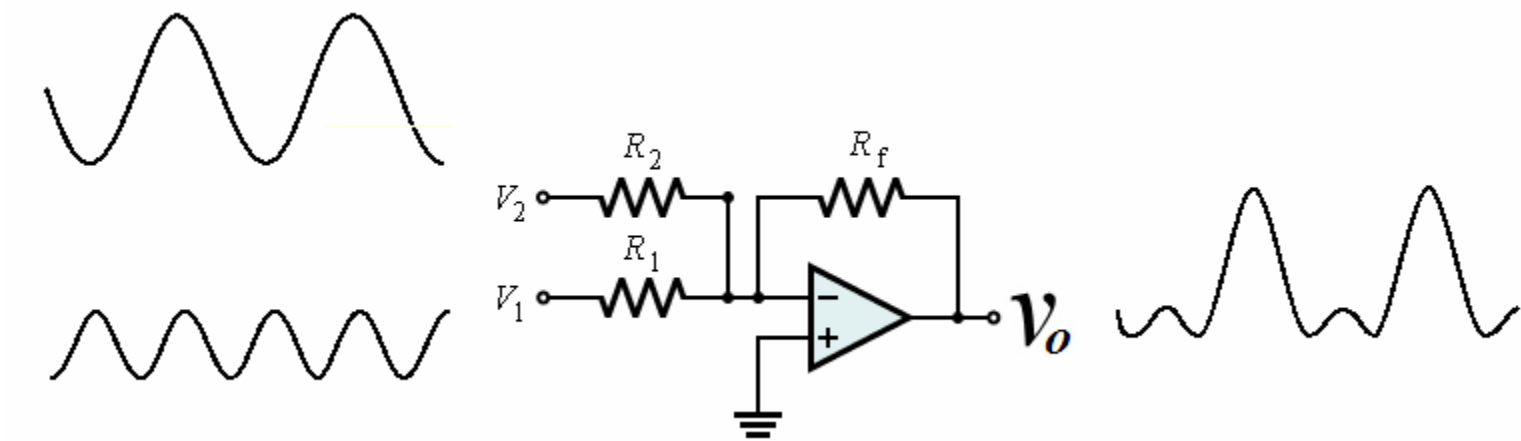
Síntese Espectral



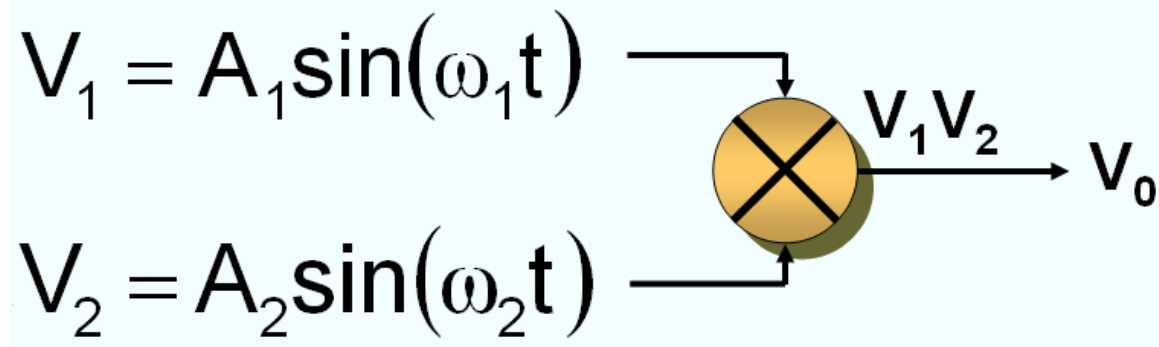
Análise Espectral



CIRCUITO SOMADOR



MULTIPLICAÇÃO DE SINAIS SENOIDAIS



$$\cos(X + Y) = \cos X \cos Y - \sin X \sin Y$$

$$\cos X \cos Y = 1/2 [\cos(X + Y) + \cos(X - Y)]$$

$$-\sin X \sin Y = [\cos(X + Y) - \cos(X - Y)]$$

Multiplicador Analógico

$$v_1 = \cos(\omega_1 t)$$

$$v_2 = \cos(\omega_2 t)$$

$$v_o = v_1 v_2$$

$$v_o = v_1 v_2 = \cos(\omega_1 t) \cos(\omega_2 t)$$

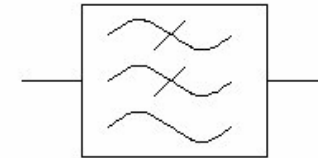
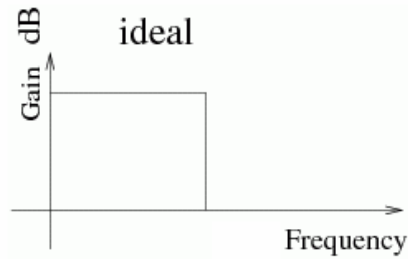
$$v_o = \frac{1}{2} [\cos((\omega_1 - \omega_2)t) + \cos((\omega_1 + \omega_2)t)]$$

Filtros Elétricos

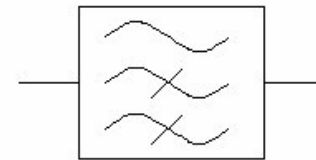
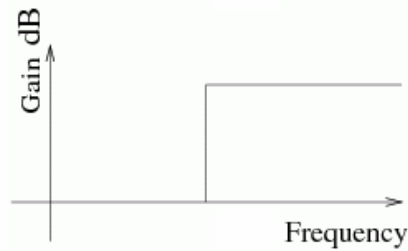
- *Os filtros elétricos constituem uma das aplicações mais comuns da eletrônica, sendo amplamente utilizados na aquisição e processamento de sinais de áudio, vídeo e de dados, em sistemas de alimentação, telecomunicações, controle e instrumentação.*
- *Possui um vasto e diversificado repertório de aplicações em praticamente todos os setores da eletrônica.*

Classificação e Simbologia

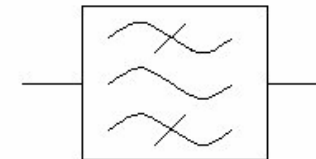
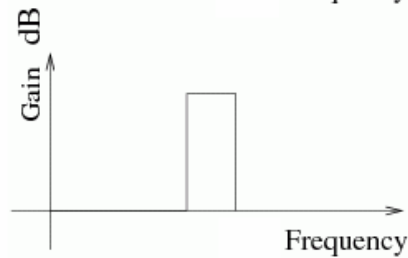
Passa-baixa



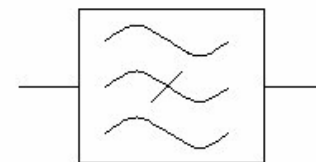
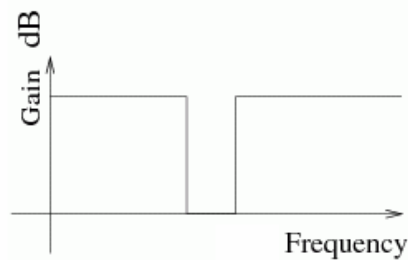
Passa-alta



Passa-faixa



Rejeita-faixa



Filtros Elétricos

Malha seletiva de frequência.

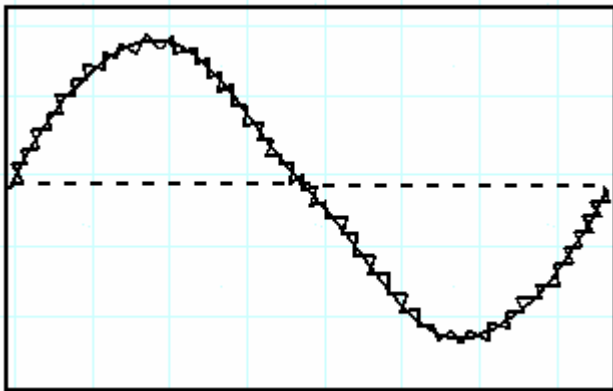
Um filtro atenua a quantidade de energia presente em certas frequências ou faixas de frequências

Deixam passar ou amplificam as frequências desejadas e atenuam as indesejáveis.

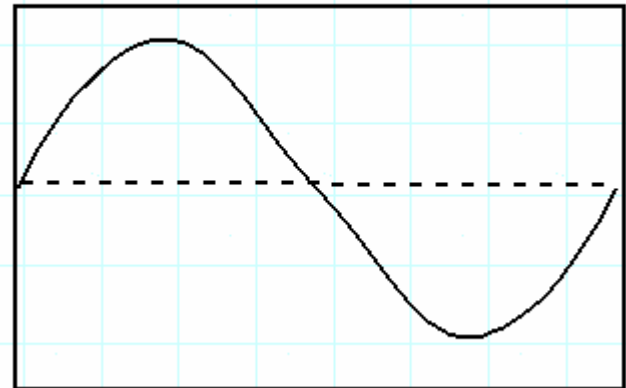
A quantidade de atenuação para cada frequência varia de filtro para filtro.

Funcionalidade de um filtro elétrico

- Um filtro elétrico é um circuito capaz de separar algumas frequências de outras quando misturadas.



filtragem

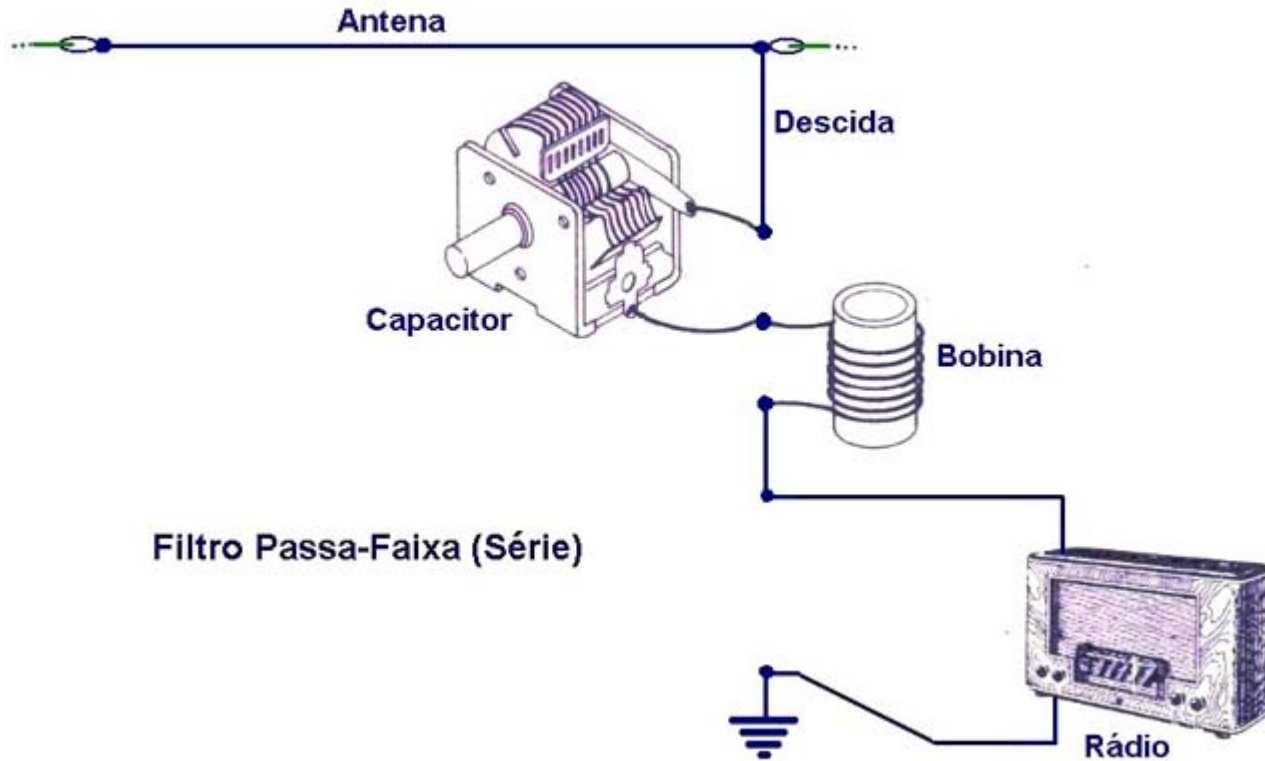


Parâmetros

- Para extrair o conteúdo de informação fundamental de um sinal é necessário um dispositivo que selecione as frequências de interesse que compõe o sinal.
- Este dispositivo é denominado de filtro, cuja resposta em frequência é caracterizada por uma faixa de passagem, por uma faixa de rejeição, as quais estão separadas por uma faixa de transição ou faixa de guarda.
- **Filtro Analógico** → *sinais analógicos*
- **Filtro Digital** → *sinais digitais*

FILTRO PASSA - FAIXA (série)

CONFIGURAÇÃO : EM SÉRIE COM O RÁDIO



Filtros Elétricos

- **Passivos (R, L, C)**
- **Ativos (R, L, C + Dispositivo Ativo)**

Dispositivo Ativo → Produz Ganho

Transistor, Amplificador Operacional

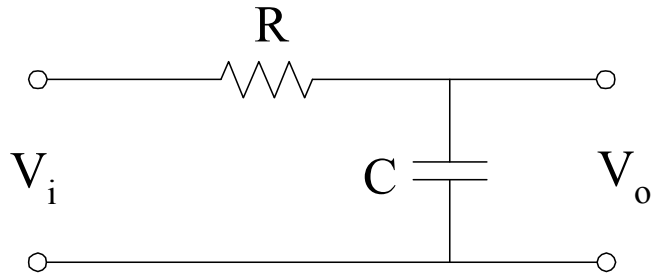
Função de Transferência
Diagramas de Bode



Hendrik Wade Bode

Engenheiro americano (1905-1982)
Pioneiro da moderna teoria de controle

Filtro Passa-Baixa Primeira Ordem



$$V_o = \frac{X_C}{X_C + R} V_i$$

$$V_o = \frac{\frac{1}{j\omega C}}{\frac{1}{j\omega C} + R} V_i = \frac{1}{1 + j\omega CR} V_i$$

$$H(j\omega) = \frac{1}{1 + j\frac{\omega}{\omega_0}} \quad \omega_0 = \frac{1}{RC}$$

$$H(s) = \frac{\omega_0}{s + \omega_0}$$

$$s = j\omega$$

$$\phi(\omega) = -\tan^{-1}\left(\frac{\omega}{\omega_0}\right)$$

$$H(j\omega) = \frac{1}{1 + j\omega CR}$$

$$V_o = \frac{1}{1 + j\omega CR} V_i$$

$$\omega = \frac{1}{RC}$$

$$V_o = \frac{1}{1 + j} V_i$$

$$|V_o| = \frac{1}{\sqrt{1^2 + 1^2}} |V_i| = \frac{1}{\sqrt{2}} |V_i|$$

$$\omega_c = \omega_o = \frac{1}{RC}$$

$\omega \rightarrow 0 \Rightarrow |V_o| = |V_i| \leftarrow \text{max. value}$
 $\omega \rightarrow \infty \Rightarrow |V_o| = 0 \leftarrow \text{min. value}$

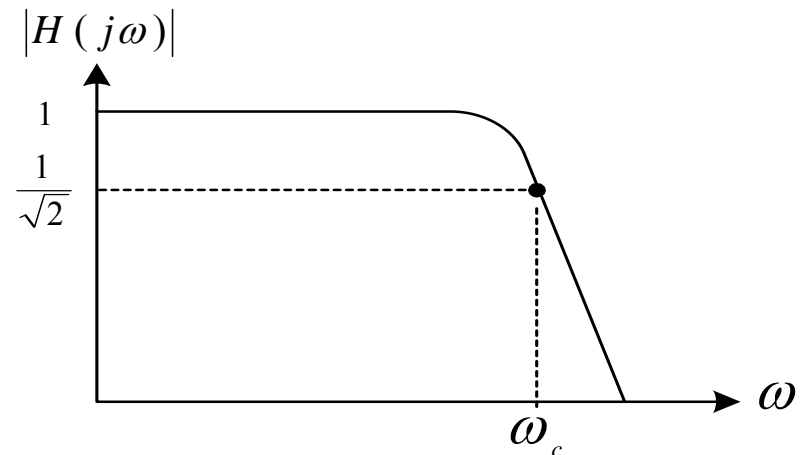
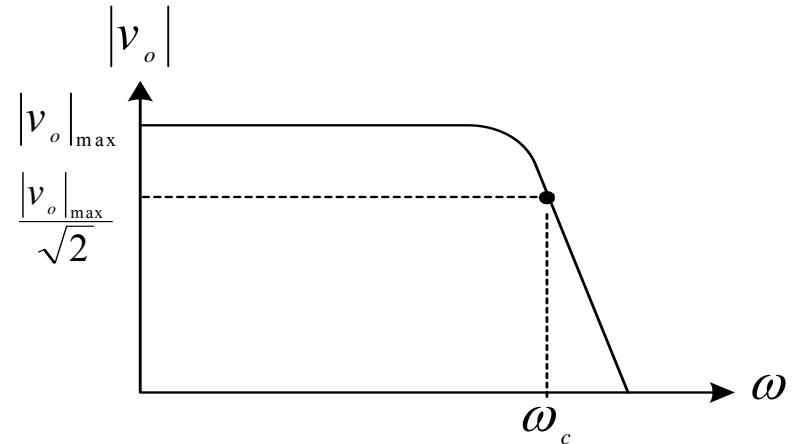


Diagrama de Bode

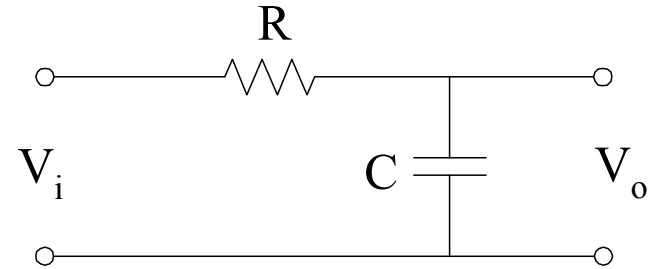
$$H(j\omega) = \frac{1}{1 + j\omega CR} = \frac{1}{1 + j\left(\frac{\omega}{\omega_o}\right)}$$

$$\Rightarrow |H(j\omega)| = \frac{1}{\sqrt{1 + \left(\frac{\omega}{\omega_o}\right)^2}}$$

$$|H(j\omega)|_{dB} = 20 \log_{10} |H(j\omega)| = 20 \log_{10} \left(\frac{1}{\sqrt{1 + \left(\frac{\omega}{\omega_o}\right)^2}} \right)$$

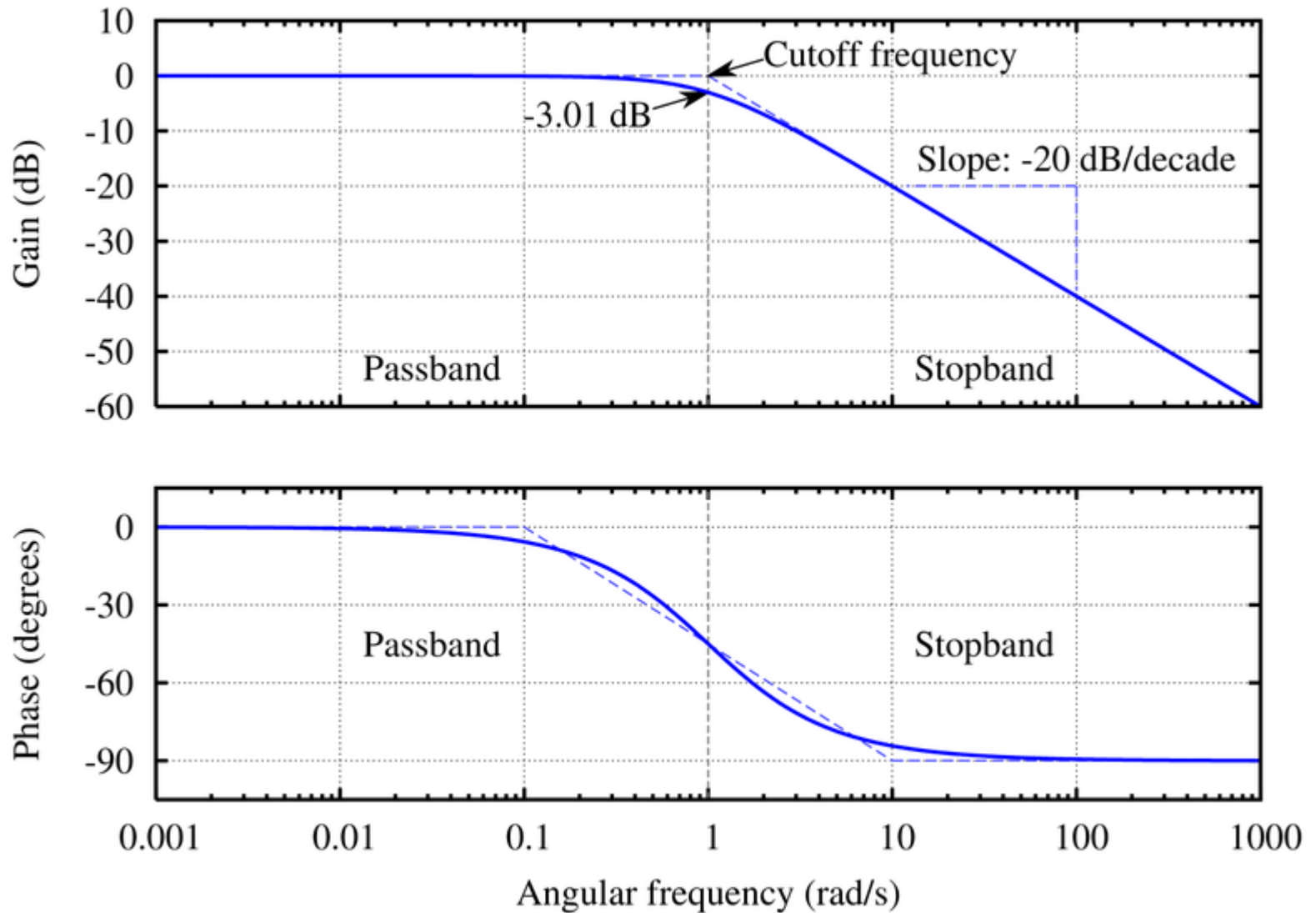
For $\omega \gg \omega_o$

$$|H(j\omega)|_{dB} \approx -20 \log_{10} \left(\frac{\omega}{\omega_o} \right)$$



filtro passa-baixa de primeira ordem

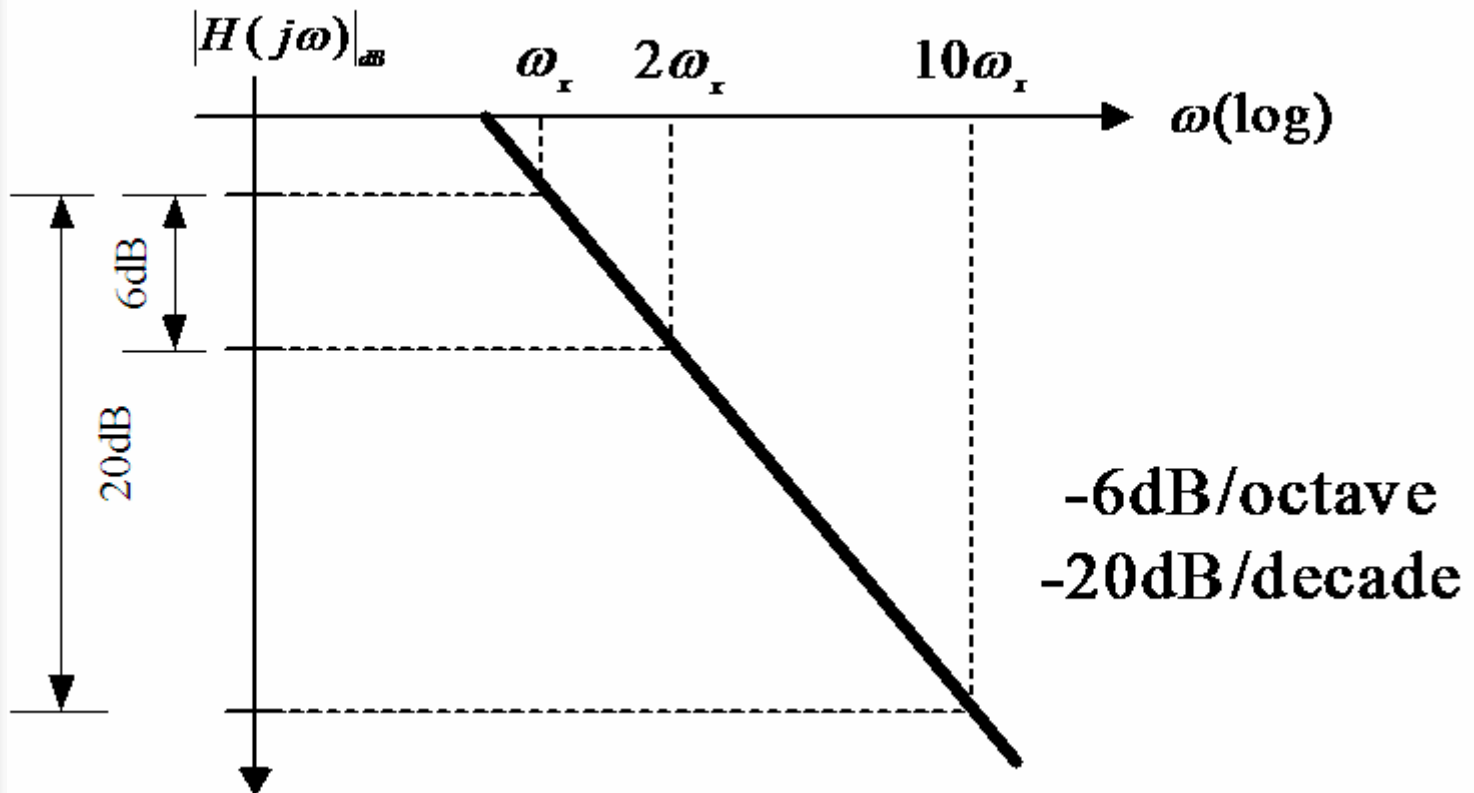
$$H(j\omega) = \frac{1}{1 + j\omega CR} = \frac{1}{1 + j\left(\frac{\omega}{\omega_o}\right)}$$



$$|H(j\omega)| \approx -20 \log_{10} \left(\frac{\omega}{\omega_o} \right)$$

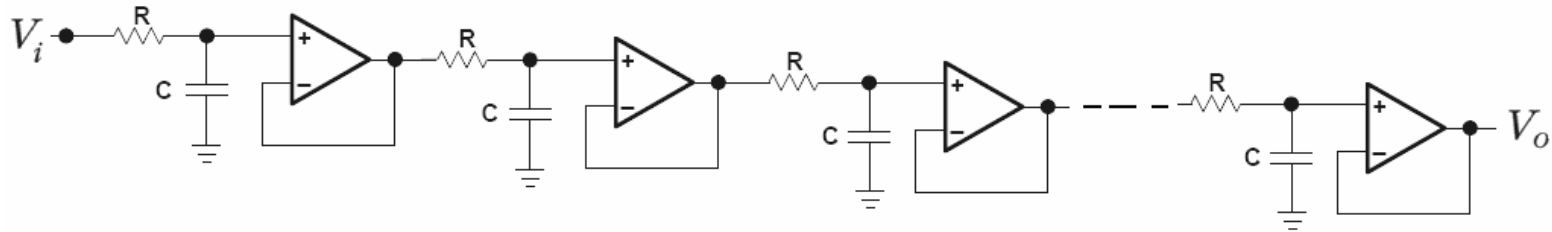
Uma oitava: $\frac{\omega}{\omega_o} = \frac{2}{1}$ $|H(j\omega)| \approx -6dB$

Uma década: $\frac{\omega}{\omega_o} = \frac{10}{1}$ $|H(j\omega)| \approx -20dB$



Resposta em frequência

N filtros passa baixa em série



$$\omega_0 = \frac{1}{RC}$$

Frequência de corte individual

$$\omega_C = \omega_0 \sqrt{2^{\frac{1}{N}} - 1}$$

Frequência de corte total

Frequência de corte equivalente a N filtros passa baixa colocados em série com frequência de corte individual f_0

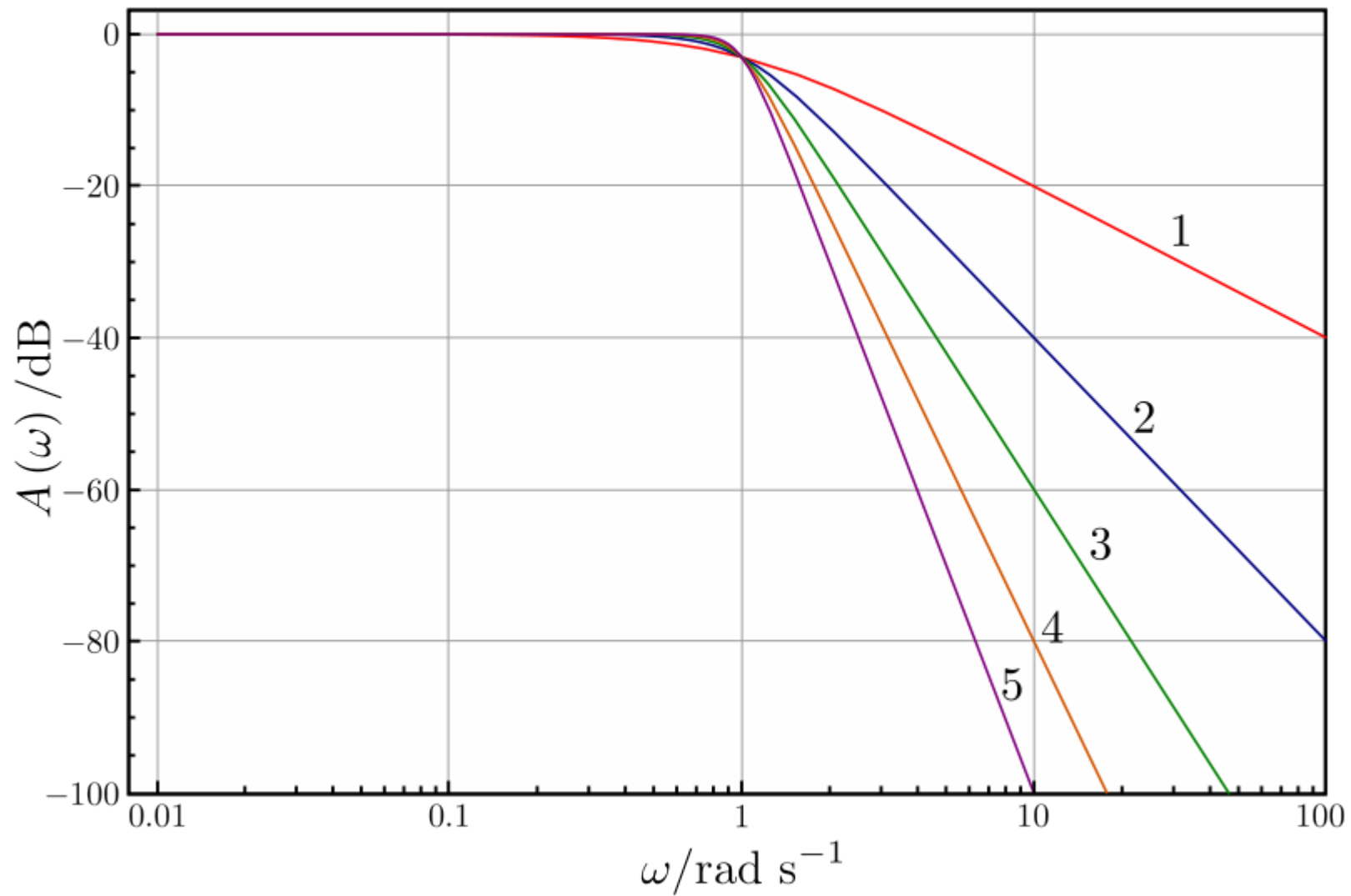
$$\omega_c = \omega_0 \sqrt{2^{\frac{1}{n}} - 1}$$

$$f_c = f_0 \sqrt{2^{\frac{1}{N}} - 1}$$

Frequência de corte equivalente a N filtros passa alta colocados em série com frequência de corte individual f_0

$$f_c = \frac{f_0}{\sqrt{2^{\frac{1}{N}} - 1}}$$

Ordem do Filtro



ELETRÔNICA

Filtros Eletrônicos
Resposta em Frequência

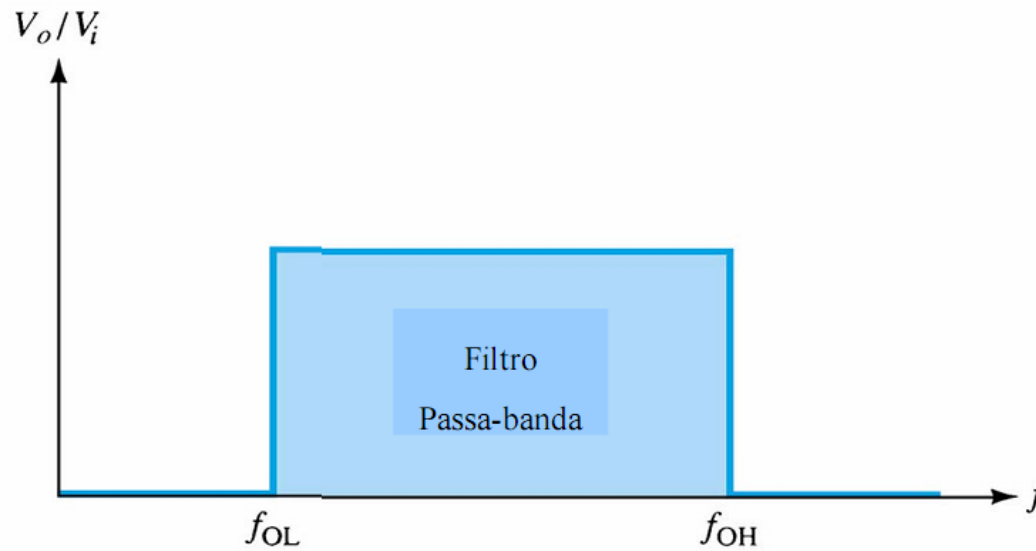
AOC



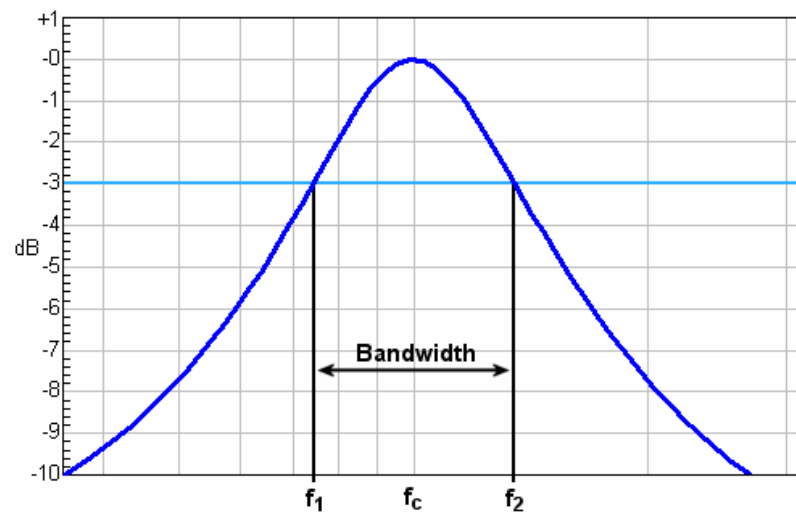
Filtro Passa-Faixa

Ideal x Real

IDEAL

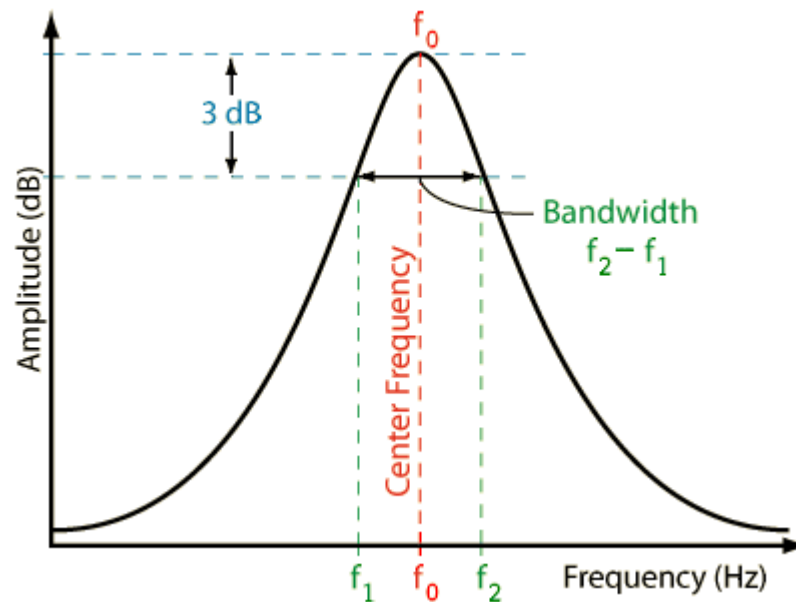


REAL

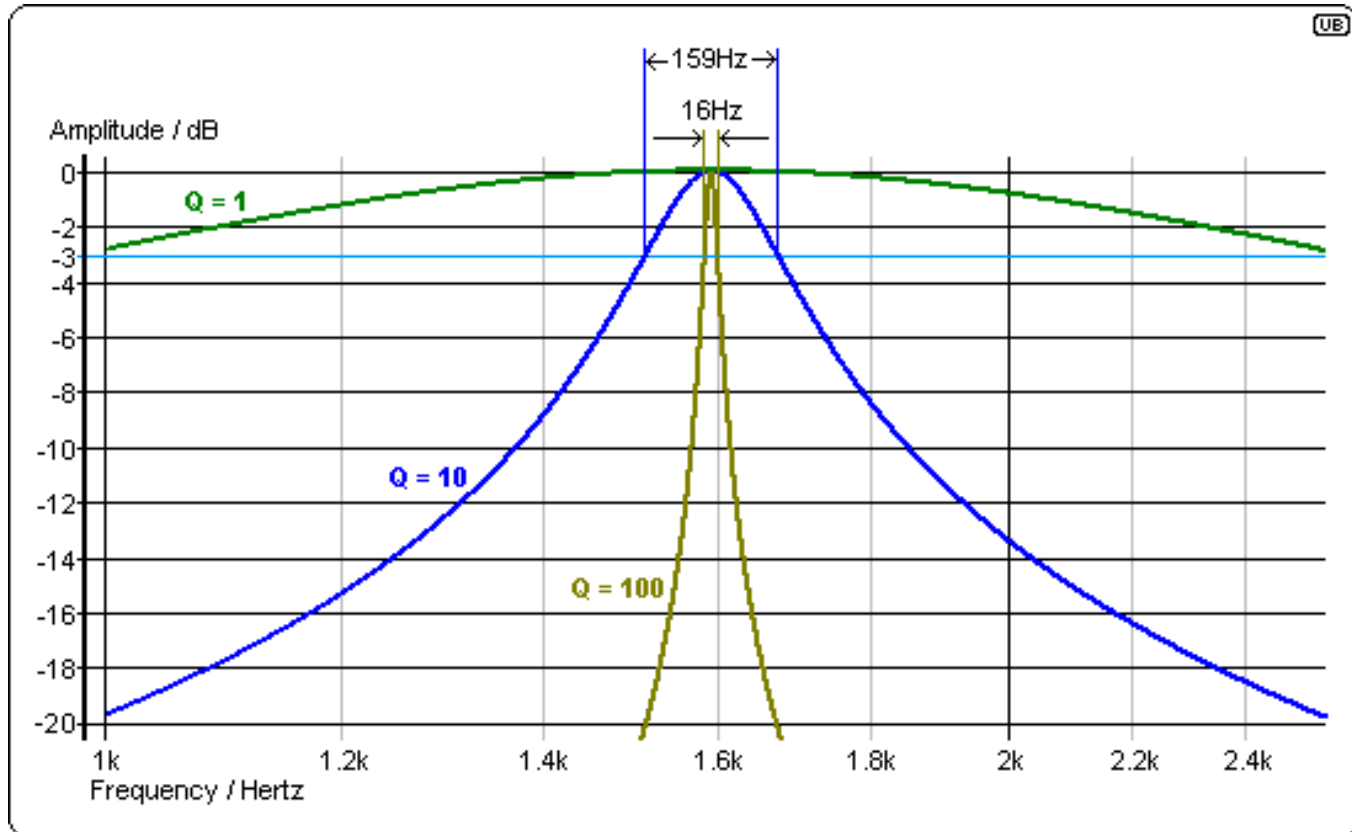


Seletividade

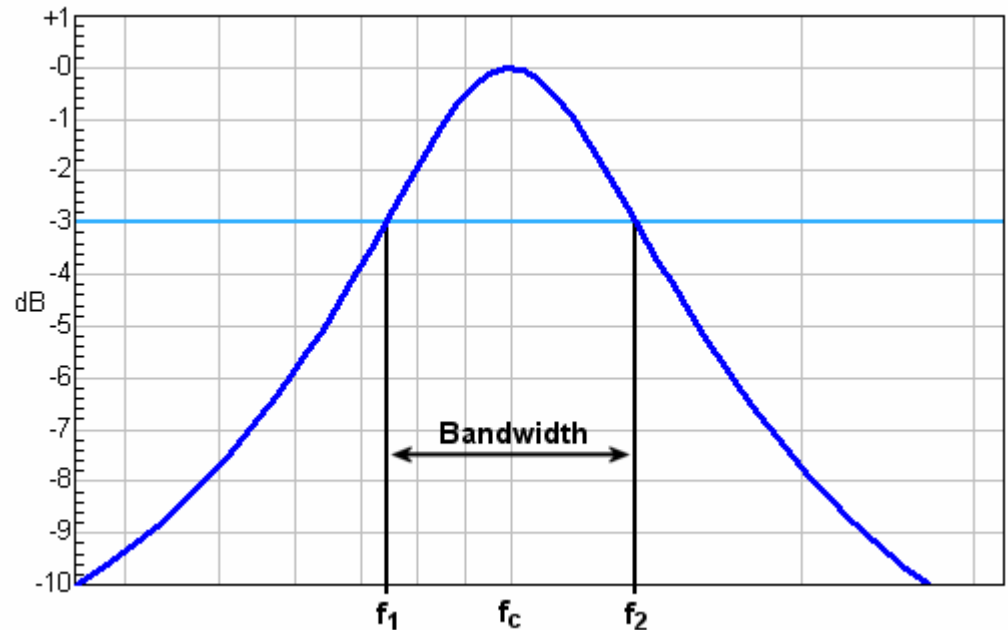
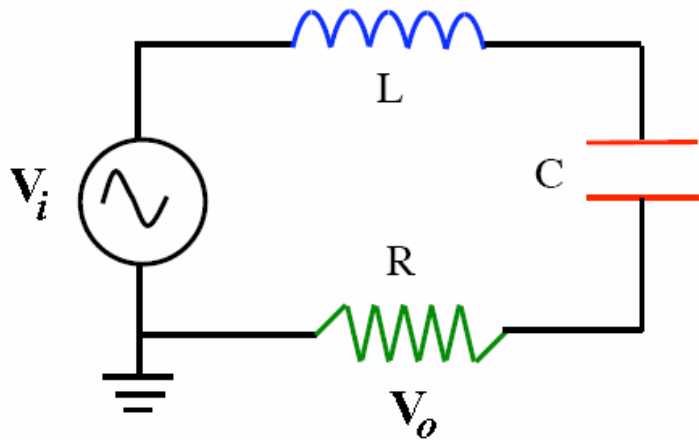
- *Seletividade* é a propriedade que o circuito possui em distinguir, num dado espectro de frequências, uma determinada frequência em relação às demais.



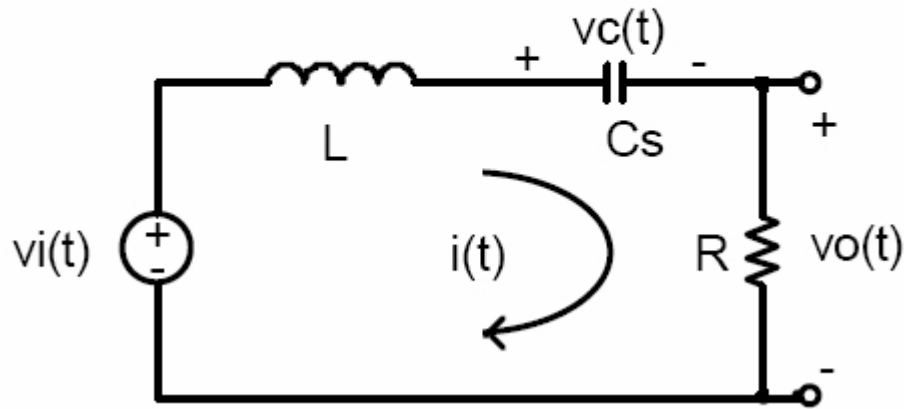
Fator de Qualidade $\rightarrow Q$



- *Um **filtro passa-faixa** é um dispositivo que permite a passagem das frequências de uma certa faixa e rejeita (atenua) as frequências fora dessa faixa.*
- *Um exemplo de um **filtro passa-faixa** analógico é o circuito **RLC**.*



Filtro LC série



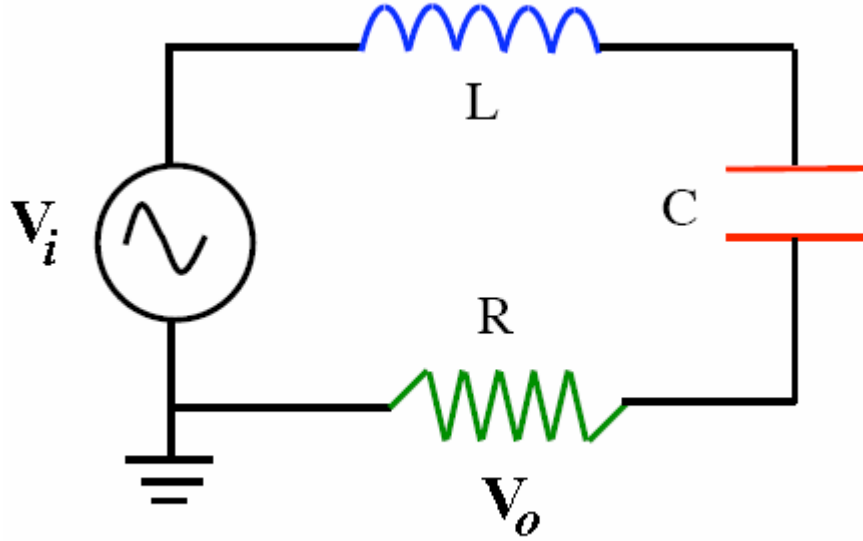
$$G(s) = \frac{V_o(s)}{V_i(s)} = \frac{s}{\frac{L}{R}s^2 + s + \frac{1}{R.C_s}}$$

$$H(s) = \frac{V_o(s)}{V_i(s)} = \frac{s \frac{R}{L}}{s^2 + s \frac{R}{L} + \frac{1}{LC}}$$

$$\frac{V_{OUT}}{V_{IN}} = \frac{Z_R}{Z_L + Z_R + Z_C}$$

$$\frac{V_{OUT}}{V_{IN}} = \frac{R}{j\omega L + R + 1/j\omega C}$$

$$\frac{V_{OUT}}{V_{IN}} = \frac{R}{R + j(\omega L - 1/\omega C)}$$



$$H(s) = \frac{V_o(s)}{V_i(s)} = \frac{s \frac{R}{L}}{s^2 + s \frac{R}{L} + \frac{1}{LC}}$$

$$Q = \frac{\omega_o L}{R} \longrightarrow \frac{\omega_0}{Q} = \frac{R}{L}$$

$$\omega_0 = \sqrt{\frac{1}{LC}} \longrightarrow \omega_0^2 = \frac{1}{LC}$$

$$H(s) = \frac{V_o(s)}{V_i(s)} = \frac{s \frac{\omega_0}{Q}}{s^2 + s \frac{\omega_0}{Q} + \omega_0^2}$$

Parâmetros do filtro passa faixa

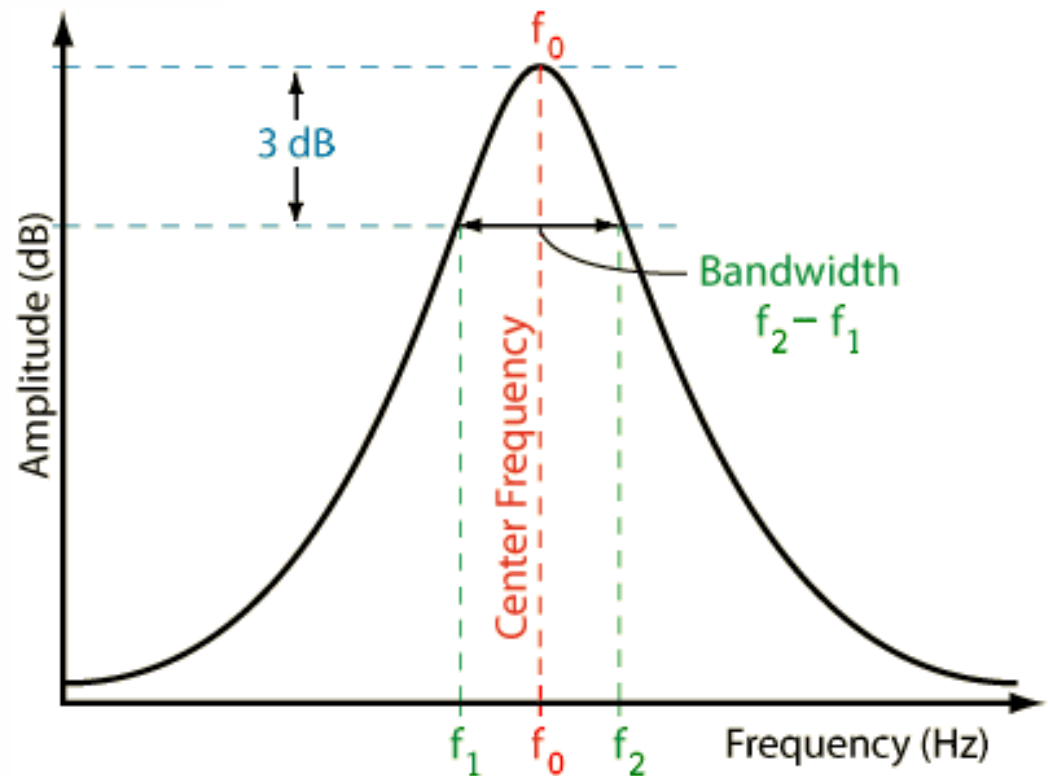
$$\Delta f = f_2 - f_1$$

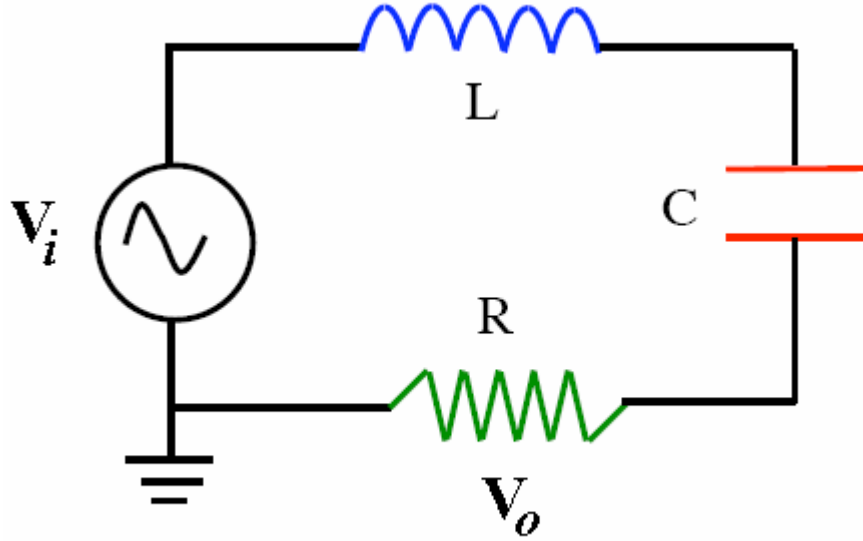
$$f_0 = \sqrt{f_1 \cdot f_2} \quad (\text{média geométrica})$$

$$Q = \frac{f_0}{\Delta f}$$

$$f_2 = \frac{f_0}{2Q} \left(1 + \sqrt{1 + 4Q^2} \right)$$

$$f_1 = \frac{f_0}{2Q} \left(\sqrt{1 + 4Q^2} - 1 \right)$$





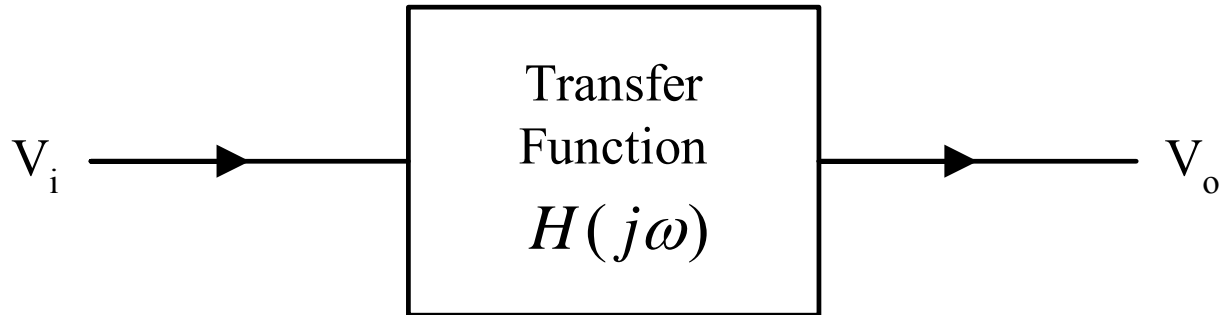
$$H(s) = \frac{V_o(s)}{V_i(s)} = \frac{s \frac{R}{L}}{s^2 + s \frac{R}{L} + \frac{1}{LC}}$$

$$H(s) = \frac{V_o(s)}{V_i(s)} = \frac{s \frac{\omega_0}{Q}}{s^2 + s \frac{\omega_0}{Q} + \omega_0^2}$$

$$Q = \frac{\omega_o L}{R} \longrightarrow \frac{\omega_0}{Q} = \frac{R}{L}$$

$$\omega_0 = \sqrt{\frac{1}{LC}} \longrightarrow \omega_0^2 = \frac{1}{LC}$$

Função de Transferência $H(j\omega)$



$$H(j\omega) = \frac{V_o(j\omega)}{V_i(j\omega)}$$

$$|H| = \sqrt{\text{Re}(H)^2 + \text{Im}(H)^2}$$

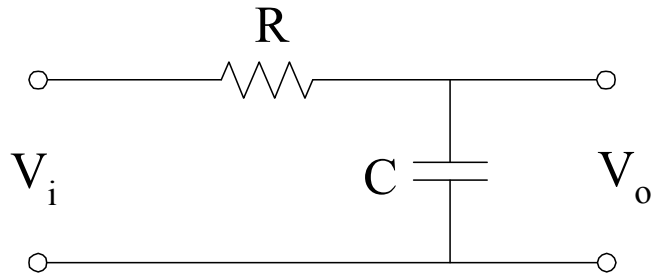
$$H = \text{Re}(H) + j \text{Im}(H)$$

$$\angle H = \tan^{-1} \left(\frac{\text{Im}(H)}{\text{Re}(H)} \right) \quad \text{Re}(H) > 0$$

$$\angle H = 180^\circ + \tan^{-1} \left(\frac{\text{Im}(H)}{\text{Re}(H)} \right) \quad \text{Re}(H) < 0$$

Filtro Passa Baixa

Primeira ordem



$$V_o = \frac{X_C}{X_C + R} V_i$$

$$V_o = \frac{\frac{1}{j\omega C}}{\frac{1}{j\omega C} + R} V_i = \frac{1}{1 + j\omega CR} V_i$$

$$H(j\omega) = \frac{1}{1 + j\frac{\omega}{\omega_0}} \quad \text{where} \quad \omega_0 = \frac{1}{RC}$$

$$H(s) = \frac{\omega_0}{s + \omega_0}$$

$$s = j\omega$$

$$\phi(\omega) = -\tan^{-1}\left(\frac{\omega}{\omega_0}\right)$$

$$H(j\omega) = \frac{1}{1 + j\omega CR}$$

$$V_o = \frac{1}{1 + j\omega CR} V_i$$

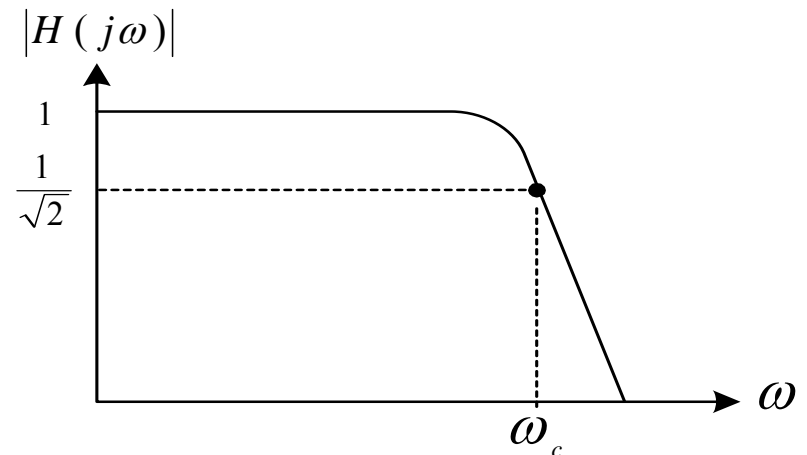
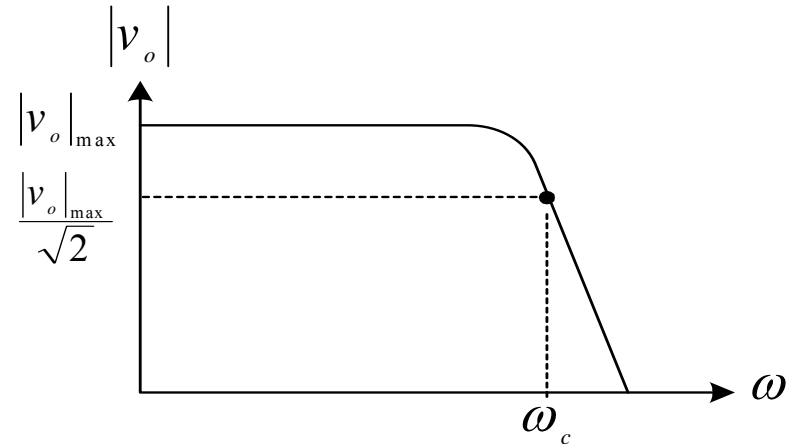
$$\omega = \frac{1}{RC}$$

$$V_o = \frac{1}{1 + j} V_i$$

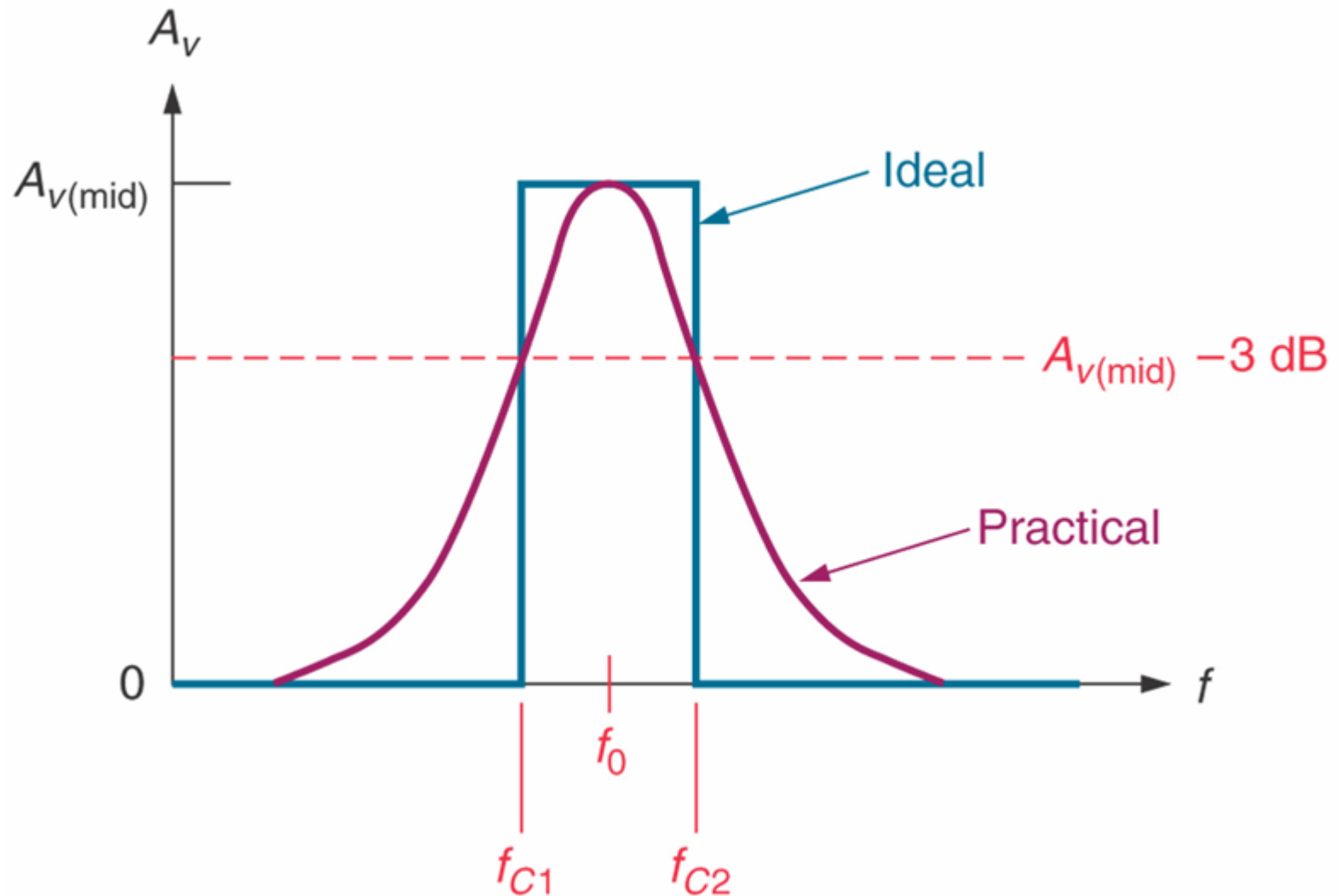
$$|V_o| = \frac{1}{\sqrt{1^2 + 1^2}} |V_i| = \frac{1}{\sqrt{2}} |V_i|$$

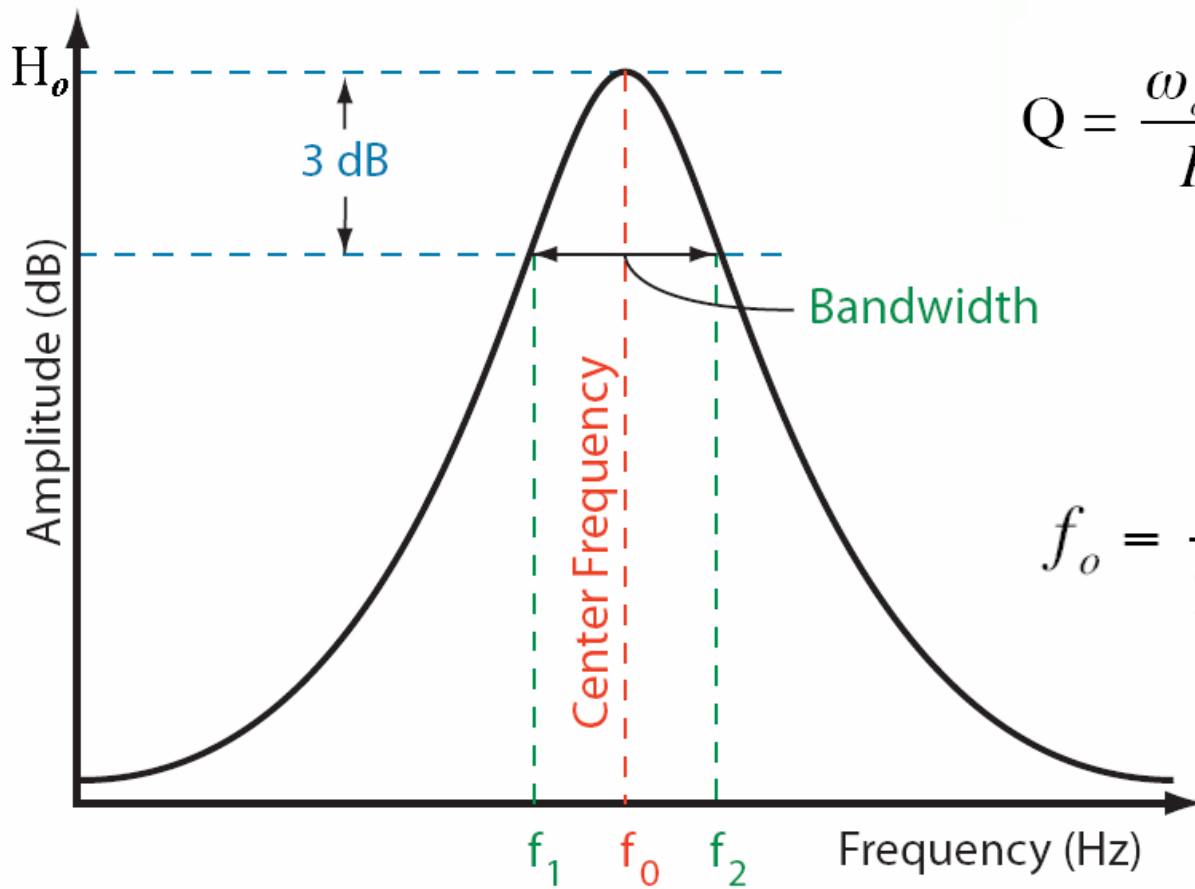
$$\omega_c = \omega_o = \frac{1}{RC} \quad (\text{frecuencia de corte})$$

$\omega \rightarrow 0 \Rightarrow |V_o| = |V_i| \leftarrow \text{max. value}$
 $\omega \rightarrow \infty \Rightarrow |V_o| = 0 \leftarrow \text{min. value}$

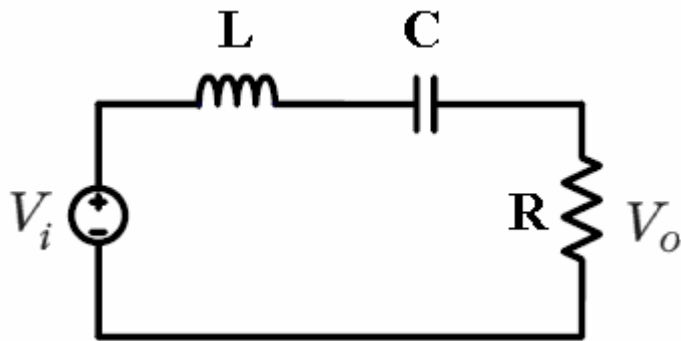


Resposta em frequência do filtro passa faixa

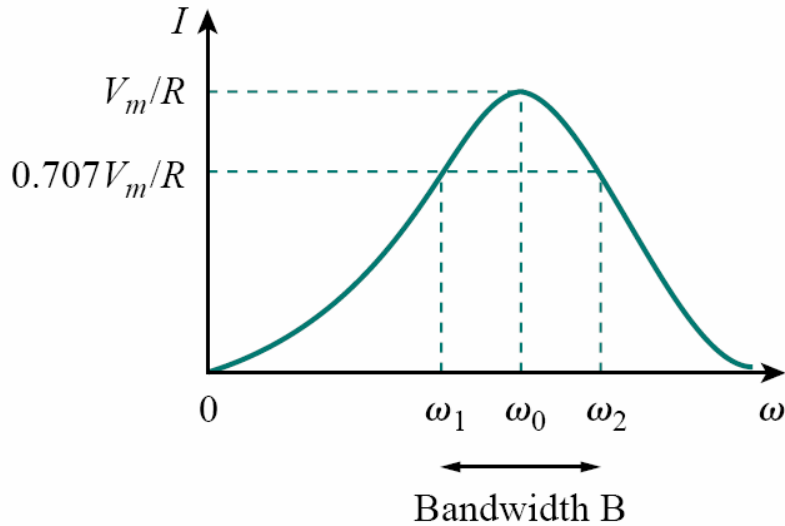




$$\frac{V_o}{V_i} = H_o \frac{\left(\frac{R}{L}\right)s}{s^2 + \left(\frac{R}{L}\right)s + \frac{1}{LC}}$$



$$\frac{V_o}{V_i} = H_o \frac{\left(\frac{\omega_o}{Q}\right)s}{s^2 + \left(\frac{\omega_o}{Q}\right)s + \omega_o^2}$$



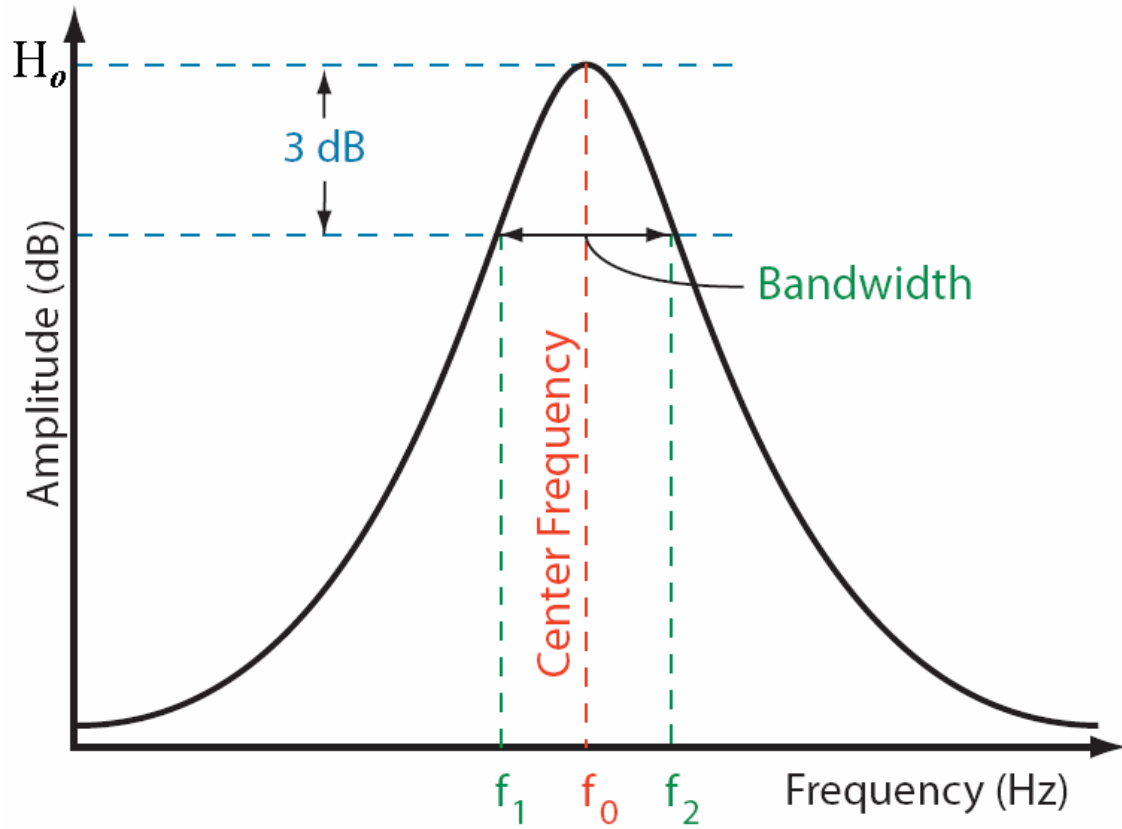
$$Q = \frac{\omega_0 L}{R} = \frac{1}{\omega_0 CR} = \frac{\omega_0}{B}$$

$$B = \omega_2 - \omega_1 = \frac{R}{L} = \frac{\omega_0}{Q}$$

$$\omega_1 = -\frac{R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \frac{1}{LC}}$$

$$\omega_2 = \frac{R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \frac{1}{LC}}$$

$$\omega_0 = \sqrt{\omega_1 \omega_2}$$



$$\frac{V_o}{V_i} = H_o \frac{\left(\frac{\omega_o}{Q}\right)s}{s^2 + \left(\frac{\omega_o}{Q}\right)s + \omega_o^2}$$

$$\frac{V_o}{V_i} = H_o \frac{\frac{1}{Q} \left(\frac{s}{\omega_o}\right)}{\left(\frac{s}{\omega_o}\right)^2 + \frac{1}{Q} \left(\frac{s}{\omega_o}\right) + 1}$$

$$|H(j\omega)| = \frac{\omega \frac{R}{L}}{\sqrt{\left(\frac{1}{LC} - \omega^2\right)^2 + \left(\omega \frac{R}{L}\right)^2}}$$

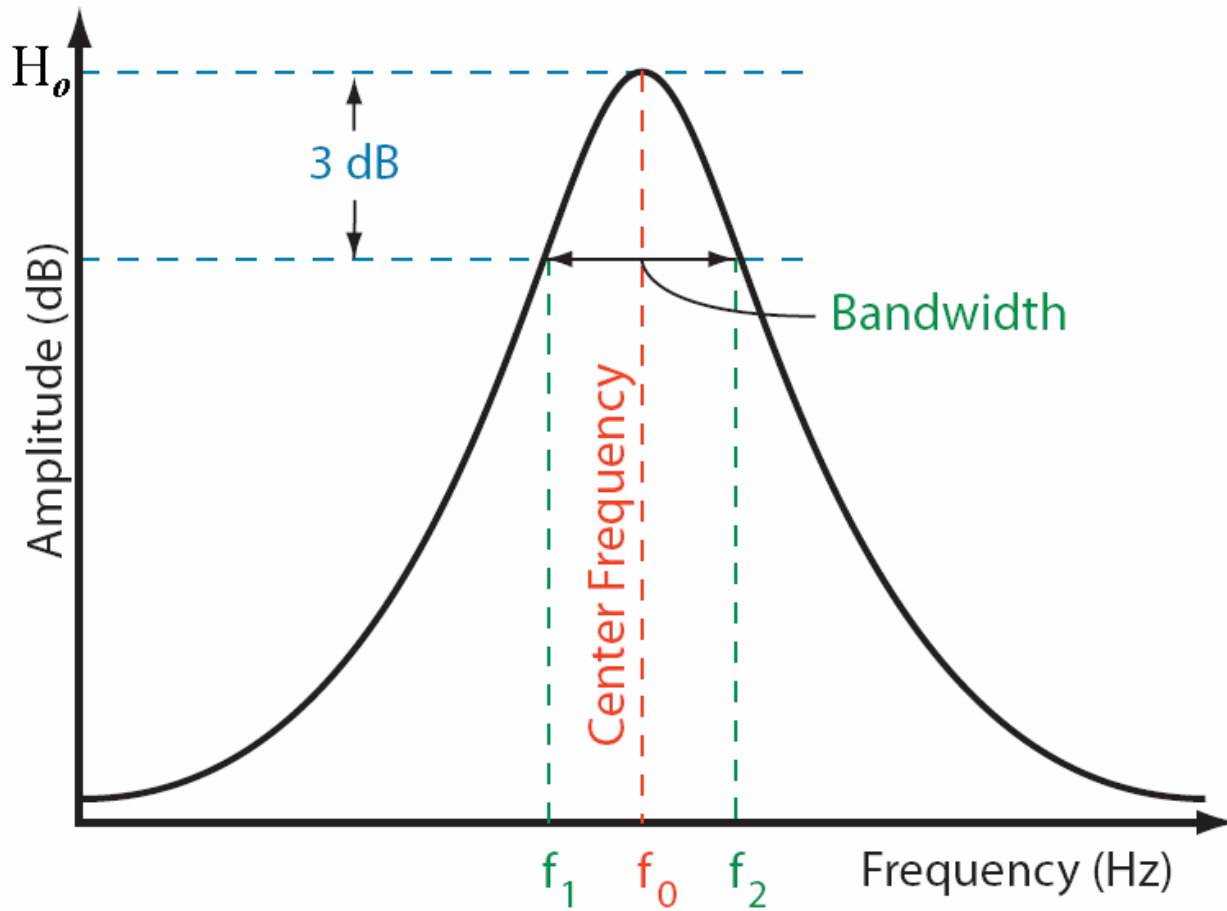
$$\theta(j\omega) = 90^\circ - \tan^{-1} \left(\frac{\omega \frac{R}{L}}{\frac{1}{LC} - \omega^2} \right)$$

$$\frac{1}{\sqrt{2}} = \frac{1}{\sqrt{\left(\omega_c \frac{L}{R} - \frac{1}{\omega_c RC}\right)^2 + 1}}$$

$$\pm 1 = \omega_c \frac{L}{R} - \frac{1}{\omega_c RC}$$

$$\omega_{c1} = -\frac{R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \left(\frac{1}{LC}\right)}$$

$$\omega_{c2} = \frac{R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \left(\frac{1}{LC}\right)}$$



$$Q = \frac{f_0}{f_2 - f_1}$$

$$f_1 = f_0 \left(\sqrt{1 + \frac{1}{4Q^2}} - \frac{1}{2Q} \right)$$

$$f_2 = f_0 \left(\sqrt{1 + \frac{1}{4Q^2}} + \frac{1}{2Q} \right)$$

N – número de oitavas

$$Q = \frac{\sqrt{2^N}}{2^{\frac{N}{2}} - 1}$$

$$f_0 = \sqrt{f_1 f_2}$$

N – número de oitavas

$$f_0 = \sqrt{f_1 f_2}$$

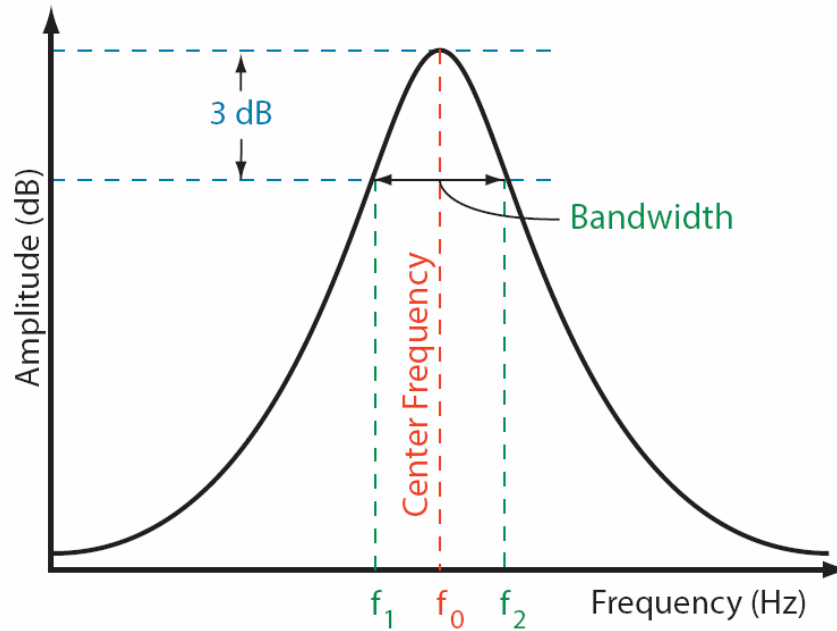
$$f_2 = y f_1,$$

$$y = 2^N,$$

$$f_2 = 2^N f_1$$

$$N = \frac{\log y}{\log 2}$$

$$Q = \frac{f_0}{f_2 - f_1}$$

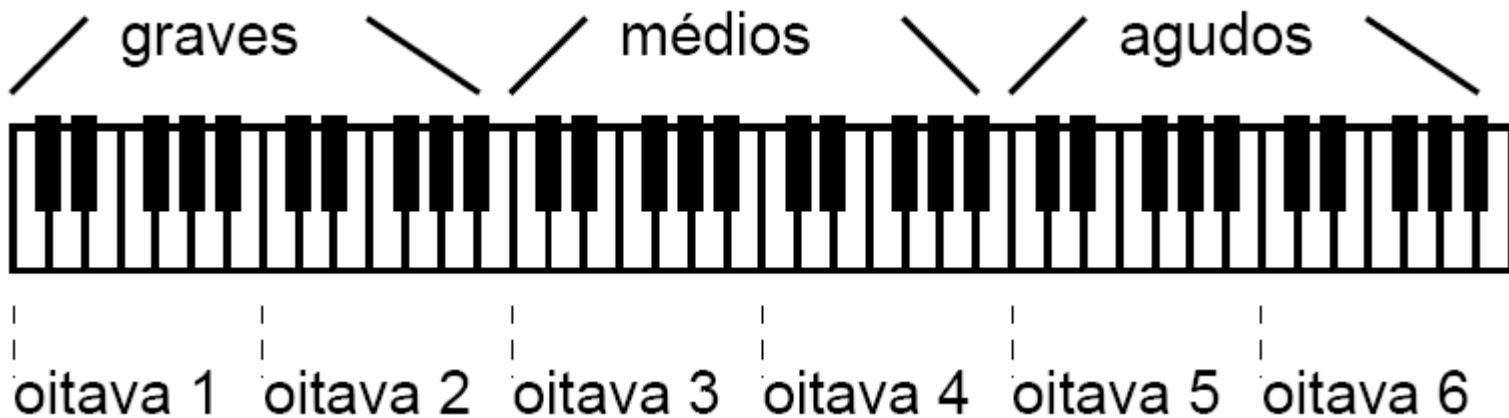
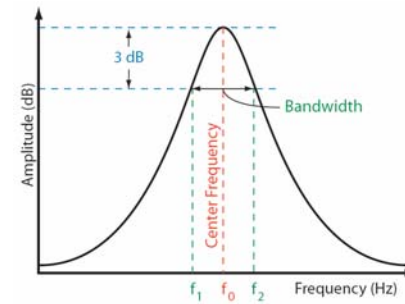
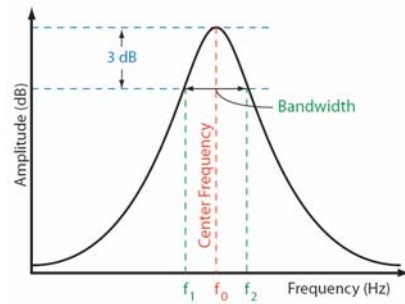
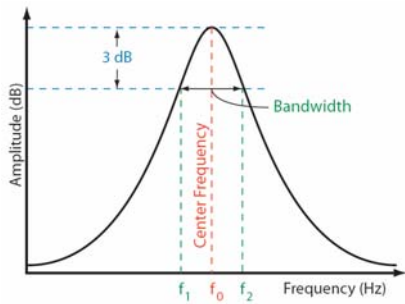


$$f_0 = \sqrt{f_1 (2^N f_1)}$$

$$f_0 = \sqrt{2^N} f_1.$$

$$Q = \frac{\sqrt{2^N} f_1}{2^N f_1 - f_1}$$

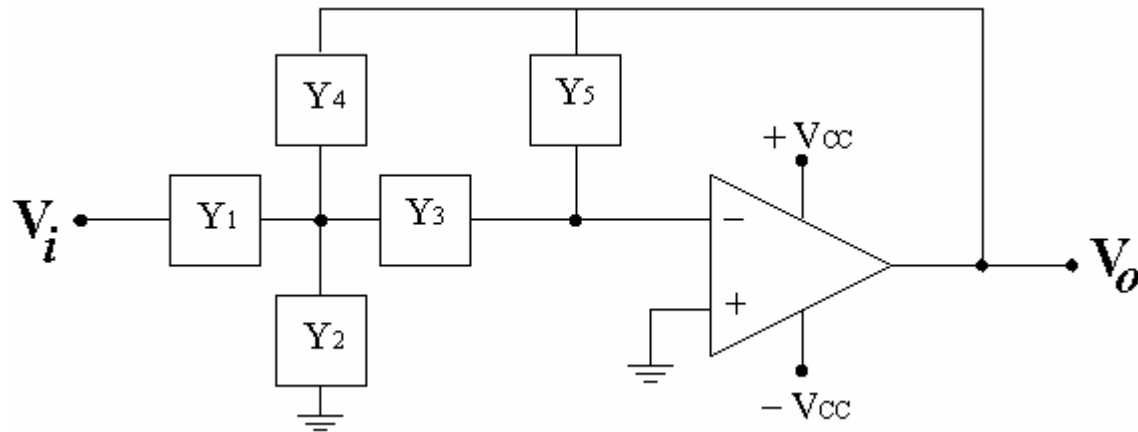
$$Q = \frac{\sqrt{2^N}}{2^N - 1}$$



BW (octaves)	Q
2.0	0.667
1.0	1.414
2/3	2.145
1/2	2.871
1/3	4.318
1/6	8.651
1/10	14.424
1/30	43.280

Filtro Ativo

Configuração Realimentação Múltipla Multiple-Feedback Filter



$$\frac{V_o}{V_i} = - \frac{Y_1 Y_3}{Y_5 (Y_1 + Y_2 + Y_3 + Y_4) + Y_3 Y_4}$$

Filtro Passa Baixa

$$\frac{V_O(s)}{V_I(s)} = \frac{A_{LP} \omega_n^2}{s^2 + s \omega_n / Q + \omega_n^2}$$

Filtro Passa Alta

$$\frac{V_{HP}(s)}{V_I(s)} = \frac{A_{HP} s^2}{s^2 + s \omega_n / Q + \omega_n^2}$$

Filtro Passa Faixa

$$\frac{V_{BP}(s)}{V_I(s)} = \frac{A_{BP} (\omega_n / Q) s}{s^2 + s \omega_n / Q + \omega_n^2}$$

Filtro Rejeita Faixa

$$\frac{V_{BR}(s)}{V_I(s)} = \frac{A_{BR} (s^2 + \omega_n^2)}{s^2 + s \omega_n / Q + \omega_n^2}$$

$$\frac{V_0}{V_i} = \frac{Y_1 Y_3}{Y_5 (Y_1 + Y_2 + Y_3 + Y_4) + Y_3 Y_4}$$

Filtro Passa Baixa

$$\frac{V_O(s)}{V_I(s)} = \frac{A_{LP} \omega_n^2}{s^2 + s \omega_n / Q + \omega_n^2}$$

Filtro Passa Alta

$$\frac{V_{HP}(s)}{V_I(s)} = \frac{A_{HP} s^2}{s^2 + s \omega_n / Q + \omega_n^2}$$

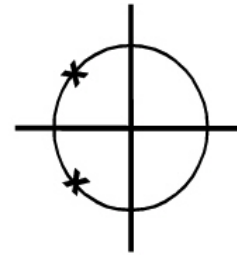
Filtro Passa Faixa

$$\frac{V_{BP}(s)}{V_I(s)} = \frac{A_{BP} (\omega_n / Q) s}{s^2 + s \omega_n / Q + \omega_n^2}$$

Filtro Rejeita Faixa

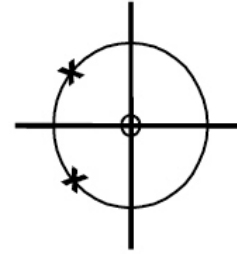
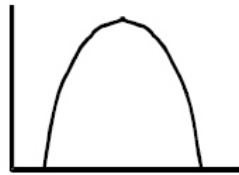
$$\frac{V_{BR}(s)}{V_I(s)} = \frac{A_{BR} (s^2 + \omega_n^2)}{s^2 + s \omega_n / Q + \omega_n^2}$$

PASSA-BAIXA



$$\frac{\omega_0^2}{s^2 + \frac{\omega_0}{Q}s + \omega_0^2}$$

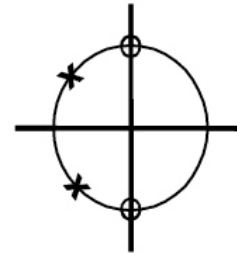
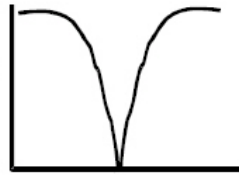
PASSA-FAIXA



$$\frac{\omega_0^3 Q s}{s^2 + \frac{\omega_0}{Q}s + \omega_0^2}$$

REJEITA-FAIXA

NOTCH



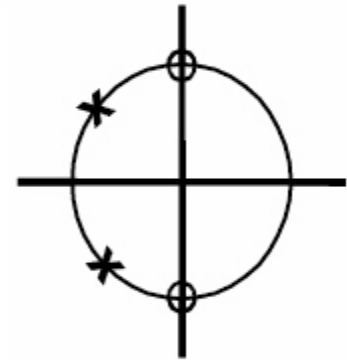
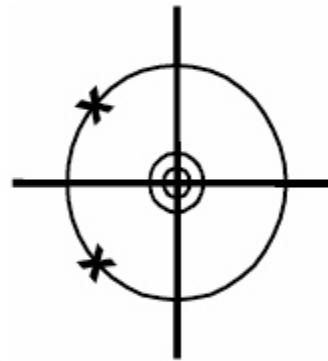
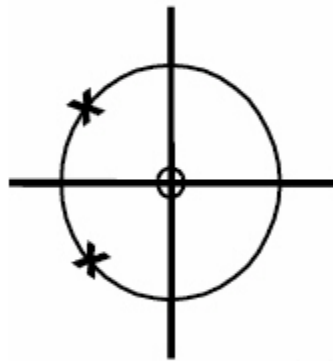
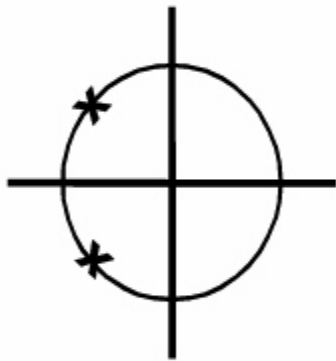
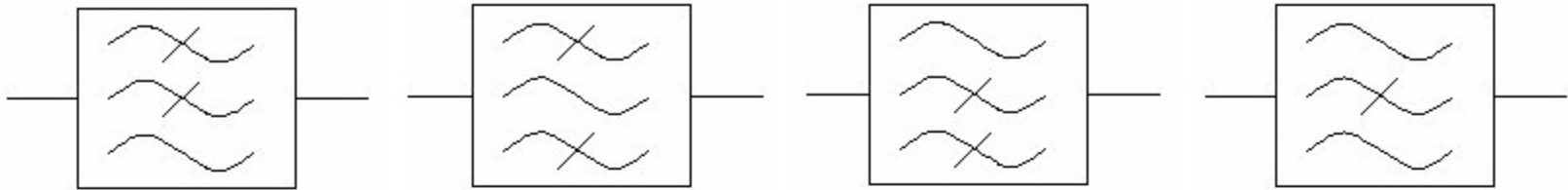
$$\frac{s^2 + \omega_z^2}{s^2 + \frac{\omega_0}{Q}s + \omega_0^2}$$

PASSA-ALTA



$$\frac{s^2}{s^2 + \frac{\omega_0}{Q}s + \omega_0^2}$$

Filtros Elétricos - Configuração de pólos e zeros



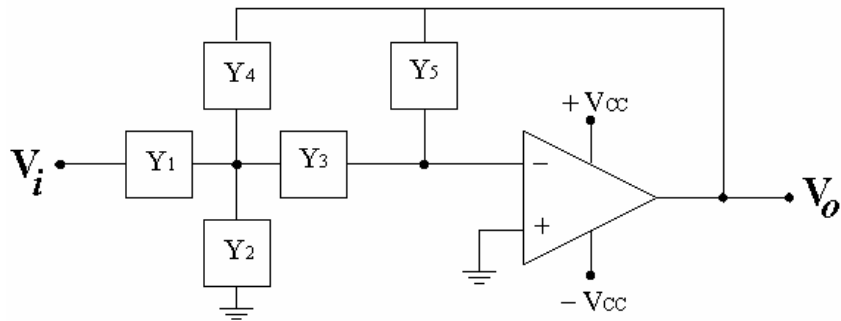
$$\frac{\omega_0^2}{s^2 + \frac{\omega_0}{Q}s + \omega_0^2}$$

$$\frac{\frac{\omega_0}{Q}s}{s^2 + \frac{\omega_0}{Q}s + \omega_0^2}$$

$$\frac{s^2}{s^2 + \frac{\omega_0}{Q}s + \omega_0^2}$$

$$\frac{s^2 + \omega_z^2}{s^2 + \frac{\omega_0}{Q}s + \omega_0^2}$$

$$\frac{V_o}{V_i} = - \frac{Y_1 Y_3}{Y_5 (Y_1 + Y_2 + Y_3 + Y_4) + Y_3 Y_4}$$



Filtro Passa Baixa

$$\frac{V_o(s)}{V_i(s)} = \frac{A_{LP} \omega_n^2}{s^2 + s \omega_n / Q + \omega_n^2}$$

Filtro Passa Alta

$$\frac{V_{HP}(s)}{V_i(s)} = \frac{A_{HP} s^2}{s^2 + s \omega_n / Q + \omega_n^2}$$

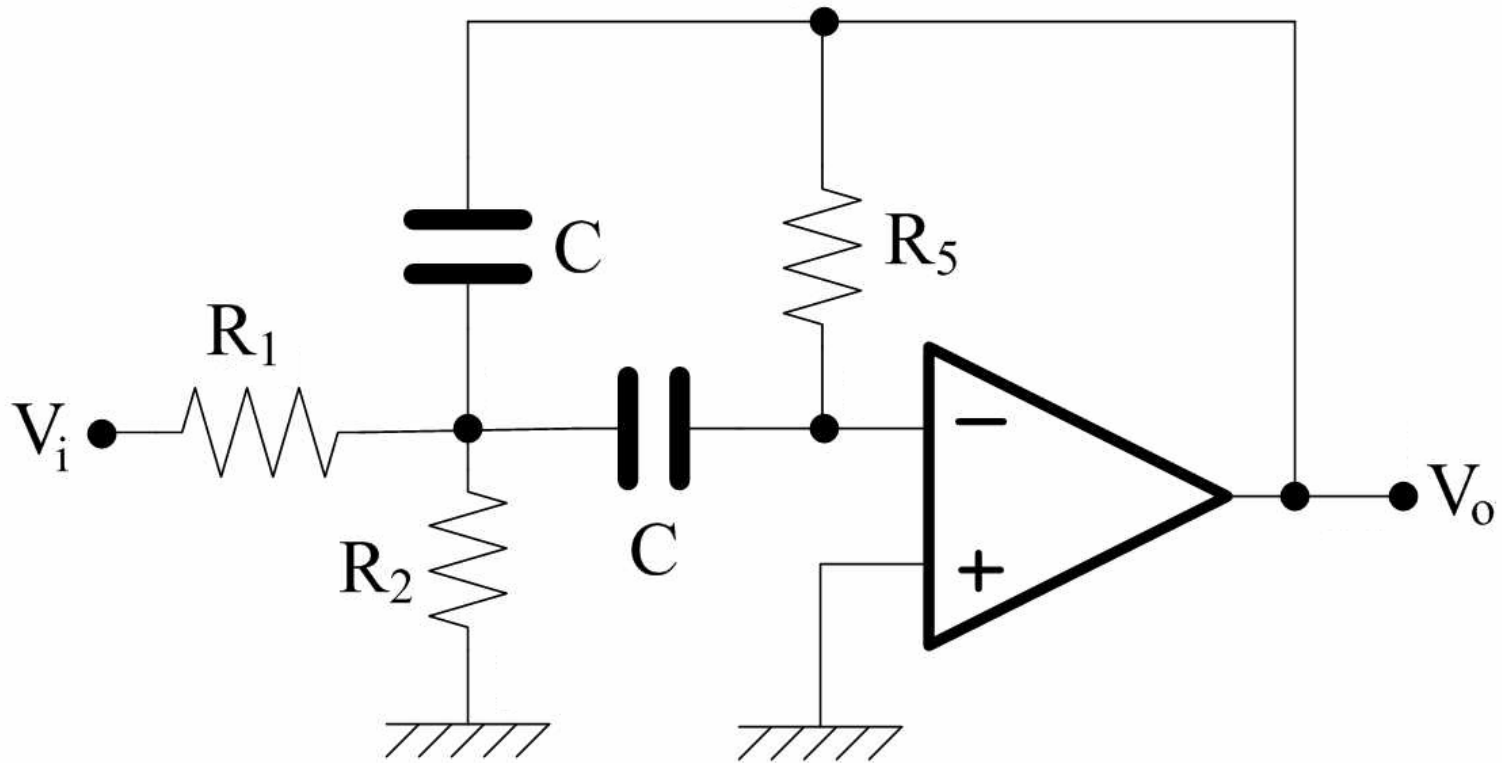
Filtro Passa Faixa

$$\frac{V_{BP}(s)}{V_i(s)} = \frac{A_{BP} (\omega_n / Q) s}{s^2 + s \omega_n / Q + \omega_n^2}$$

Filtro Rejeita Faixa

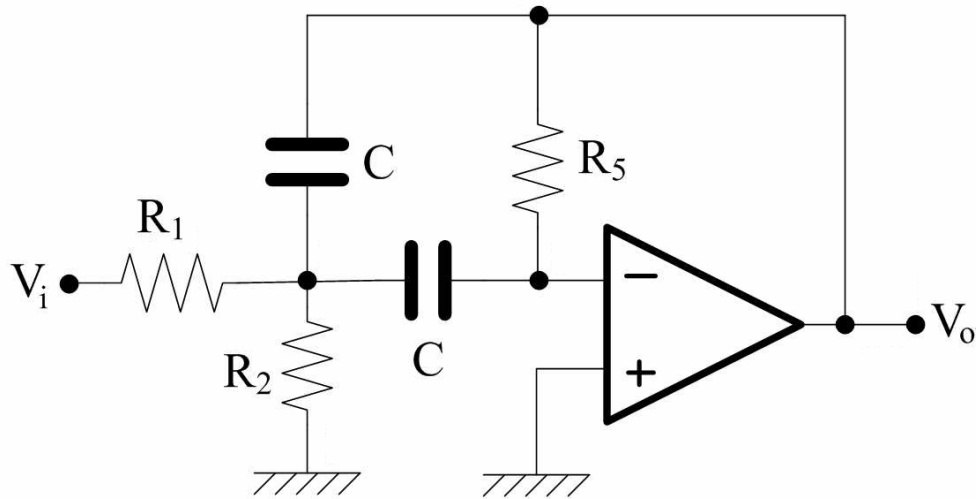
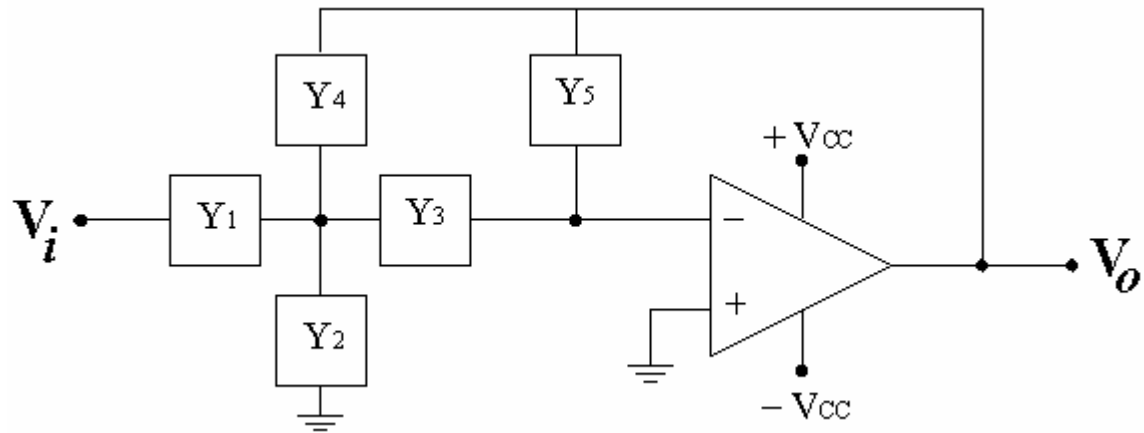
$$\frac{V_{BR}(s)}{V_i(s)} = \frac{A_{BR} (s^2 + \omega_n^2)}{s^2 + s \omega_n / Q + \omega_n^2}$$

Filtro Passa Faixa

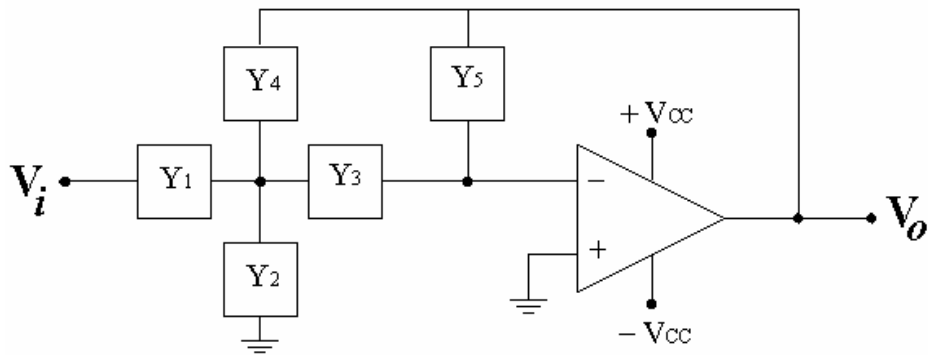


Filtro Passa Faixa

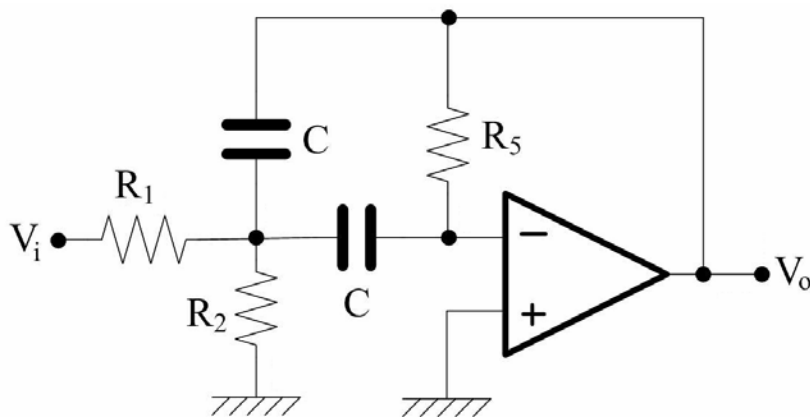
Multiple-Feedback Band-Pass Filter



Filtro Passa Faixa

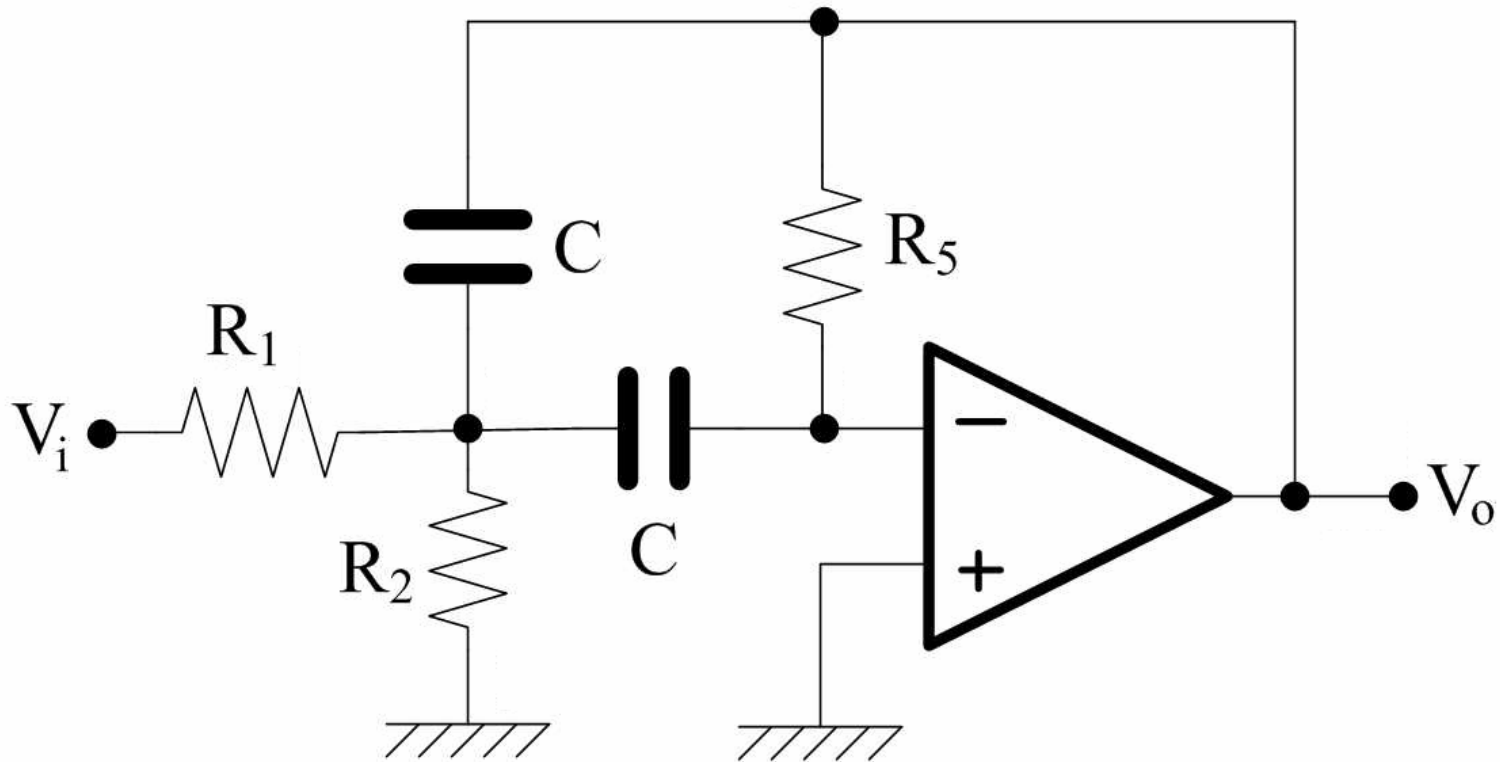


$$\frac{V_o}{V_i} = - \frac{Y_1 Y_3}{Y_5 (Y_1 + Y_2 + Y_3 + Y_4) + Y_3 Y_4}$$



$$\frac{V_o}{V_i} = \frac{\frac{1}{R_1 C} s}{s^2 + \frac{2}{R_5 C} s + \frac{(R_1 + R_2)}{R_1 R_2 R_5 C^2}}$$

Filtro Passa Faixa



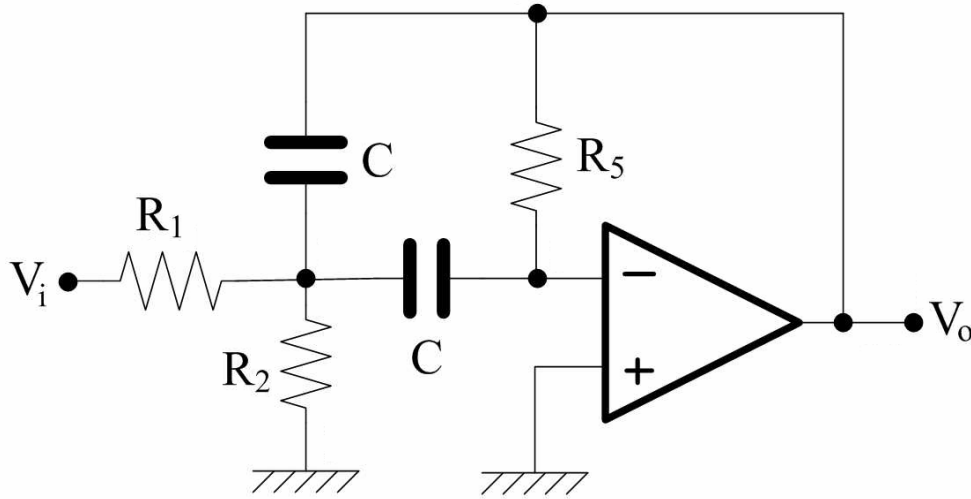
$$\frac{V_o}{V_i} = - \frac{\frac{1}{R_1 C} s}{s^2 + \frac{2}{R_5 C} s + \frac{(R_1 + R_2)}{R_1 R_2 R_5 C^2}}$$

Filtro Passa Faixa

$$\frac{V_o}{V_i} = H_o \frac{\frac{\omega_0}{Q} s}{s^2 + \frac{\omega_0}{Q} s + \omega_0^2}$$

$$\frac{V_o}{V_i} = - \frac{\frac{1}{R_1 C} s}{s^2 + \frac{2}{R_5 C} s + \frac{(R_1 + R_2)}{R_1 R_2 R_5 C^2}}$$

Filtro Passa Faixa



$$\omega_0^2 = \frac{(R_1 + R_2)}{R_1 R_2 R_5 C^2}$$

$$\frac{\omega_0}{Q} = \frac{2}{R_5 C}$$

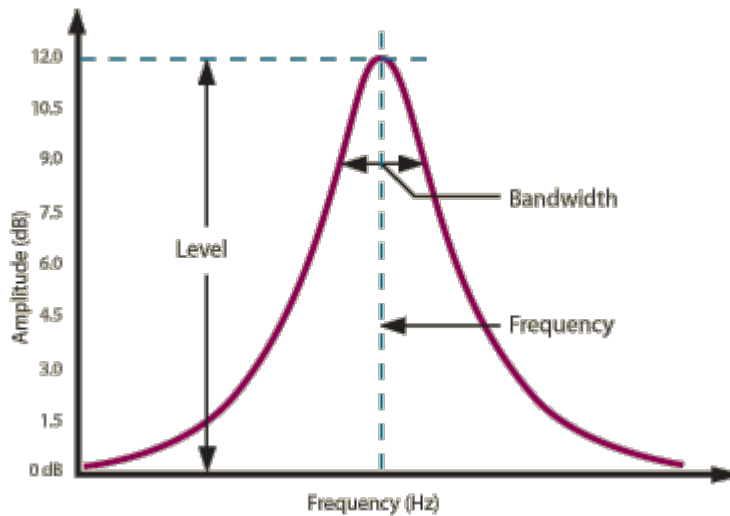
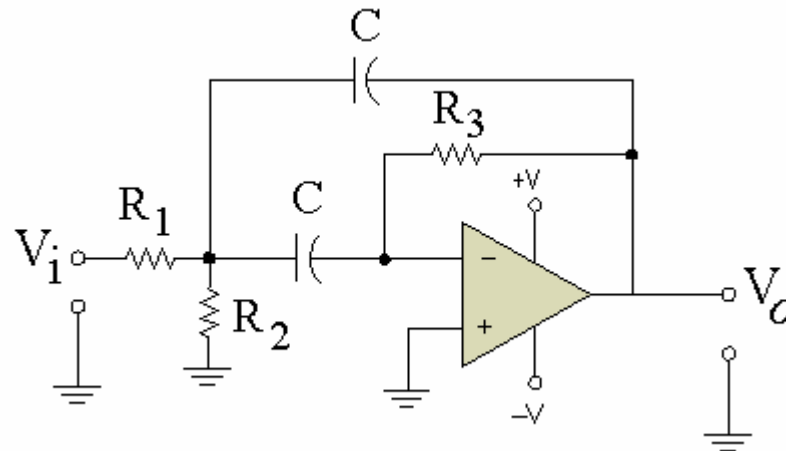
$$Q = \frac{\omega_0 R_5 C}{2}$$

$$Q = \frac{1}{2} \sqrt{\frac{R_5 (R_1 + R_2)}{R_1 R_2}}$$

$$\frac{V_o}{V_i} = - \frac{\frac{1}{R_1 C} s}{s^2 + \frac{2}{R_5 C} s + \frac{(R_1 + R_2)}{R_1 R_2 R_5 C^2}}$$

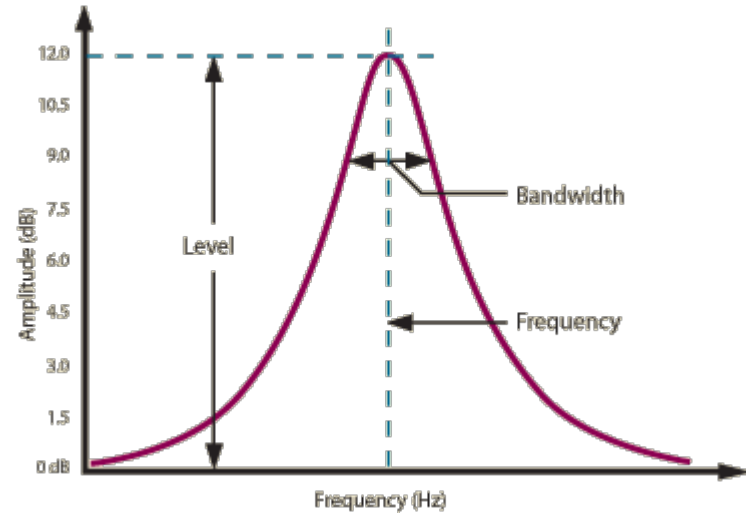
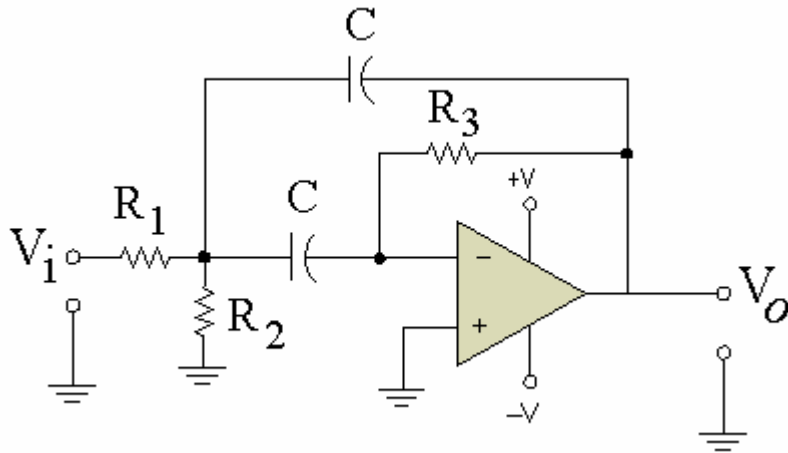
$$H_o = - \frac{R_5}{2 R_1}$$

Filtro Passa Faixa



$$f_0 = \frac{1}{2\pi C} \left[\frac{1}{R_3} \frac{R_1 + R_2}{R_1 R_2} \right]^{1/2}$$

Filtro Passa Faixa



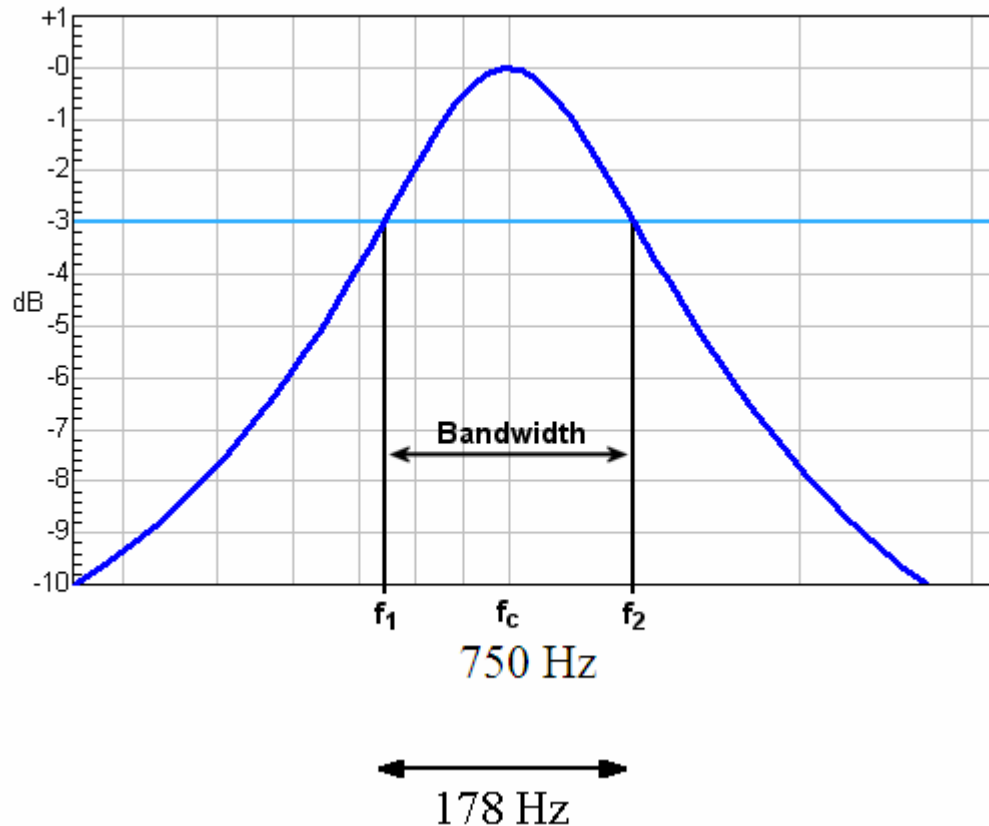
Fator de Qualidade $\rightarrow Q = \frac{1}{2} \sqrt{\frac{R_3}{R}} = \pi f_0 R_3 C \quad R = R_1 // R_2$

Frequência central $\rightarrow f_0 = \frac{1}{2\pi C \sqrt{RR_3}}$

Faixa de Passagem $\rightarrow \Delta\omega = \frac{2}{R_3 C} = \frac{\omega_0}{Q}$

Projeto

Projete um filtro passa faixa para 750 Hz, ganho de 1,32 na frequência central e faixa de passagem de 178 Hz.

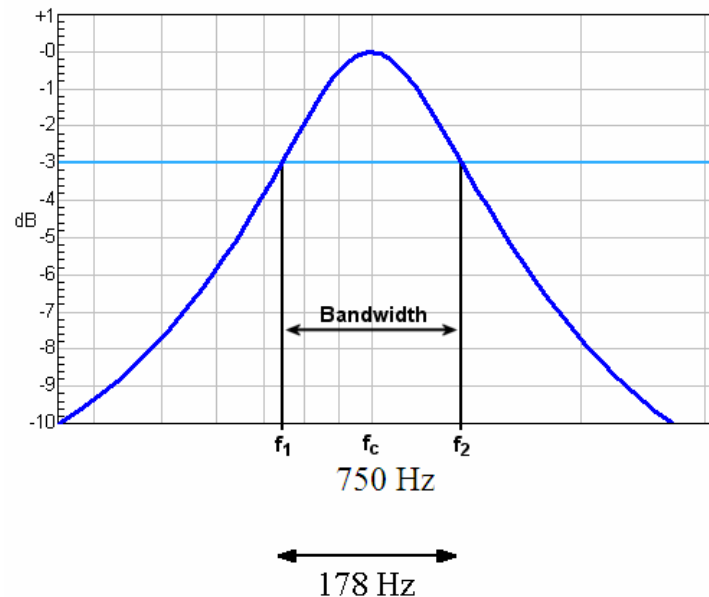


Filtro Passa Faixa

Solução

Determinação do fator de qualidade Q

$$Q = \frac{f_o}{\Delta f} = \frac{750 \text{ Hz}}{178 \text{ Hz}} = 4,2$$



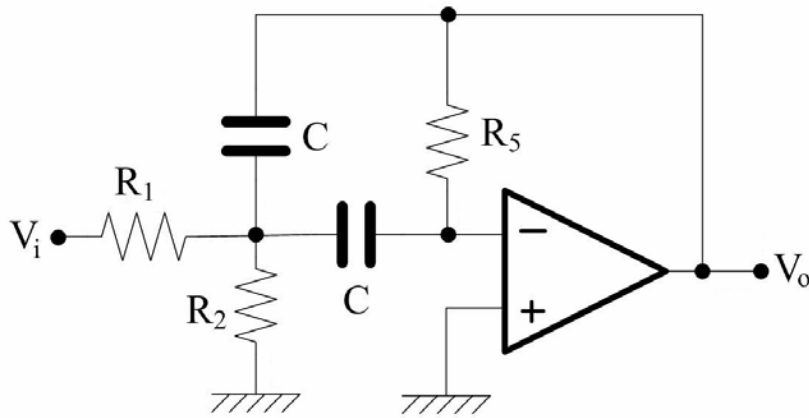
Filtro Passa Faixa

Especificações

Parâmetros para o projeto do filtro passa-faixa

- *Frequência central de 750 Hz*
- *Ganho de 1,32 na frequência central*
- *Faixa de passagem $\Delta f = 178$ Hz.*
- *Fator de qualidade $Q = 4,2$.*

Circuito do Filtro Passa-Faixa Esquemático



$$R_1 = \frac{Q}{2\pi f_o G_o C}$$

$$R_2 = \frac{Q}{2\pi f_o C (2Q^2 - G_o)}$$

$$\frac{V_o}{V_i} = - \frac{\frac{1}{R_1 C} s}{s^2 + \frac{2}{R_5 C} s + \frac{(R_1 + R_2)}{R_1 R_2 R_5 C^2}}$$

$$R_5 = 2R_1 G_o$$

Escolhendo $C = 10 \text{ nF}$

$$R_1 = \frac{Q}{2\pi f_o G_o C} = \frac{(4.2)}{(2\pi)(750 \text{ Hz})(1.32)(.01\mu\text{F})}$$

$$R_1 = 67.6 \text{ kOhm (use 68 kOhm)}$$

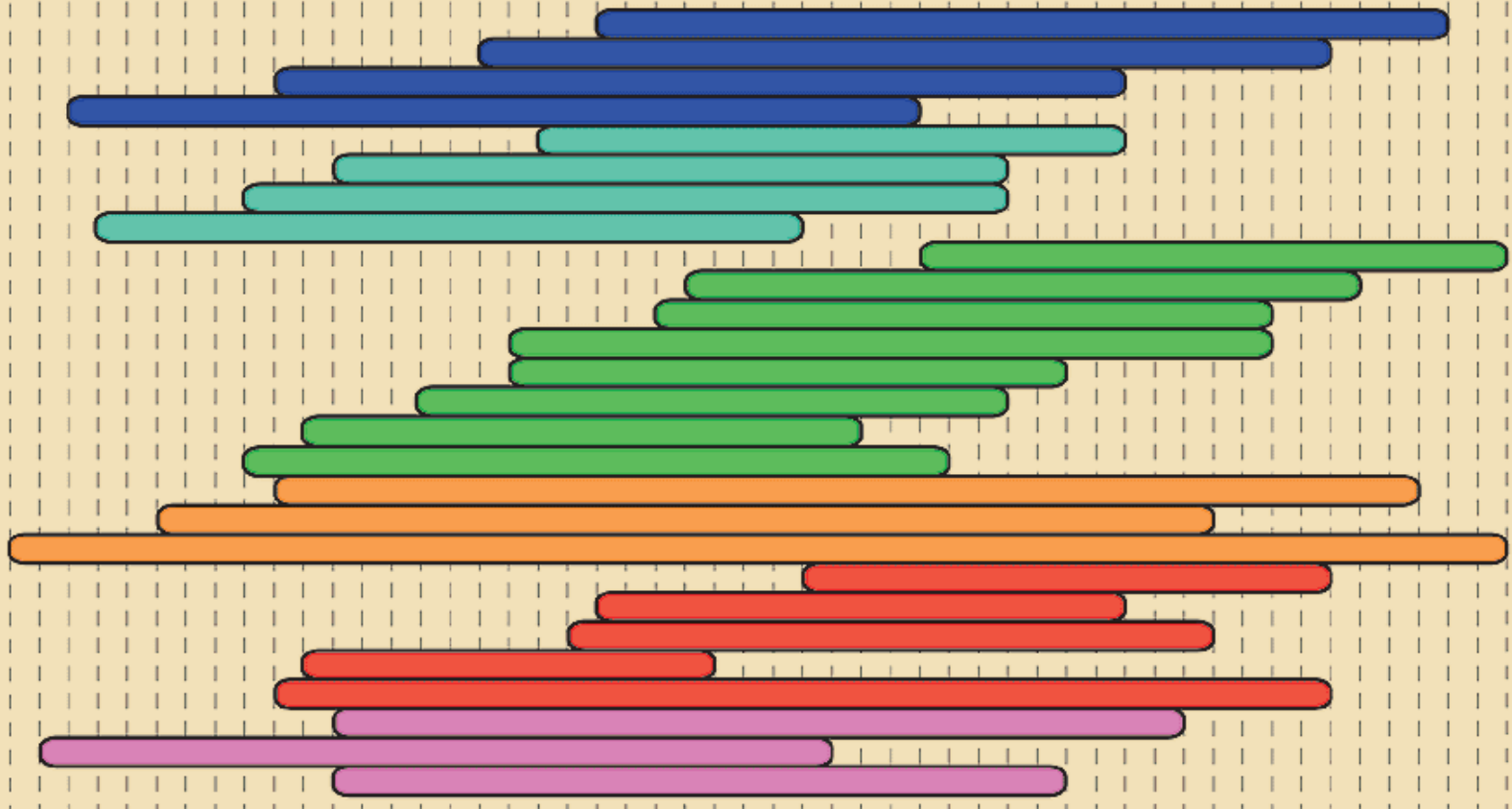
$$R_5 = 2R_1 G_o = (2)(67.6 \text{ kOhm})(1.32)$$

$$R_5 = 178 \text{ kOhm (use 180 kOhm)}$$

$$R_2 = \frac{Q}{2\pi f_o C (2Q^2 - G_o)}$$

$$R_2 = \frac{4.2}{\left[2\pi(750\text{Hz})(.01\mu\text{F})\left(2(4.2)^2 - 1.32\right)\right]}$$

$$R_2 = 2.6 \text{ k.Ohm (use 2.7 kOhm)}$$



- Violin
- Viola
- Cello
- Bass
- Trumpet
- Trombone
- French Horn
- Tuba
- Piccolo
- Flute
- Oboe
- Clarinet
- Alto Sax
- Tenor Sax
- Baritone Sax
- Bassoon
- Harp
- Harpsichord
- Piano
- Xylophone
- Glockenspiel
- Vibraphone
- Timpani
- Marimba
- Guitar
- Bass Guitar
- Voice

Faixa Espectral da voz humana

Speech

men 110–165 Hz women and children 220–330 Hz



Singing

bass 82–330 Hz

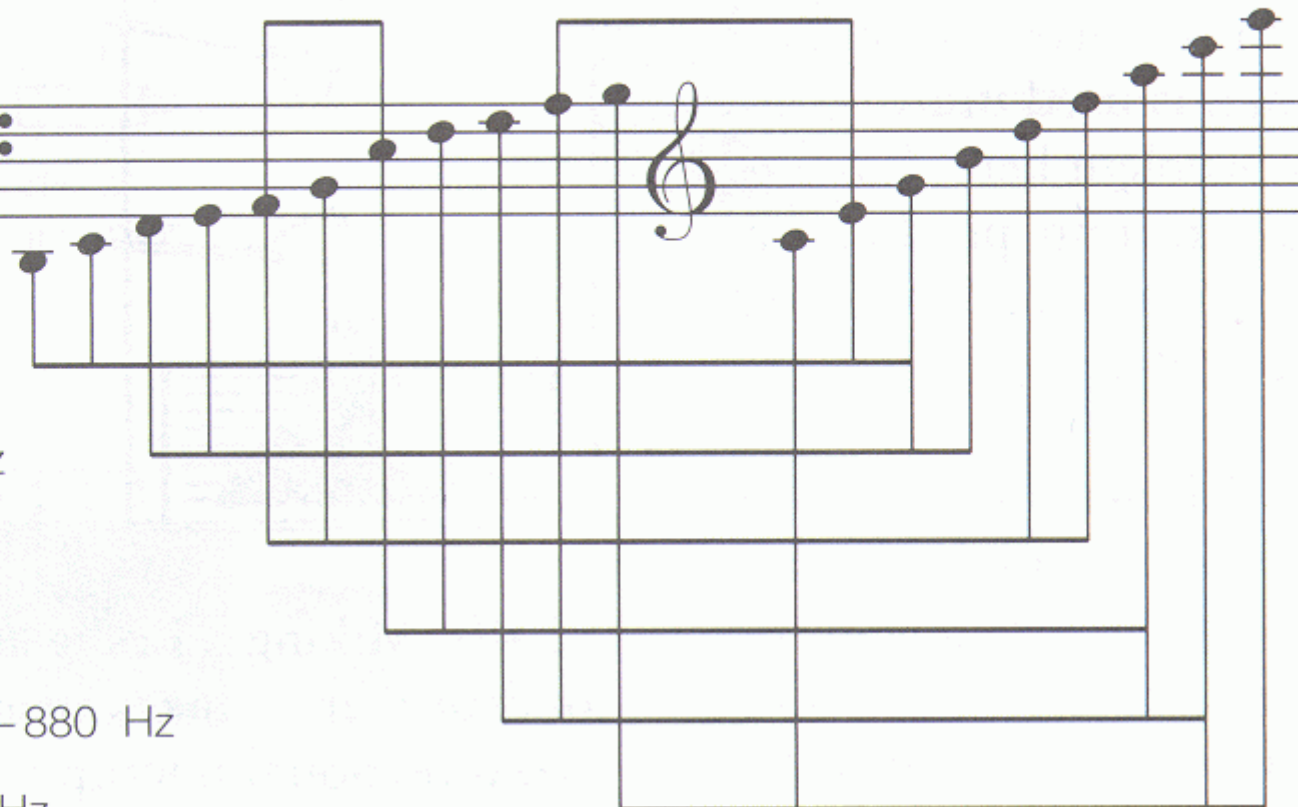
baritone 98–392 Hz

tenor 124–494 Hz

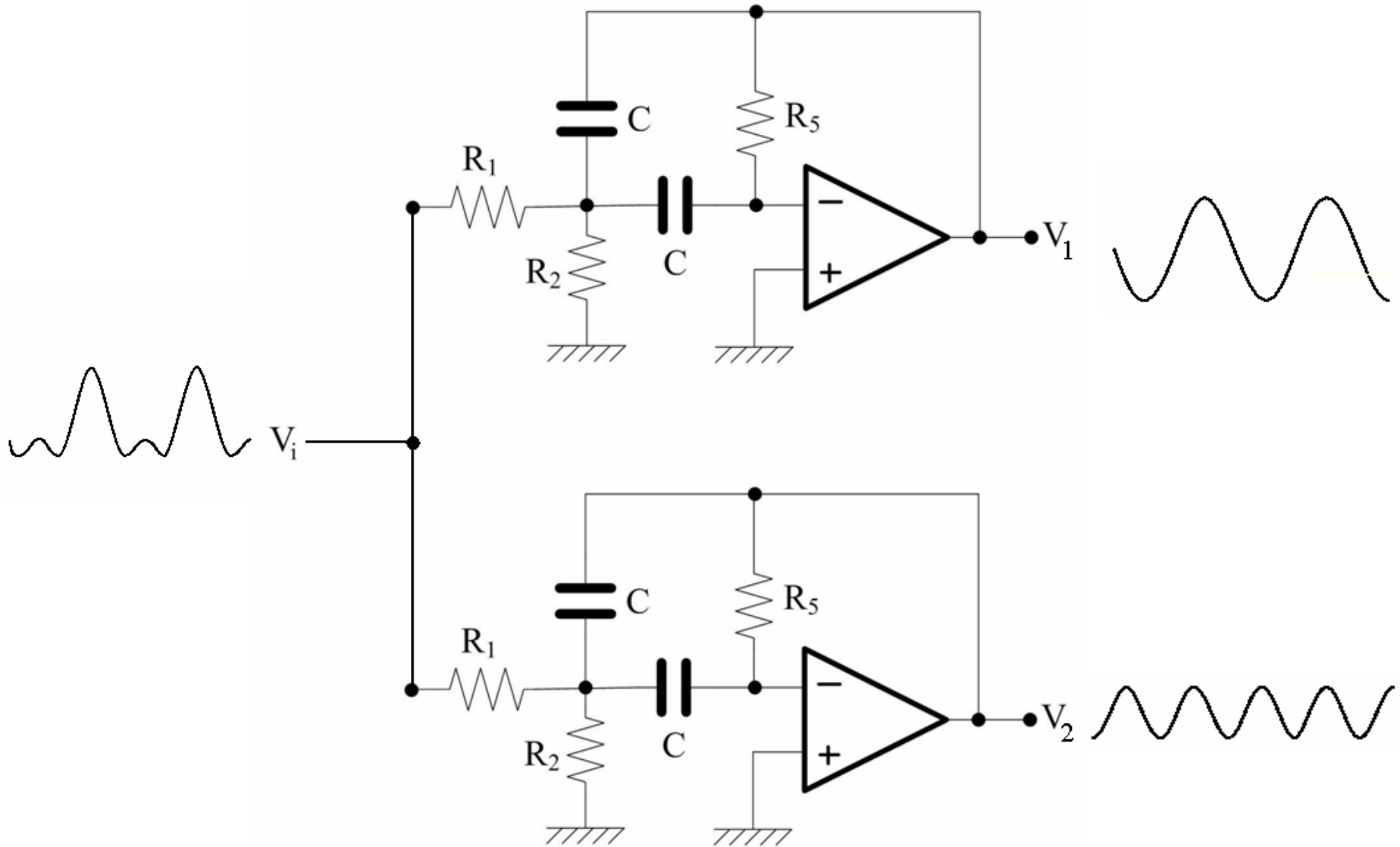
alto 175–699 Hz

mezzo-soprano 220–880 Hz

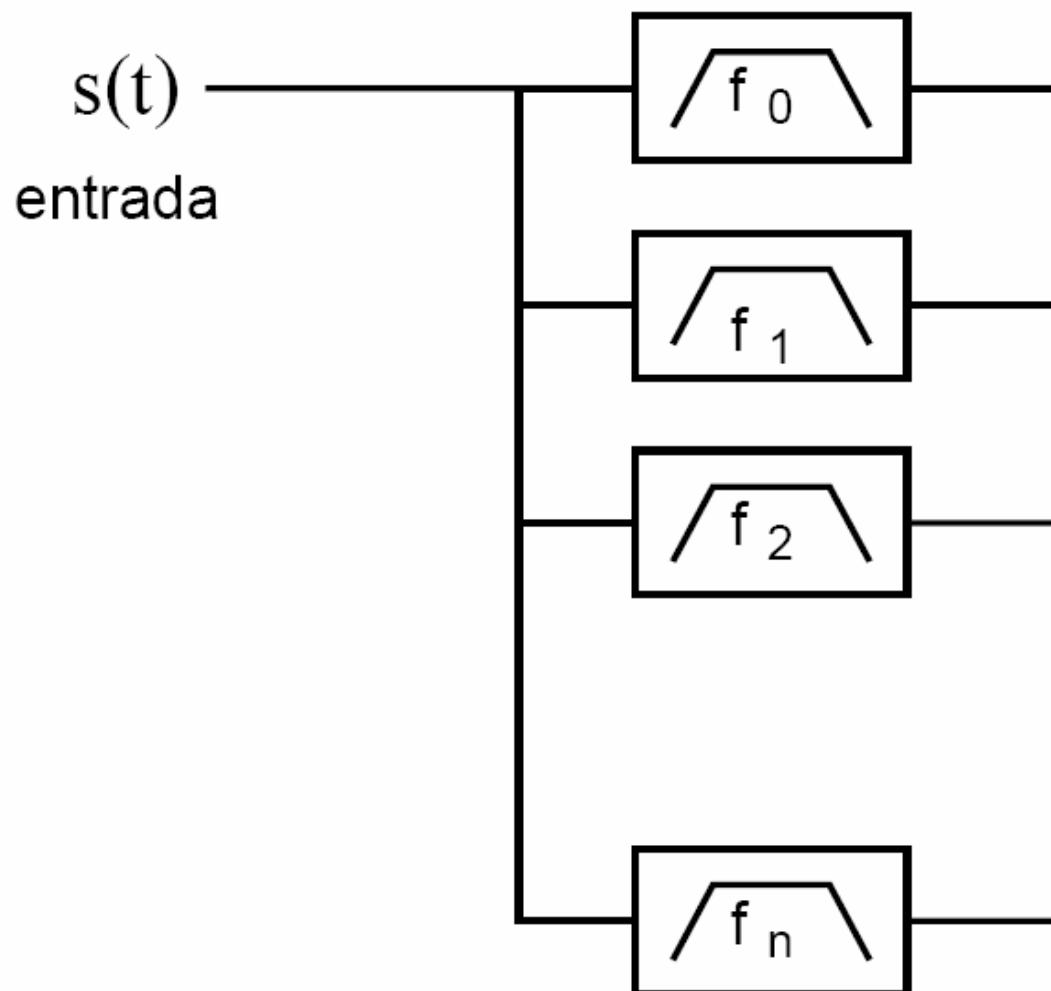
soprano 262–1047 Hz

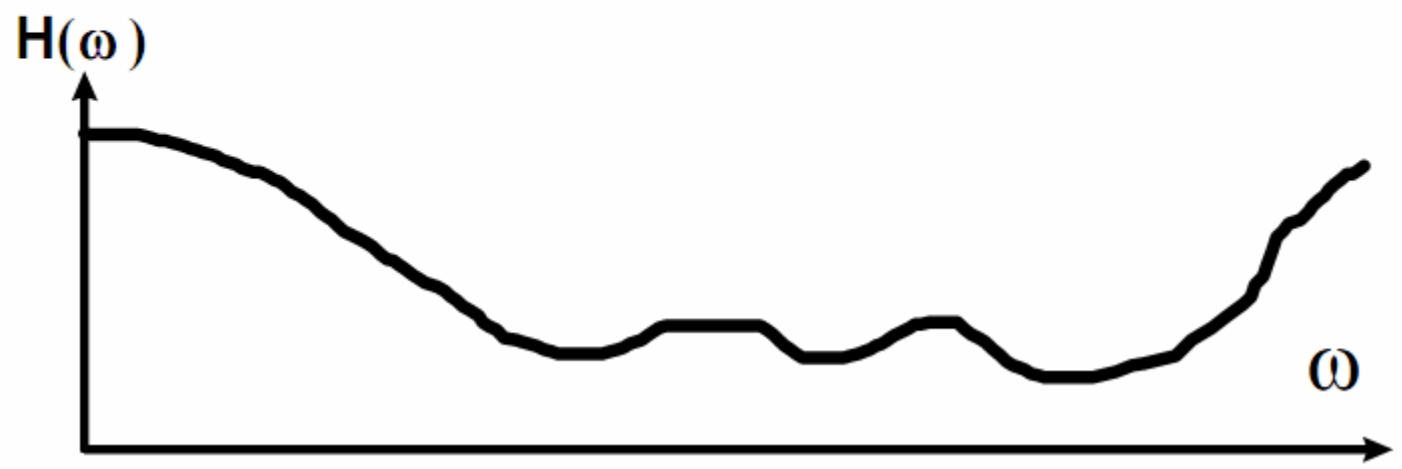
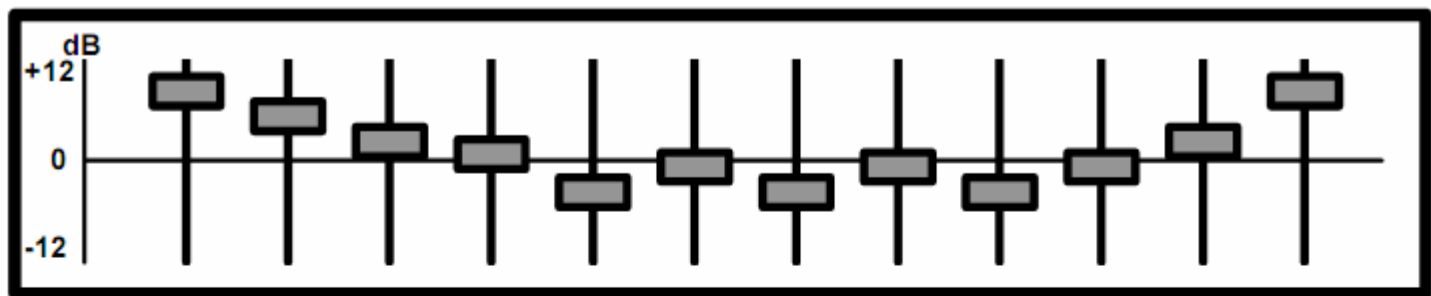
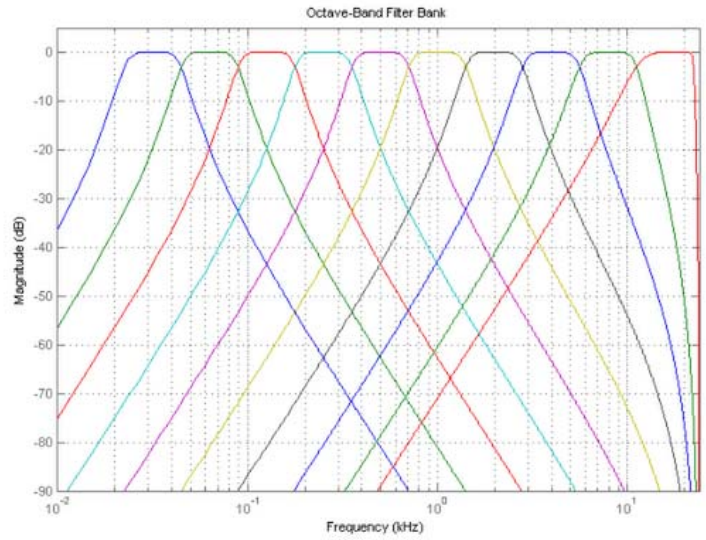


Análise Espectral



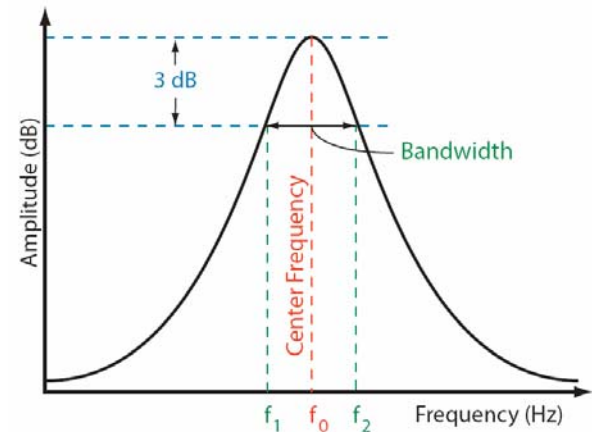
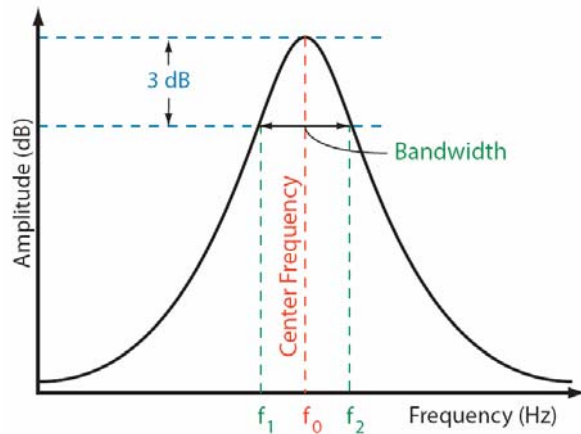
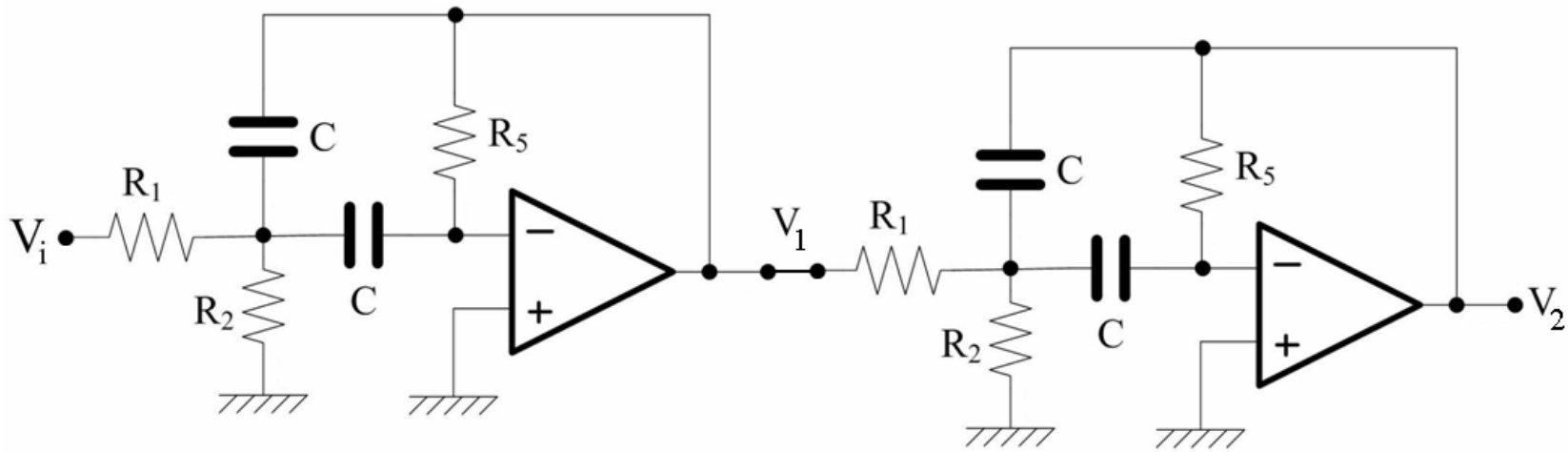
FILTROS PASSA FAIXA





Filtros passa faixas em série

Filtro Passa Faixa de Quarta Ordem



Filtros passa faixa em série
Fator de Qualidade Equivalente

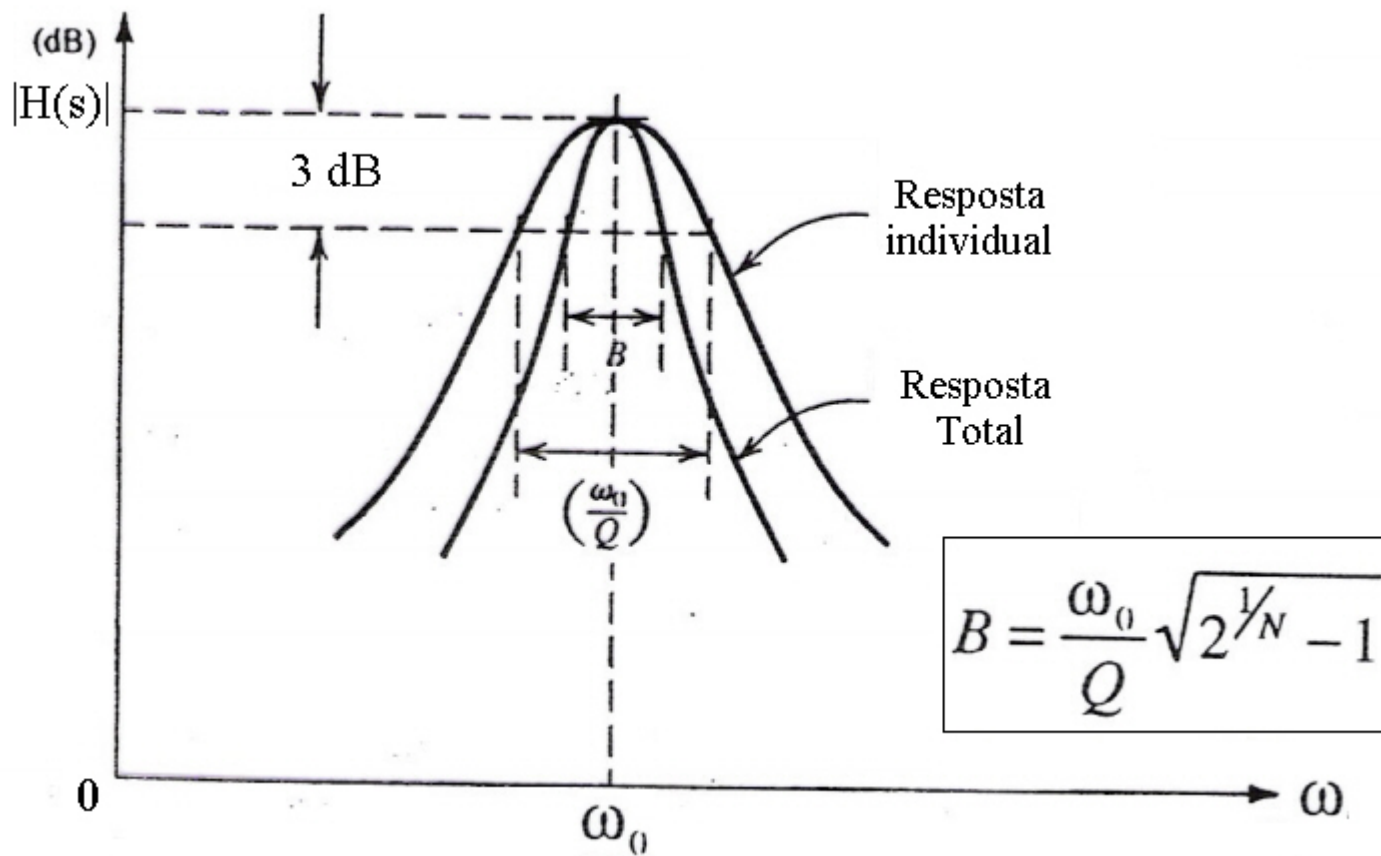
$N \rightarrow$ número de filtros em série de mesma frequência central e com fatores de qualidades iguais a Q_0

$$Q_T = \frac{Q_0}{\sqrt{\sqrt[N]{2} - 1}}$$

Fator de Qualidade Equivalente

$N = 1$	$Q = Q_0$
$N = 2$	$Q_2 = 1,55Q_0$
$N = 3$	$Q_3 = 1,96Q_0$
$N = 4$	$Q_4 = 2,3Q_0$
$N = 5$	$Q_5 = 2,6Q_0$

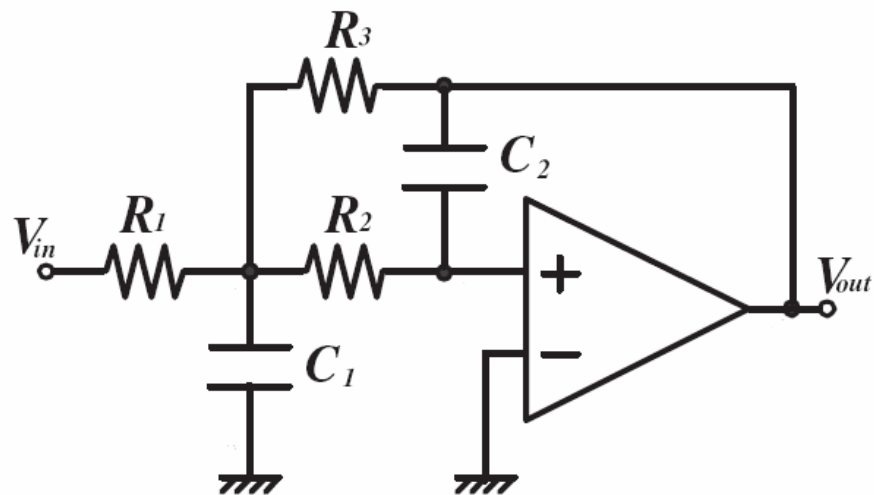
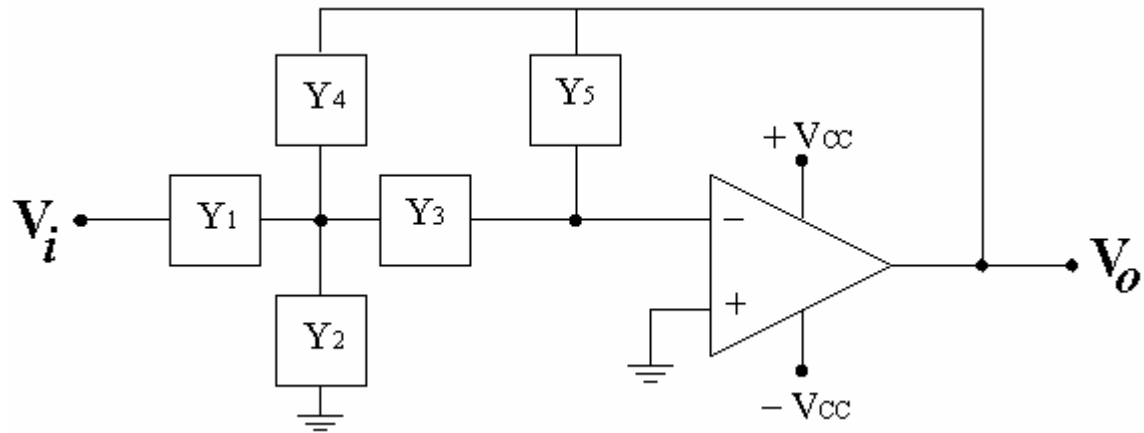
$$Q_T = \frac{Q_0}{\sqrt{\sqrt[N]{2} - 1}}$$



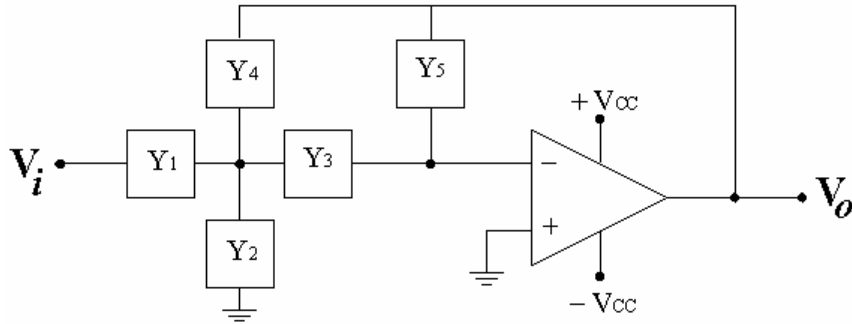
$$\Delta\omega = \frac{\omega_0}{Q} \sqrt{2^{\frac{1}{N}} - 1}$$

$$\Delta f = \frac{f_0}{Q} \sqrt{2^{\frac{1}{N}} - 1}$$

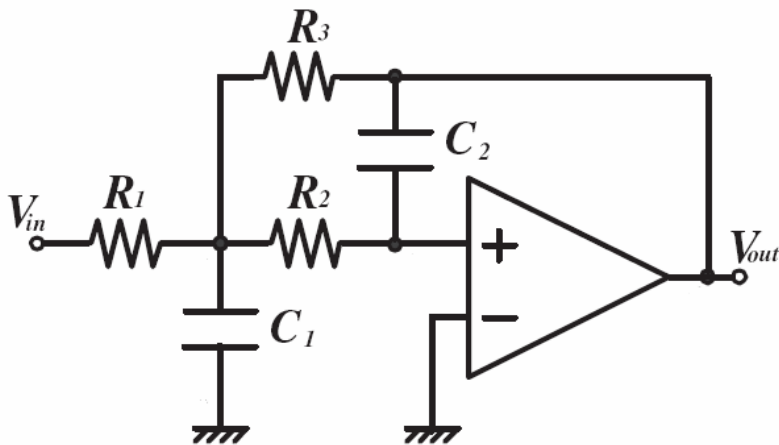
Filtro Passa Baixa



Filtro Passa Baixa

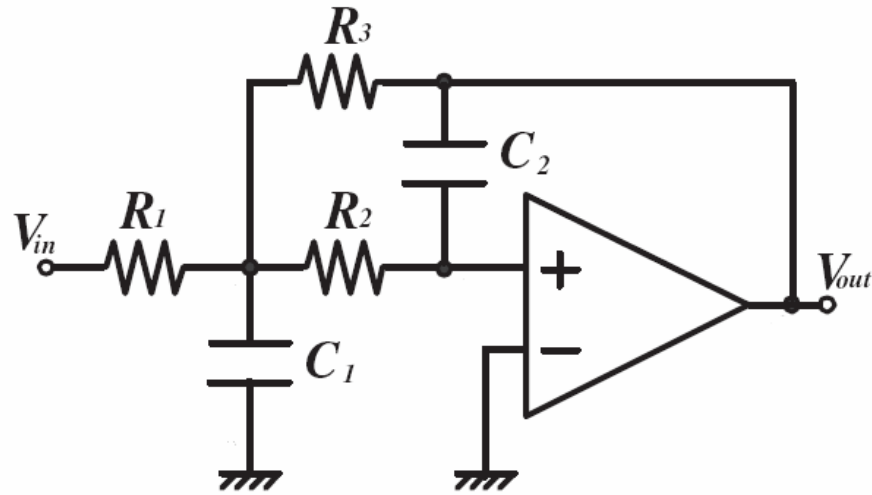


$$\frac{V_0}{V_i} = - \frac{Y_1 Y_3}{Y_5 (Y_1 + Y_2 + Y_3 + Y_4) + Y_3 Y_4}$$



$$\frac{V_0}{V_i} = - \frac{\frac{1}{R_1 R_2 C_1 C_2}}{s^2 + s \frac{1}{C_1} \left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} \right) + \frac{1}{R_2 R_3 C_1 C_2}}$$

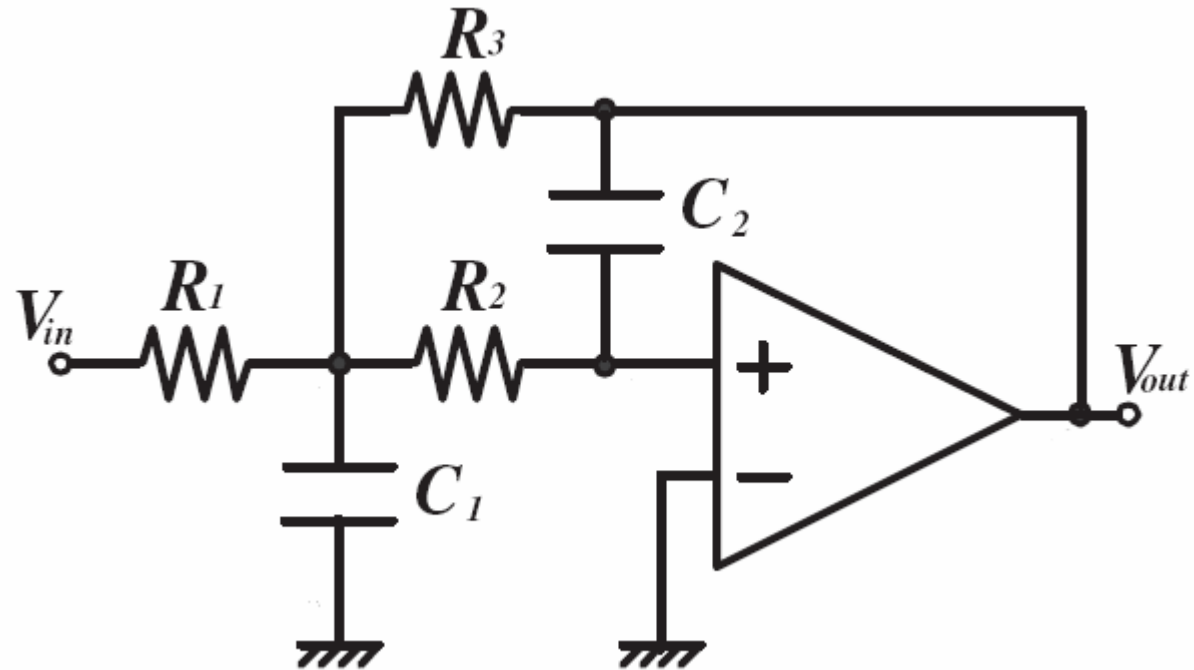
Filtro Passa Baixa



$$\frac{V_0}{V_i} = \frac{1}{s^2 + s \frac{1}{C_1} \left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} \right) + \frac{1}{R_2 R_3 C_1 C_2}}$$

$$H(s) = \frac{H_0}{s^2 + \frac{\omega_0}{Q} s + \omega_0^2}$$

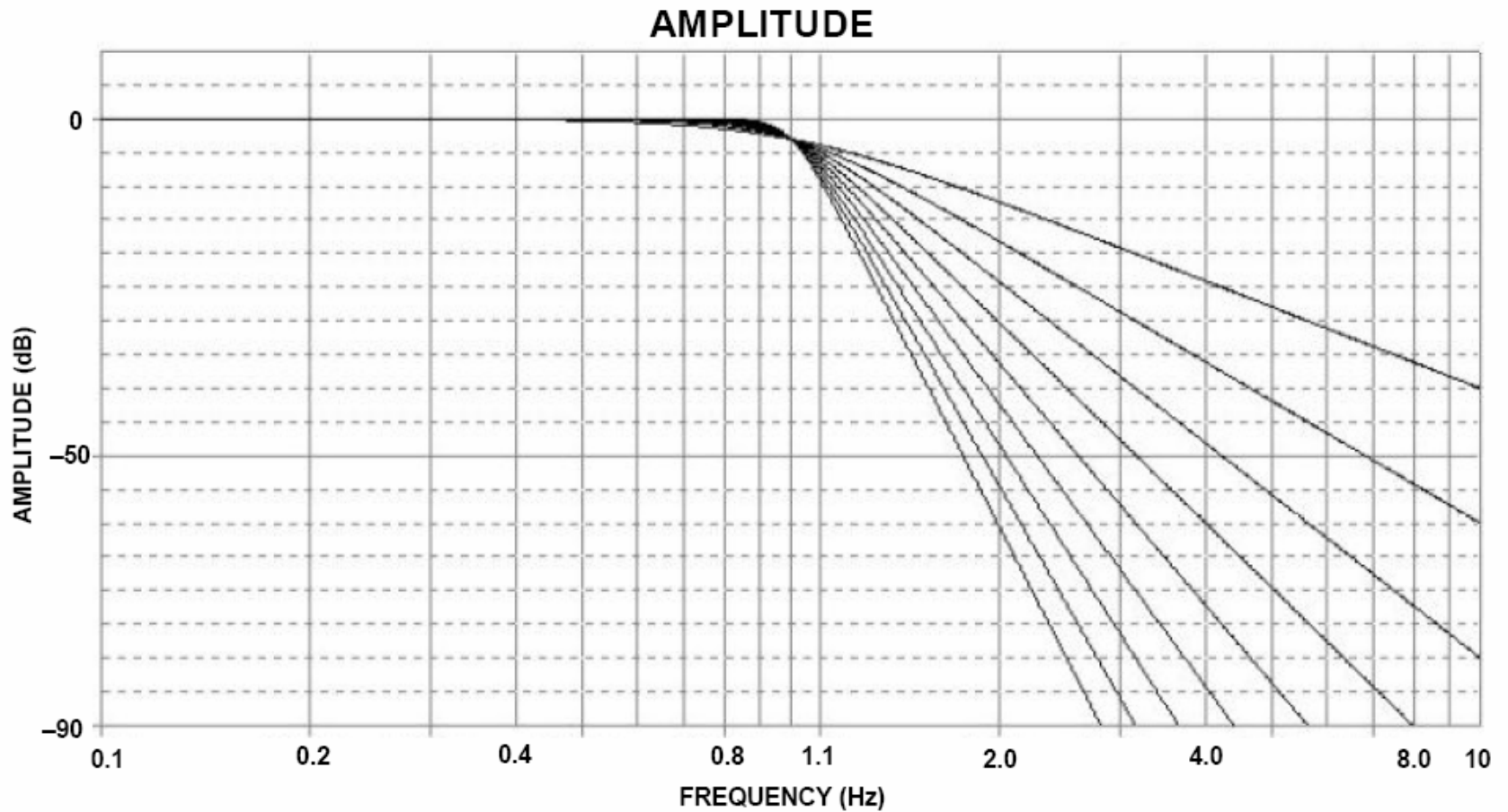
Filtro Passa Baixa



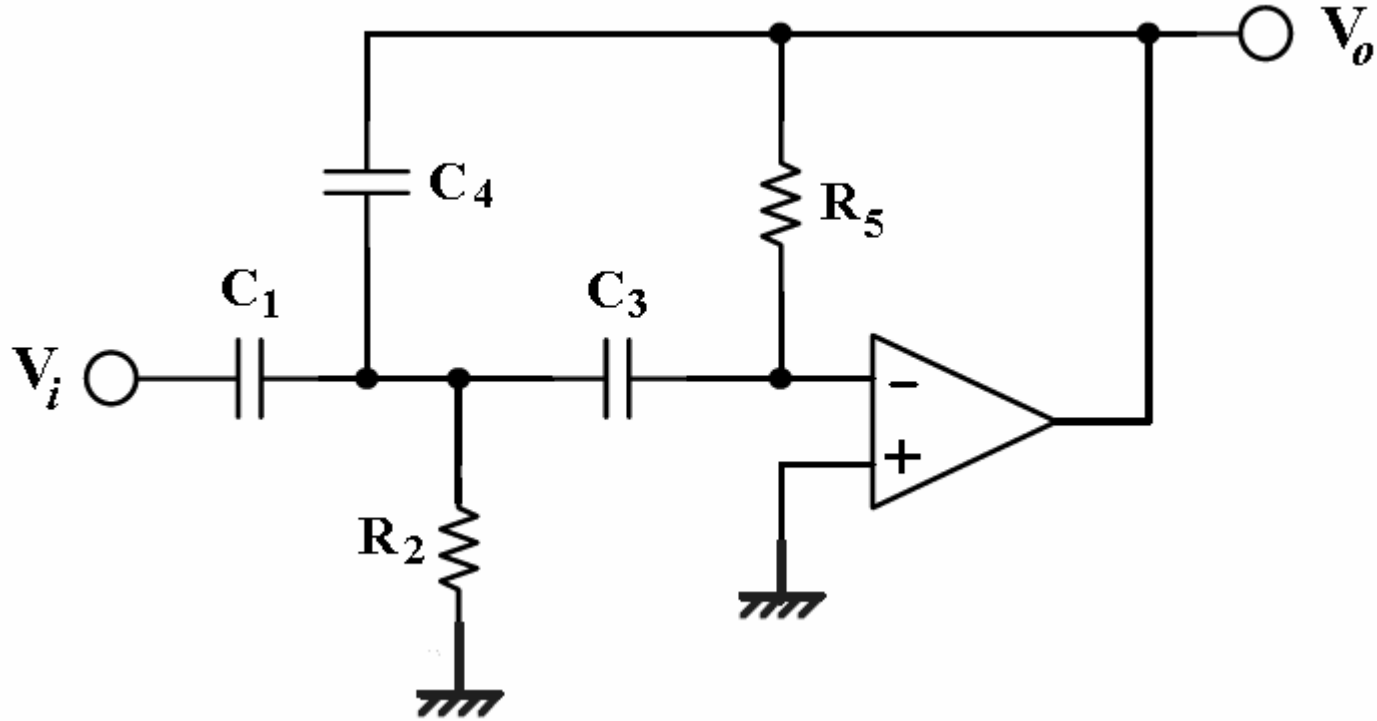
$$H(s) = \frac{H_0}{s^2 + \frac{\omega_0}{Q} s + \omega_0^2}$$

$$\omega_0 = \frac{1}{\sqrt{R_2 R_3 C_1 C_2}}$$

Ordem do Filtro Passa Baixa



Filtro Passa Alta



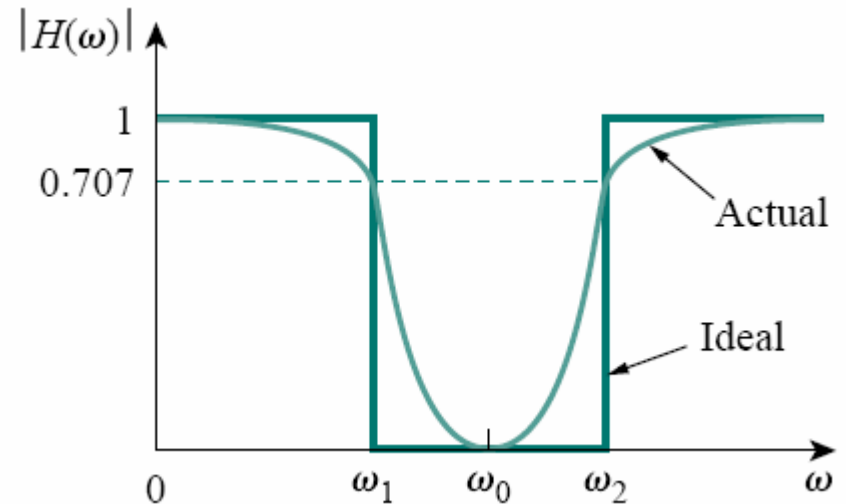
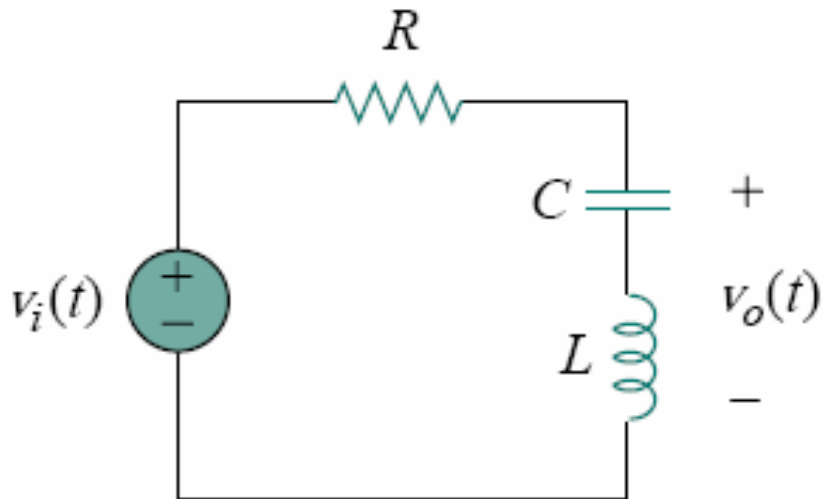
$$\frac{V_0}{V_i} = - \frac{\frac{C_1}{C_4} s^2}{s^2 + s \frac{1}{R_5} \left(\frac{1}{C_3} + \frac{1}{C_4} + \frac{C_1}{C_3 C_4} \right) + \frac{1}{R_2 R_5 C_3 C_4}}$$

$$H(s) = \frac{H_0 s^2}{s^2 + \frac{\omega_0}{Q} s + \omega_0^2}$$

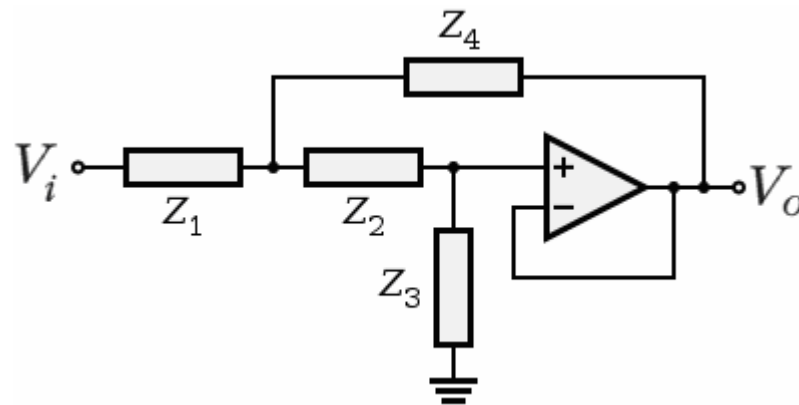
Filtro Rejeita Faixa

$$H(s) = \frac{H_0 (s^2 + \omega_z^2)}{s^2 + \frac{\omega_0}{Q} s + \omega_0^2}$$

$$H(\omega) = \frac{V_o}{V_i} = \frac{j(\omega L - 1/\omega C)}{R + j(\omega L - 1/\omega C)}$$

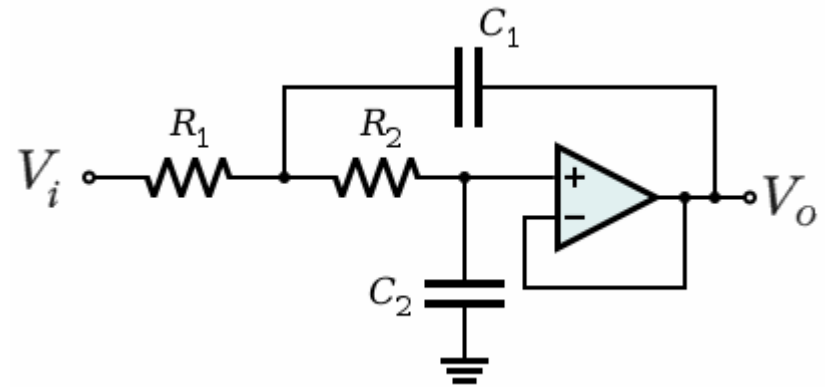
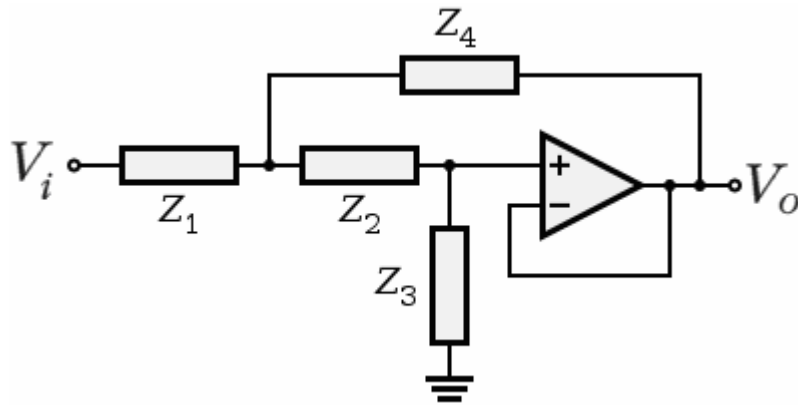


Topologia Sallen–Key



$$\frac{v_o}{v_i} = \frac{Z_3 Z_4}{Z_1 Z_2 + Z_4 (Z_1 + Z_2) + Z_3 Z_4}$$

Filtro Passa Baixa Sallen-Key

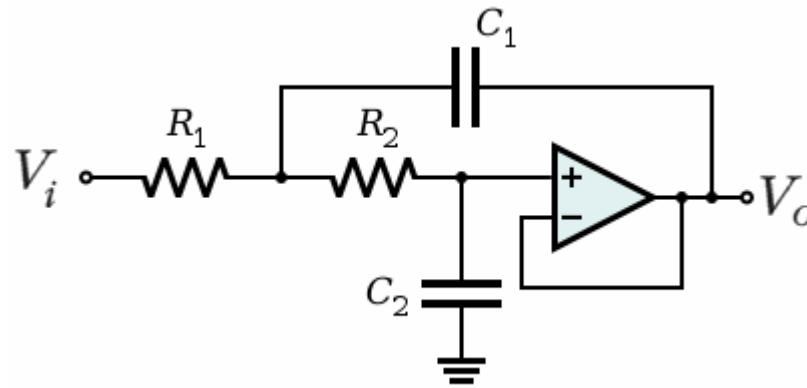


$$Z_1 = R_1, \quad Z_2 = R_2, \quad Z_3 = \frac{1}{sC_2}, \quad Z_4 = \frac{1}{sC_1}.$$

$$\frac{v_o}{v_i} = \frac{Z_3 Z_4}{Z_1 Z_2 + Z_4(Z_1 + Z_2) + Z_3 Z_4}$$

$$\frac{V_o(s)}{V_i(s)} = \frac{\frac{1}{R_1 C_1 R_2 C_2}}{s^2 + s \left(\frac{1}{R_2 C_1} + \frac{1}{R_1 C_1} \right) + \frac{1}{R_1 C_1 R_2 C_2}}$$

Filtro Passa Baixa Sallen-Key



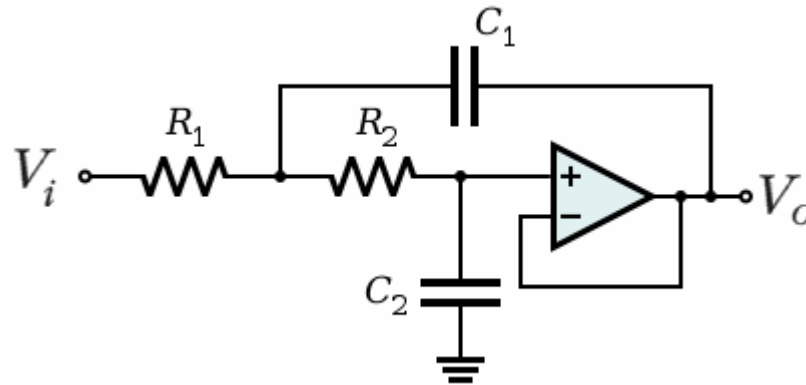
$$H(s) = \frac{\frac{1}{R_1 R_2 C_1 C_2}}{s^2 + s \frac{1}{C_1} \left(\frac{1}{R_1} + \frac{1}{R_2} \right) + \frac{1}{R_1 R_2 C_1 C_2}}$$

$$H(s) = \frac{\omega_0^2}{s^2 + s \frac{\omega_0}{Q} + \omega_0^2}$$

$$\omega_0 = \frac{1}{\sqrt{R_1 R_2 C_1 C_2}}$$

$$Q = \sqrt{\frac{C_1}{C_2}} \frac{\sqrt{R_1 R_2}}{R_1 + R_2}$$

Filtro Passa Baixa Sallen-Key

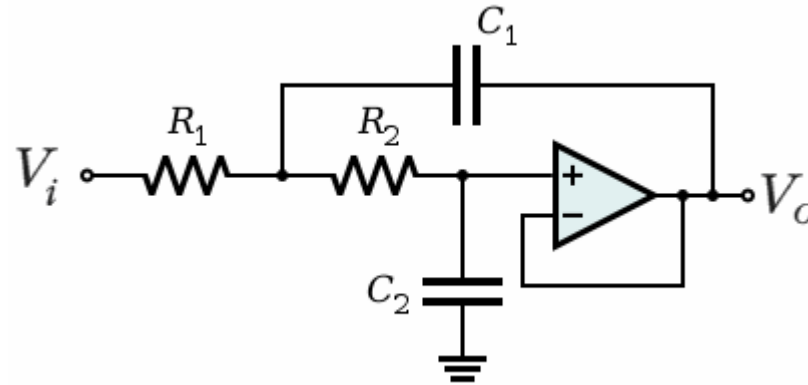


$$Z_1 = R_1, \quad Z_2 = R_2, \quad Z_3 = \frac{1}{sC_2}, \quad Z_4 = \frac{1}{sC_1}.$$

$$\frac{v_o}{v_i} = \frac{Z_3 Z_4}{Z_1 Z_2 + Z_4(Z_1 + Z_2) + Z_3 Z_4}$$

$$\frac{V_o(s)}{V_i(s)} = \frac{\frac{1}{R_1 C_1 R_2 C_2}}{s^2 + s \left(\frac{1}{R_2 C_1} + \frac{1}{R_1 C_1} \right) + \frac{1}{R_1 C_1 R_2 C_2}}$$

Filtro Passa Baixa Sallen-Key



$$f_c = \frac{1}{2\pi\sqrt{R_1 R_2 C_1 C_2}}$$

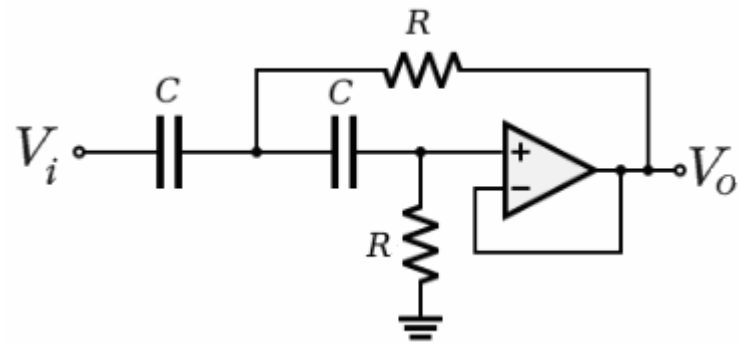
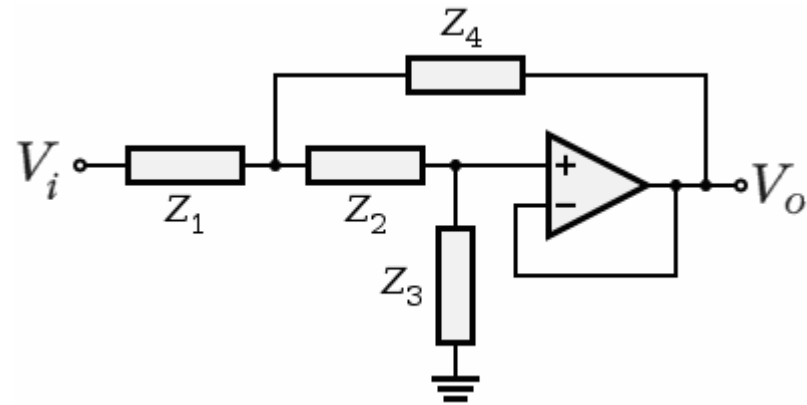
$$\frac{V_o(s)}{V_i(s)} = \frac{\frac{1}{R_1 C_1 R_2 C_2}}{s^2 + s\left(\frac{1}{R_2 C_1} + \frac{1}{R_1 C_1}\right) + \frac{1}{R_1 C_1 R_2 C_2}}$$

$$\frac{1}{Q} = \frac{\sqrt{R_1 R_2 C_1 C_2}}{C_1} \left(\frac{1}{R_1} + \frac{1}{R_2} \right).$$

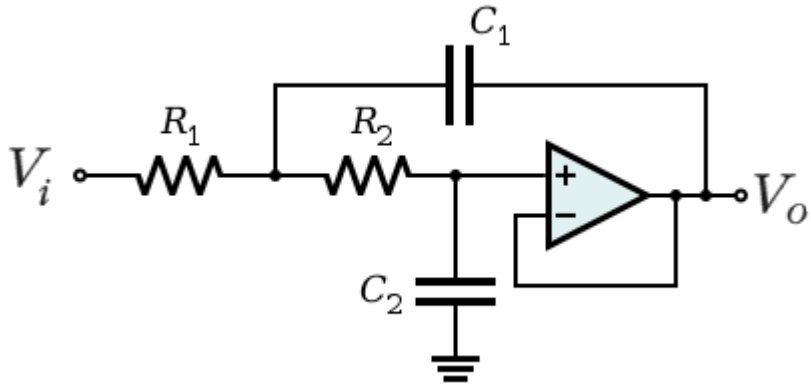
$$Q = \frac{\sqrt{R_1 R_2 C_1 C_2}}{C_2 (R_1 + R_2)}$$

$$\frac{f_c}{Q} = \frac{R_1 + R_2}{2\pi C_1 R_1 R_2}$$

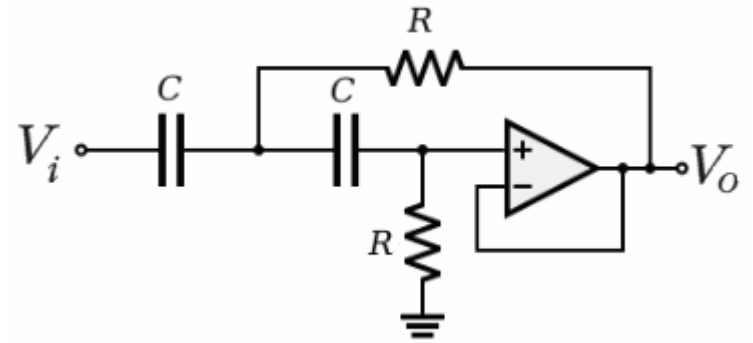
Filtro Passa Alta Sallen-Key



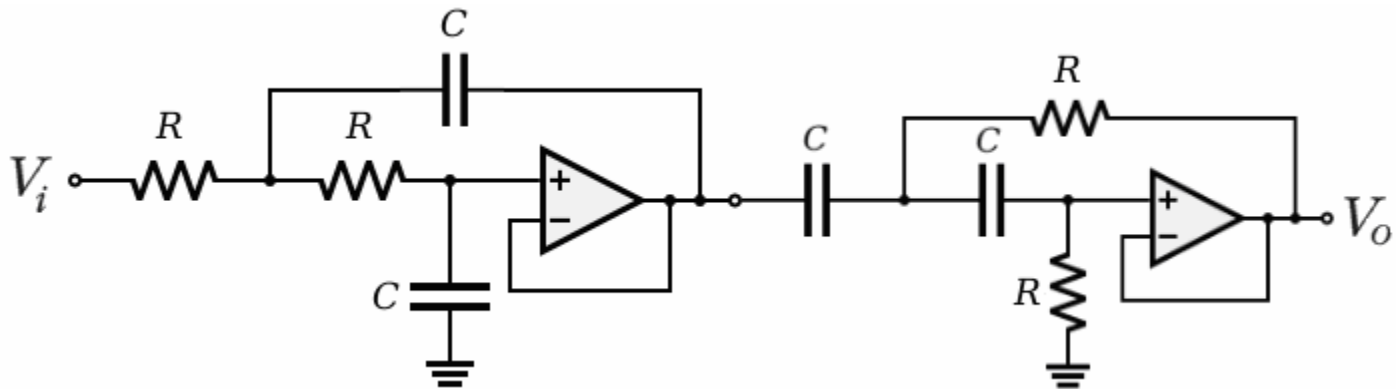
PASSA BAIXA



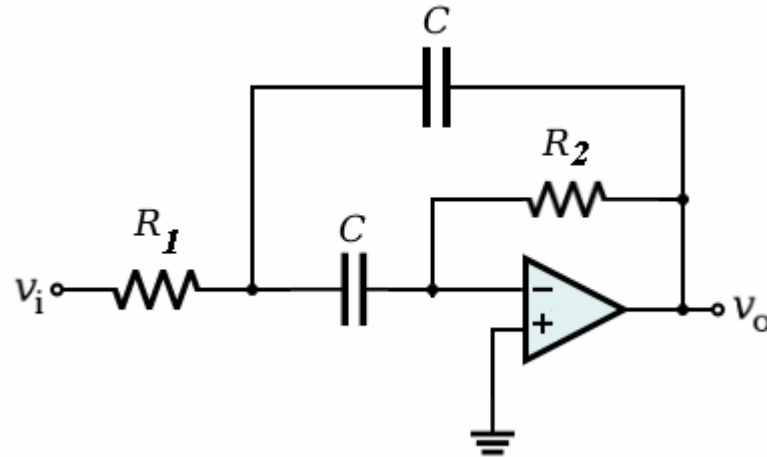
PASSA ALTA



PASSA FAIXA



Filtro Passa Faixa



$$H(s) = - \frac{s \frac{1}{R_1 C}}{s^2 + \frac{2}{R_2 C} s + \frac{1}{R_1 R_2 C^2}}$$

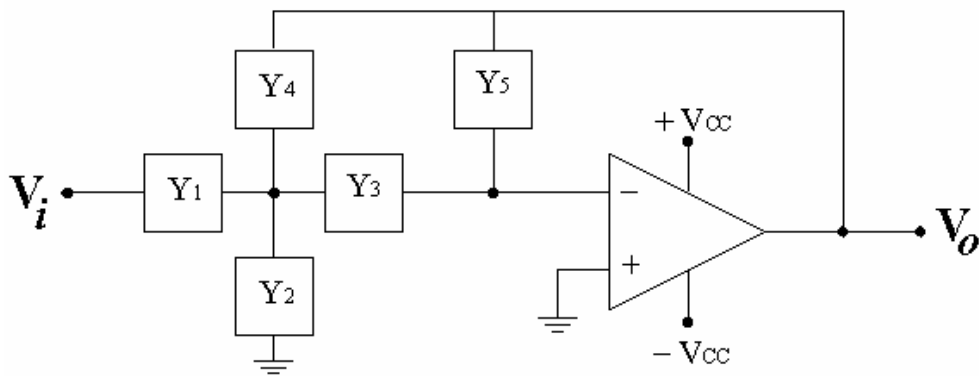
$$\frac{V_o}{V_i} = H_0 \frac{\frac{\omega_0}{Q} s}{s^2 + \frac{\omega_0}{Q} s + \omega_0^2}$$

$$H(s) = - \frac{R_2}{2R_1} \frac{s \frac{2}{R_2 C}}{s^2 + \frac{2}{R_2 C} s + \frac{1}{R_1 R_2 C^2}}$$

$$H_0 = - \frac{R_2}{2R_1} \quad \omega_0 = \frac{1}{C \sqrt{R_1 R_2}} \quad Q = \frac{1}{2} \sqrt{\frac{R_2}{R_1}}$$

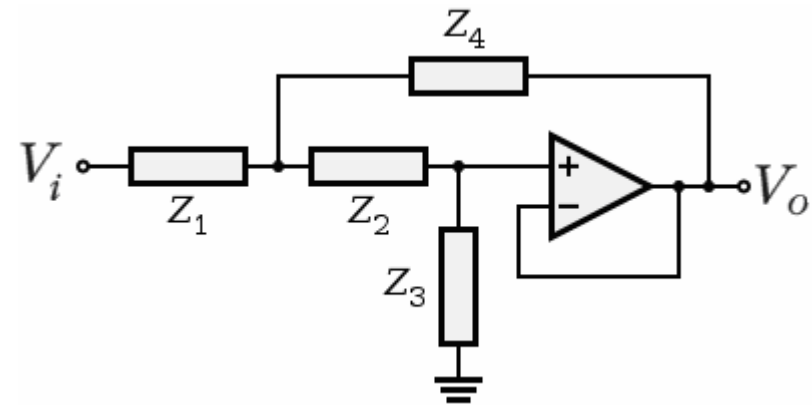
Filtros Ativos

Multipla Realimentação



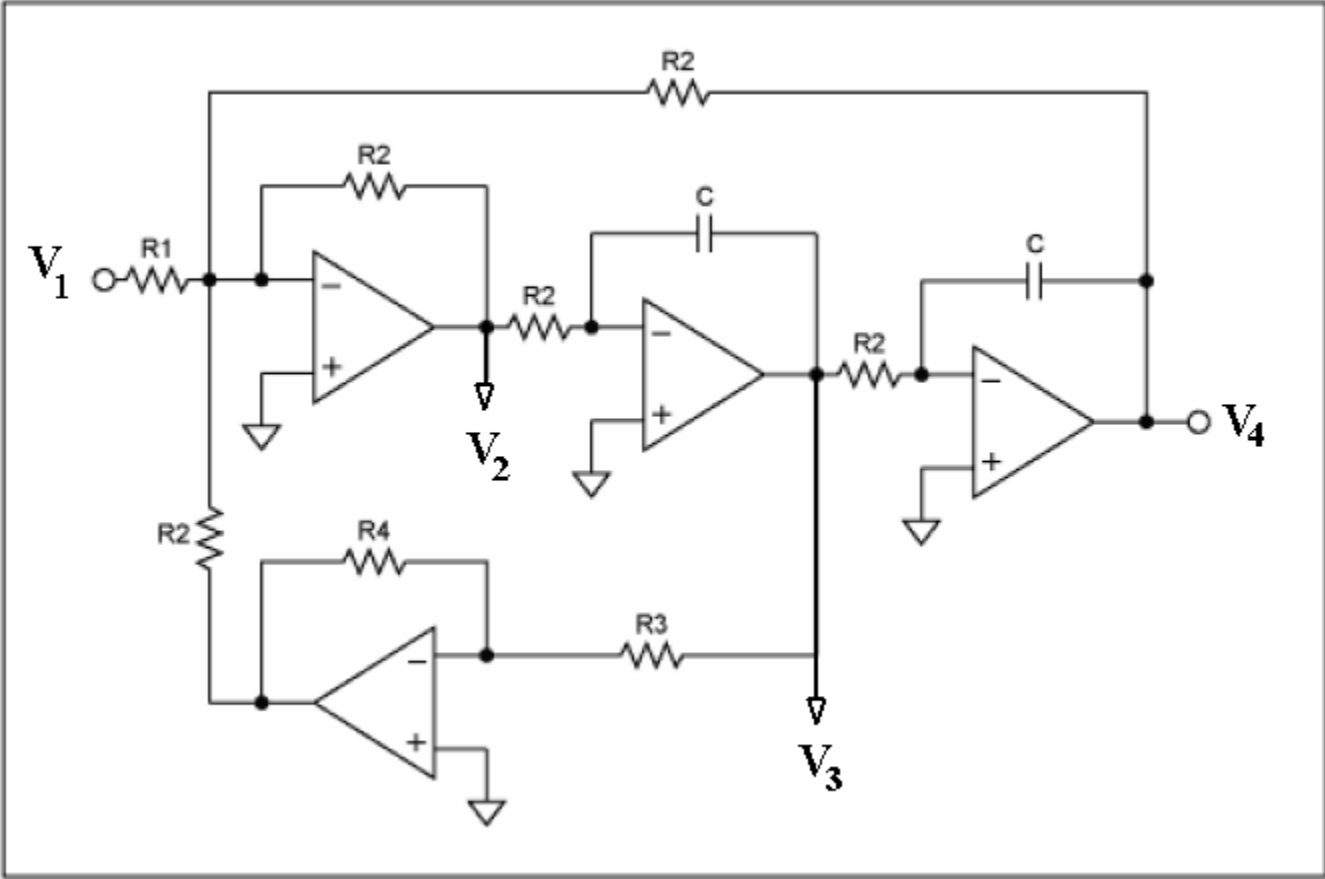
$$\frac{V_o}{V_i} = -\frac{Y_1 Y_3}{Y_5(Y_1 + Y_2 + Y_3 + Y_4) + Y_3 Y_4}$$

Sallen-Key



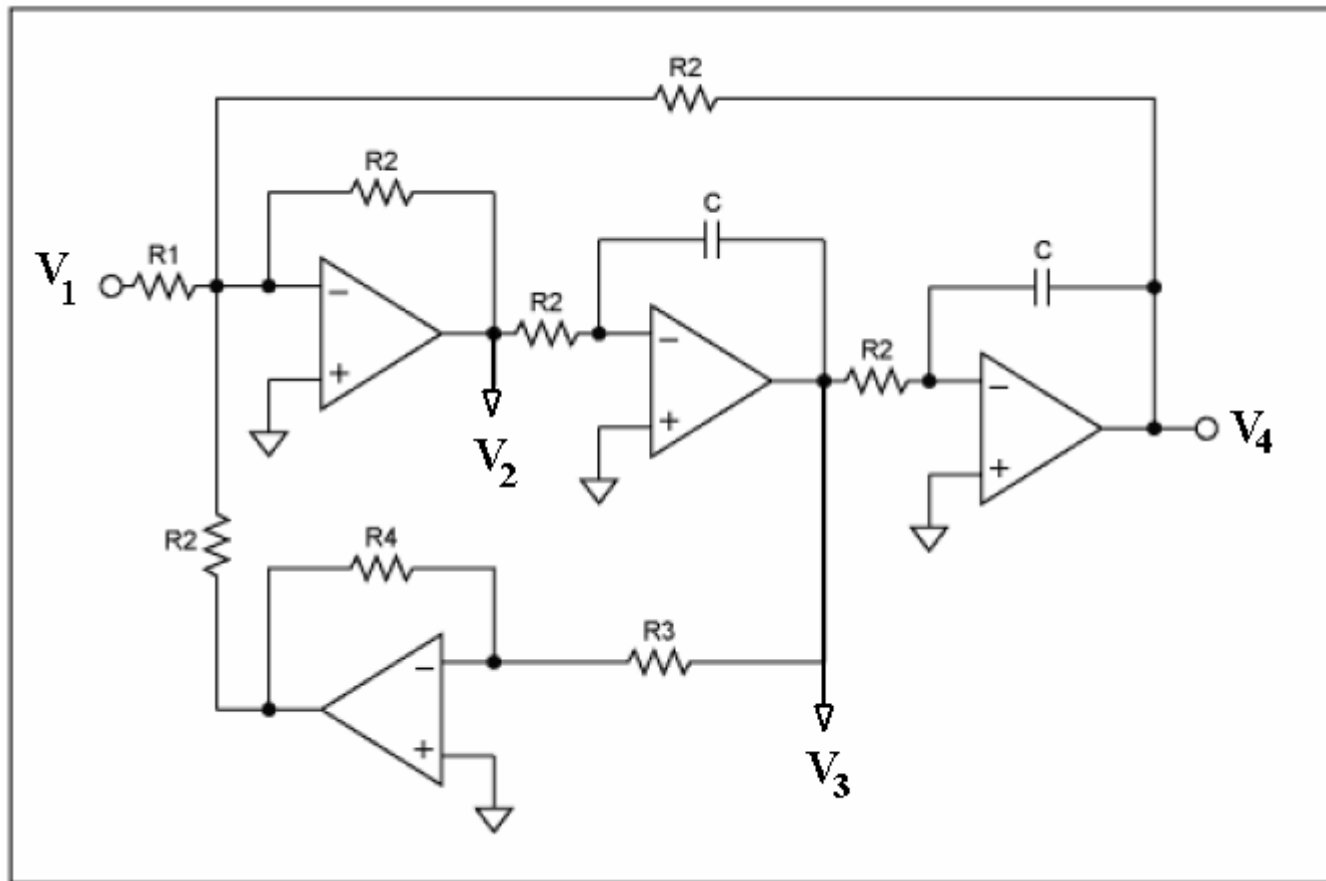
$$\frac{v_o}{v_i} = \frac{Z_3 Z_4}{Z_1 Z_2 + Z_4(Z_1 + Z_2) + Z_3 Z_4}$$

Filtro Ativo Universal

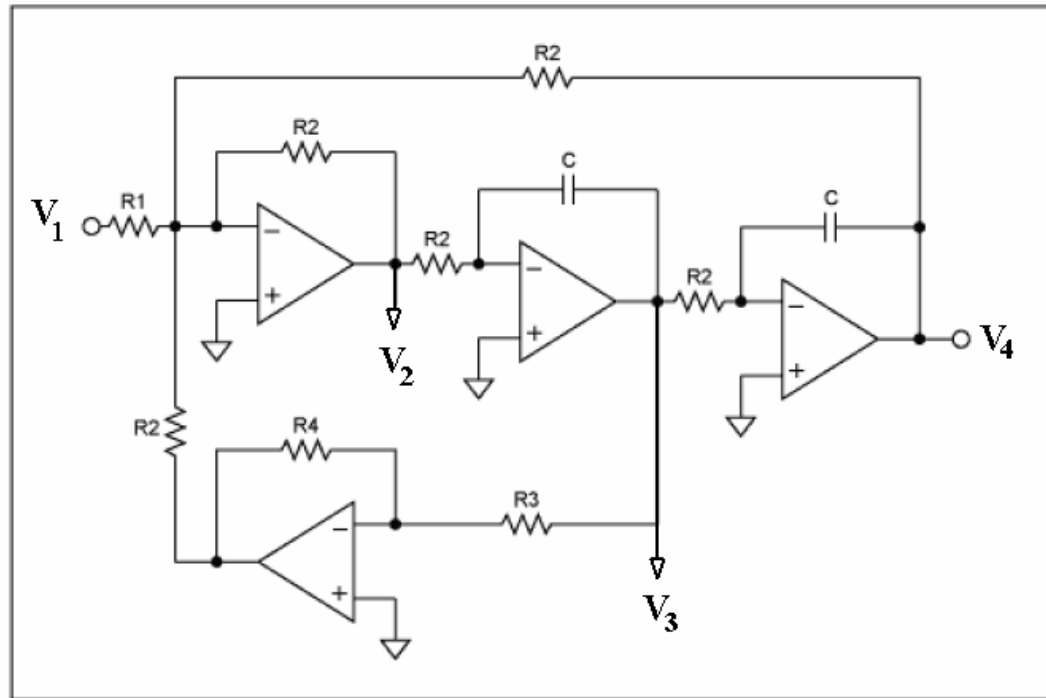


Filtro Ativo Universal *State Variable Filter*

Kerwin-Huelsman-Newcomb (KHN)



Filtro Ativo Universal Segunda Ordem



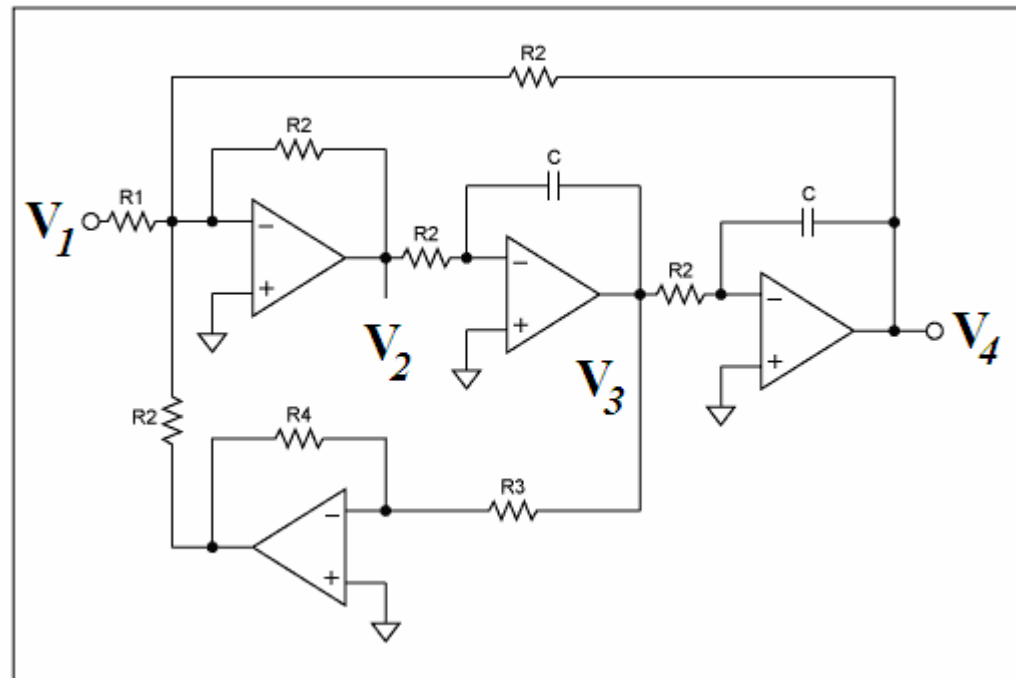
$V_2 \rightarrow$ *Filtro Passa Alta*

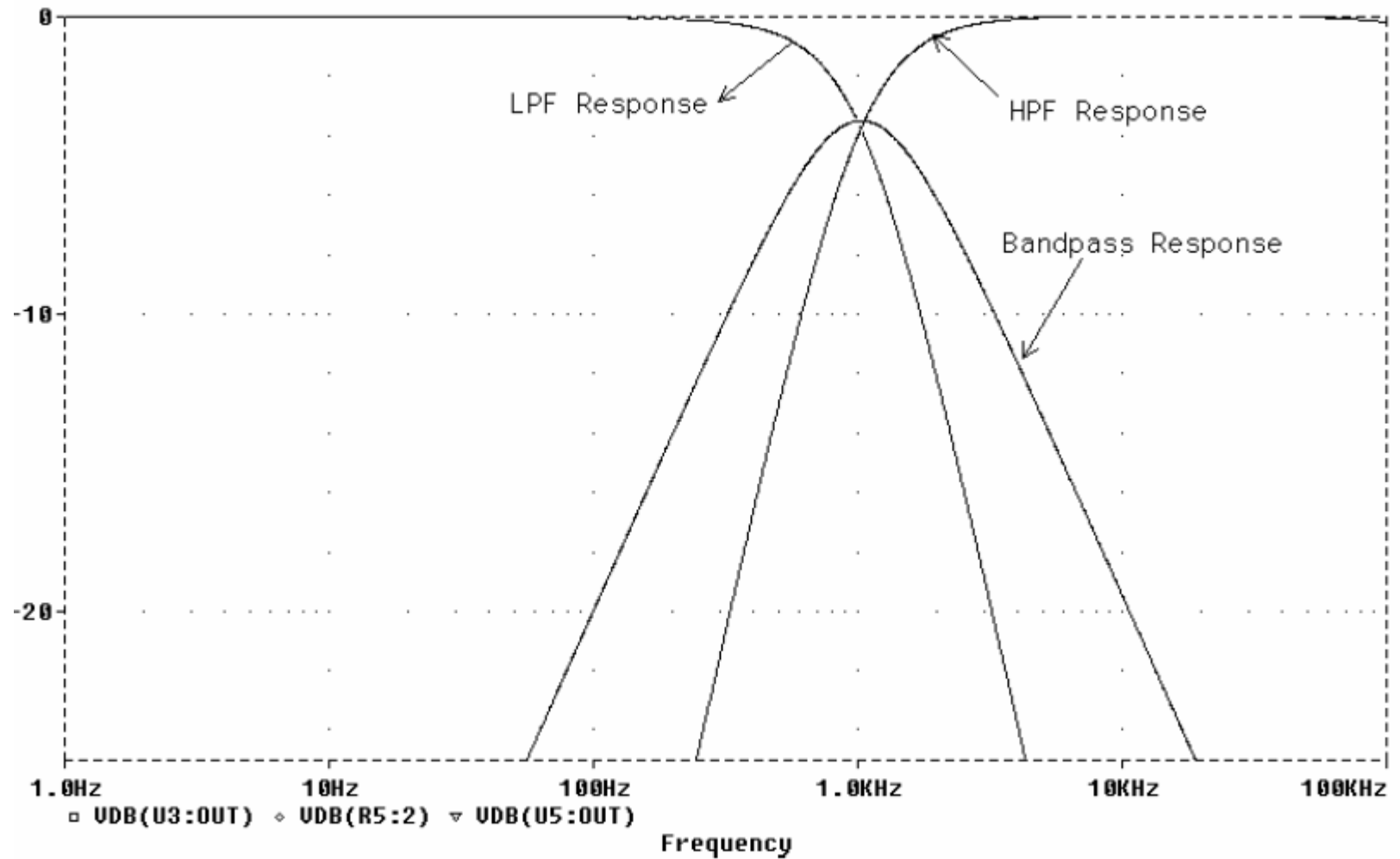
$V_3 \rightarrow$ *Filtro Passa Faixa*

$V_4 \rightarrow$ *Filtro Passa Baixa*

Filtro Ativo Universal

Produz simultaneamente saídas Passa-Baixa, Passa-Alta e Passa-Faixa a partir de uma simples entrada

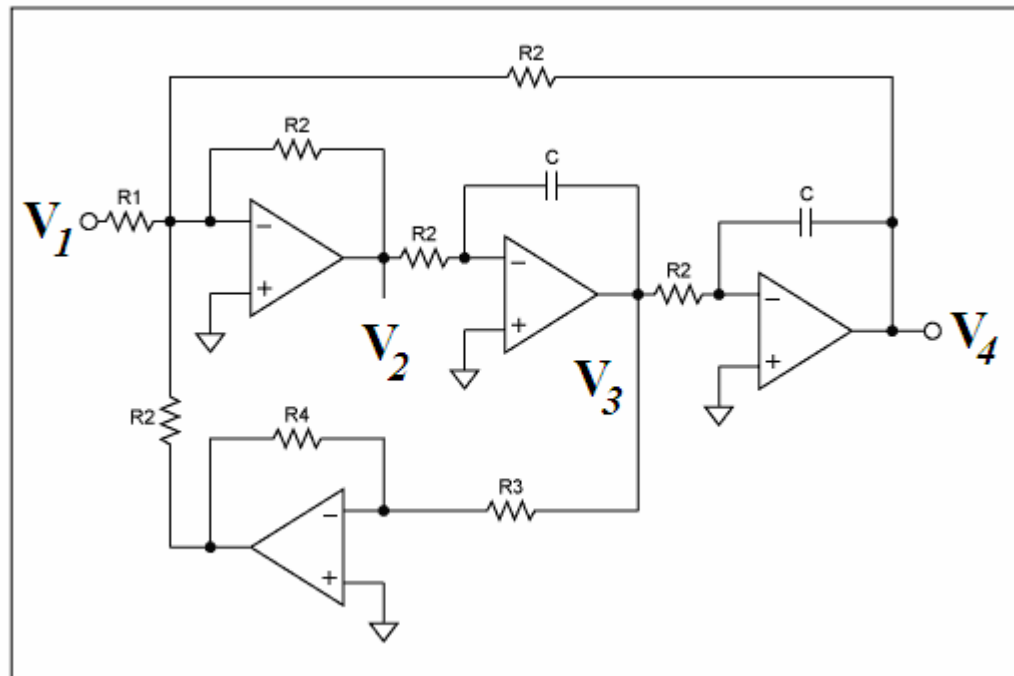




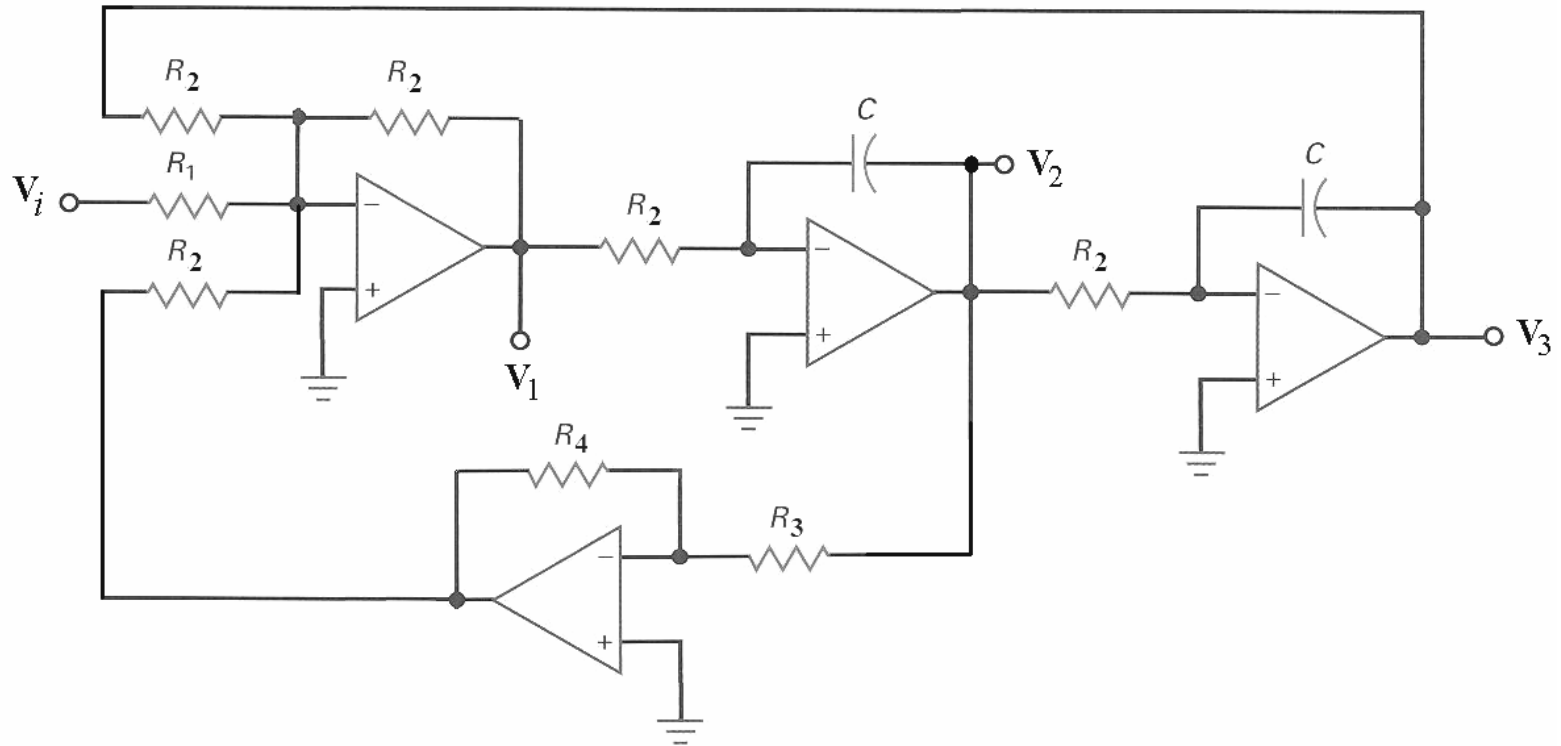
Frequency response of a state variable filter.

Filtro Ativo Universal

Além de produzir simultaneamente saídas Passa-Baixa, Passa-Alta e Passa-Faixa permite o Controle independente dos principais parâmetros do filtro: frequência de corte, fator de qualidade Q e Ganho.



Filtro Ativo Universal



Passa ALTA

$$\frac{V_1}{V_i} = - \frac{\frac{R_2}{R_1} s^2}{s^2 + \frac{R_4}{R_2 R_3 C} s + \left(\frac{1}{R_2 C} \right)^2}$$

Passa FAIXA

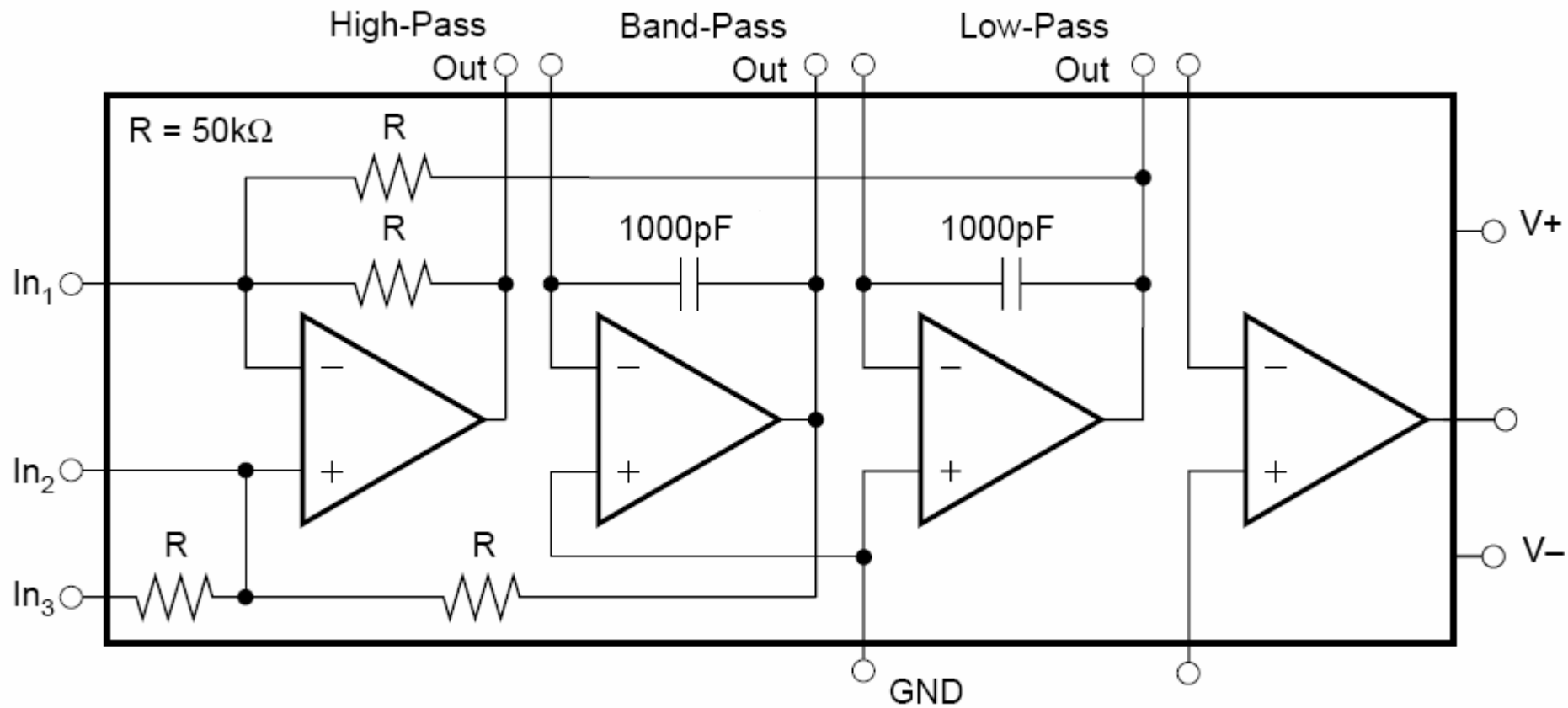
$$\frac{V_2}{V_i} = \frac{\frac{1}{R_1 C} s}{s^2 + \frac{R_4}{R_2 R_3 C} s + \left(\frac{1}{R_2 C} \right)^2}$$

Passa BAIXA

$$\frac{V_3}{V_i} = - \frac{\frac{1}{R_1 R_2 C^2}}{s^2 + \frac{R_4}{R_2 R_3 C} s + \left(\frac{1}{R_2 C} \right)^2}$$

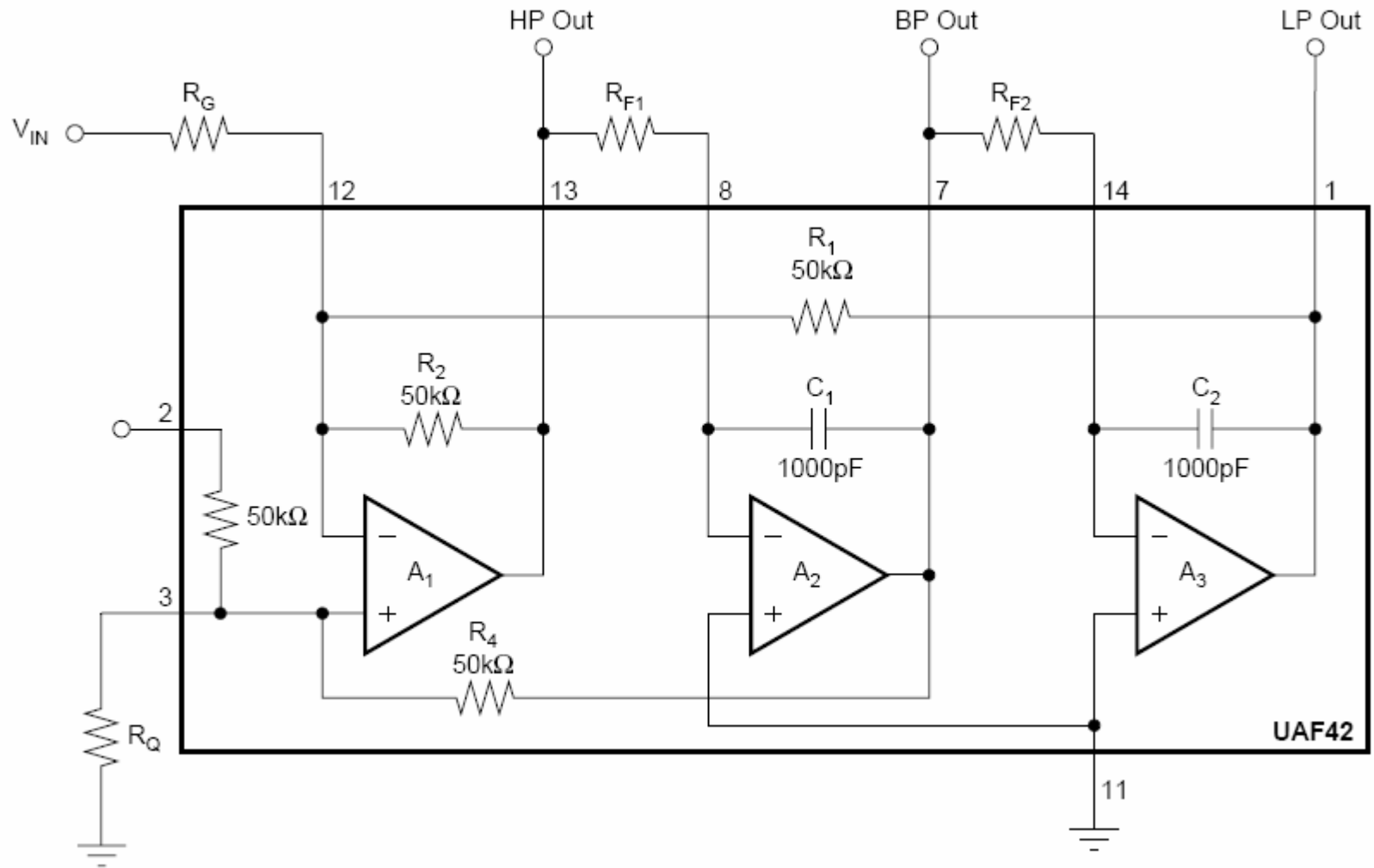
UNIVERSAL ACTIVE FILTER

UAF42



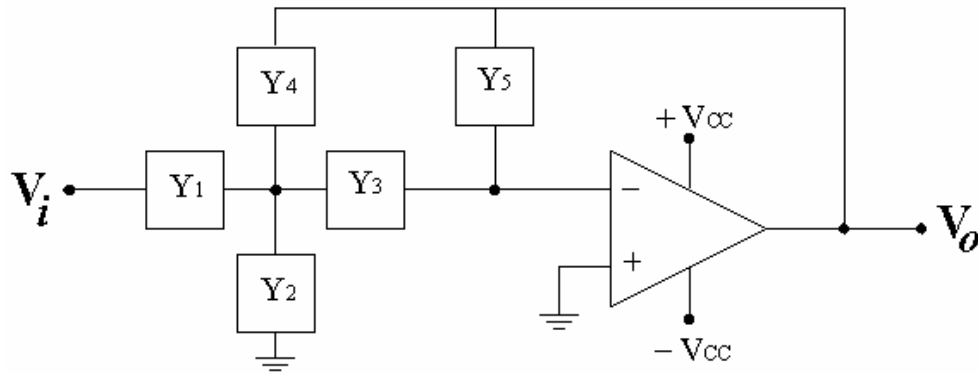
UNIVERSAL ACTIVE FILTER

UAF42



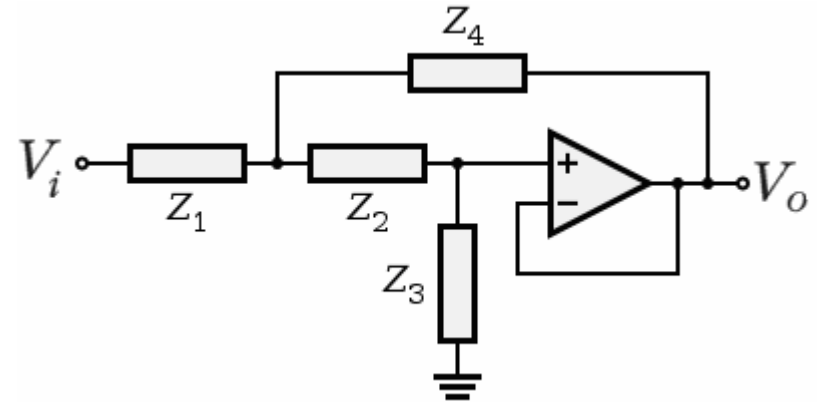
Topologias de Filtros Ativos

Multipla Realimentação



$$\frac{V_0}{V_i} = - \frac{Y_1 Y_3}{Y_5 (Y_1 + Y_2 + Y_3 + Y_4) + Y_3 Y_4}$$

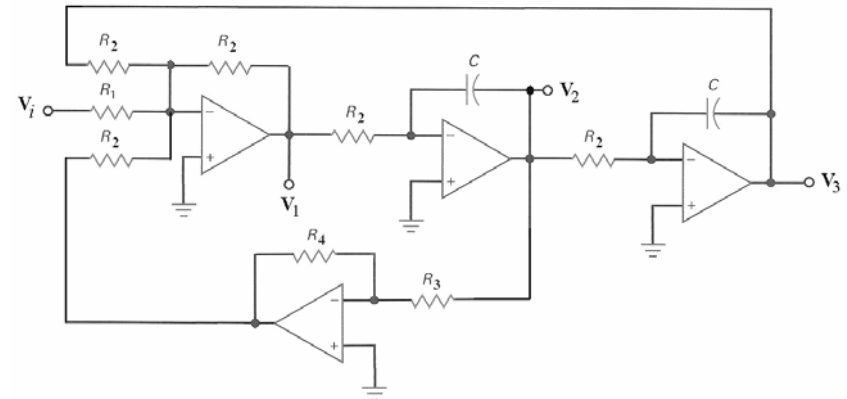
Sallen-Key



$$\frac{v_o}{v_i} = \frac{Z_3 Z_4}{Z_1 Z_2 + Z_4 (Z_1 + Z_2) + Z_3 Z_4}$$

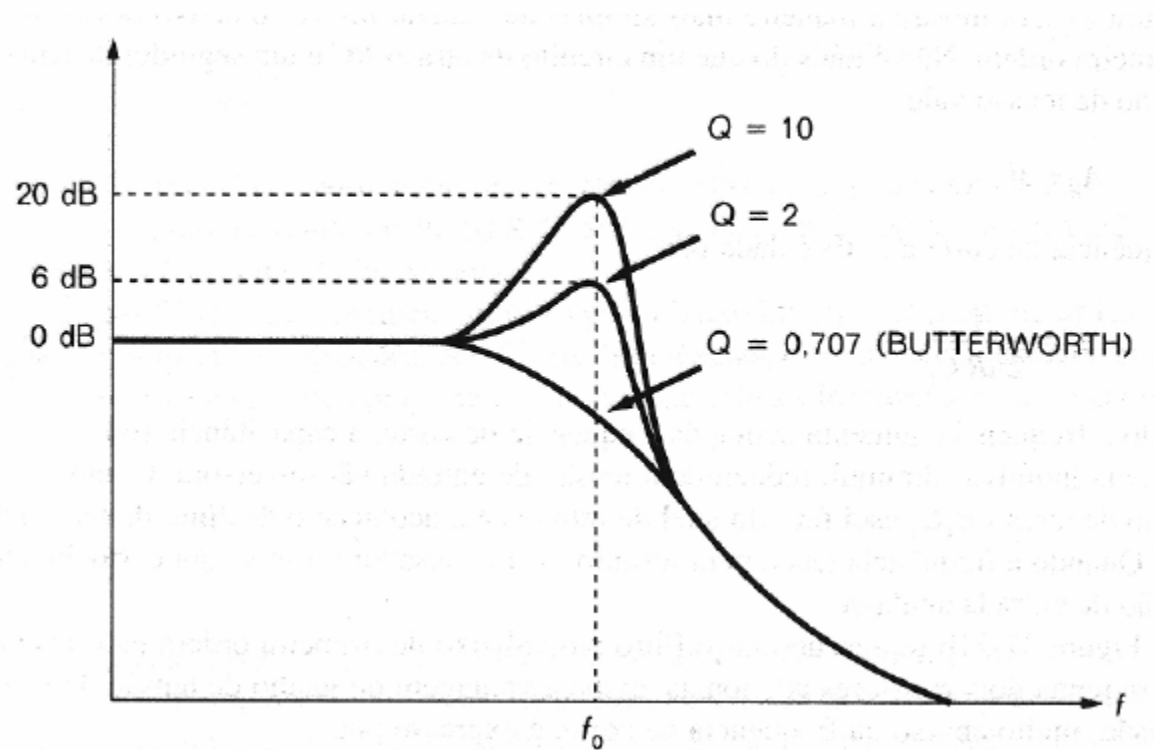
Filtro Universal

$$\frac{V_1}{V_i} = - \frac{\frac{R_2}{R_1} s^2}{s^2 + \frac{R_4}{R_2 R_3 C} s + \left(\frac{1}{R_2 C} \right)^2}$$

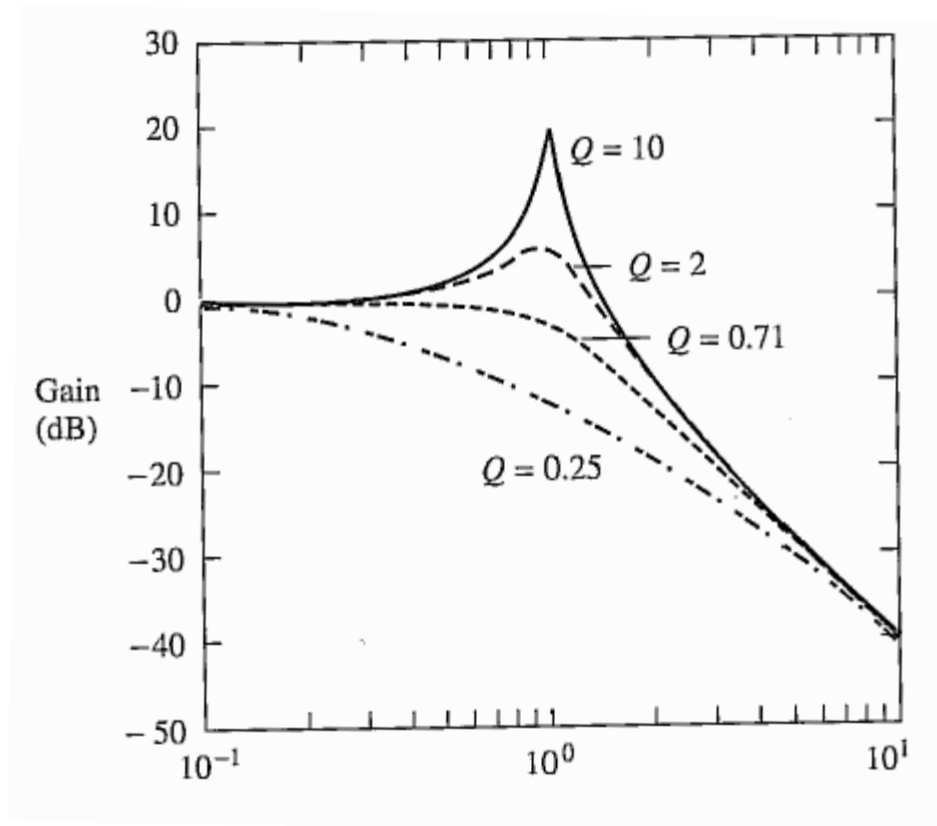


Efeito do fator de qualidade Q na resposta em frequência de um filtro passa baixa de segunda ordem.

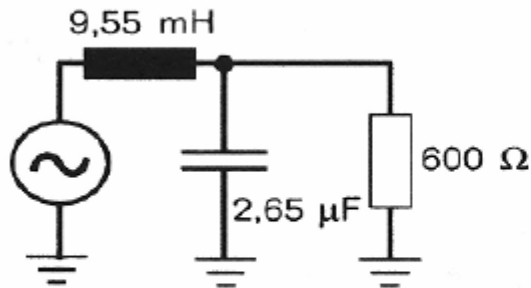
A figura seguinte resume o efeito de Q num filtro de 2ª ordem:



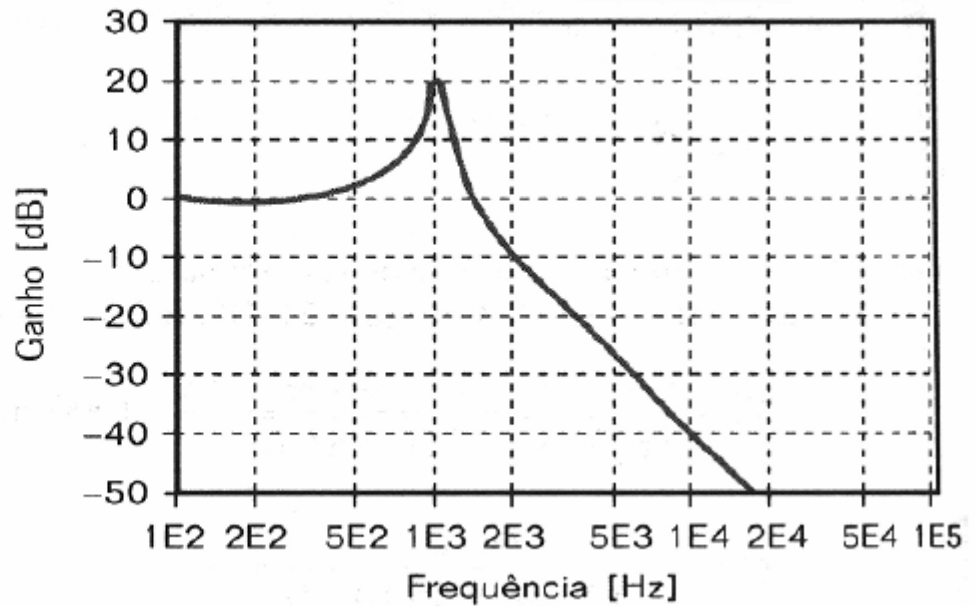
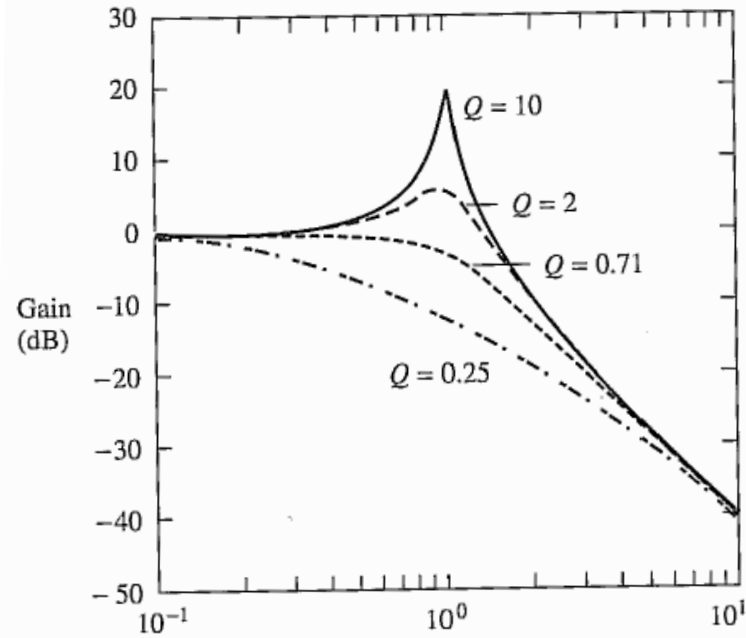
Efeito do fator de qualidade Q na resposta em frequência de um filtro passa baixa de segunda ordem.



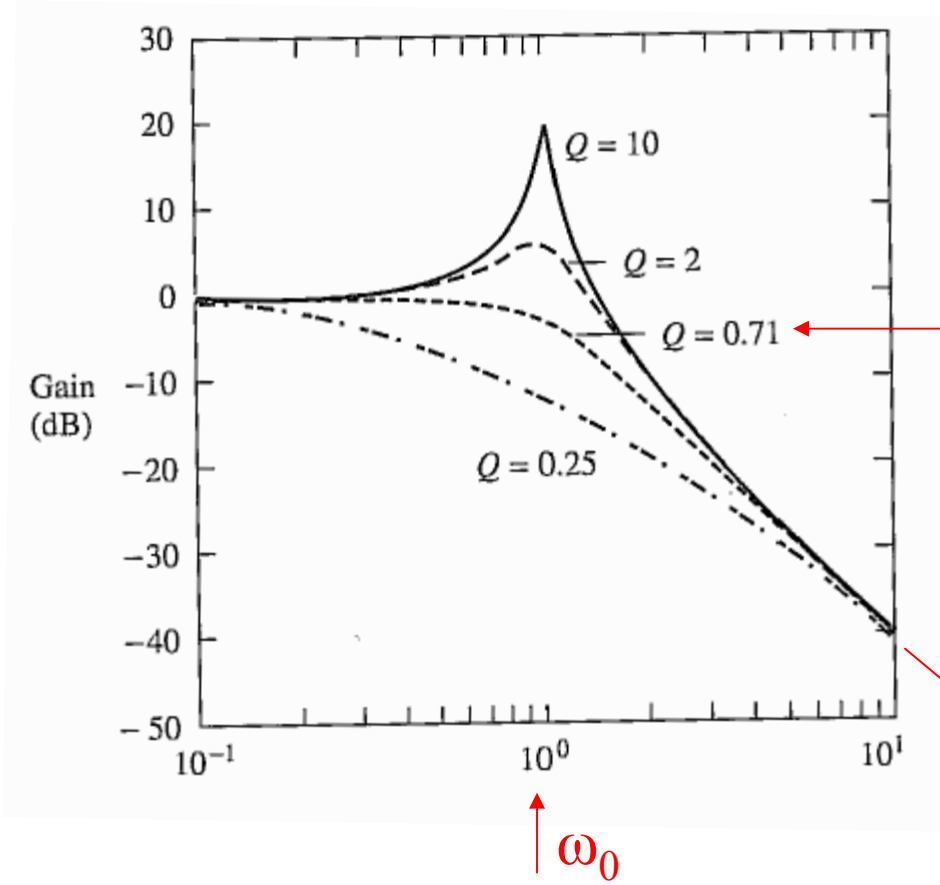
Efeito do fator de qualidade Q na resposta em frequência de um filtro passa baixa de segunda ordem.



$$f_0 = 1\text{kHz}, Q = 10$$



Filtro passa baixa de segunda ordem



$Q = 1/\sqrt{2} = 0.707$
called “Butterworth” –
RMS deviation from ideal
is minimum

High frequency roll-off
of 40db/decade since
2nd order low-pass filter