

# Slides Aulas de Eletrônica

## **Observação importante:**

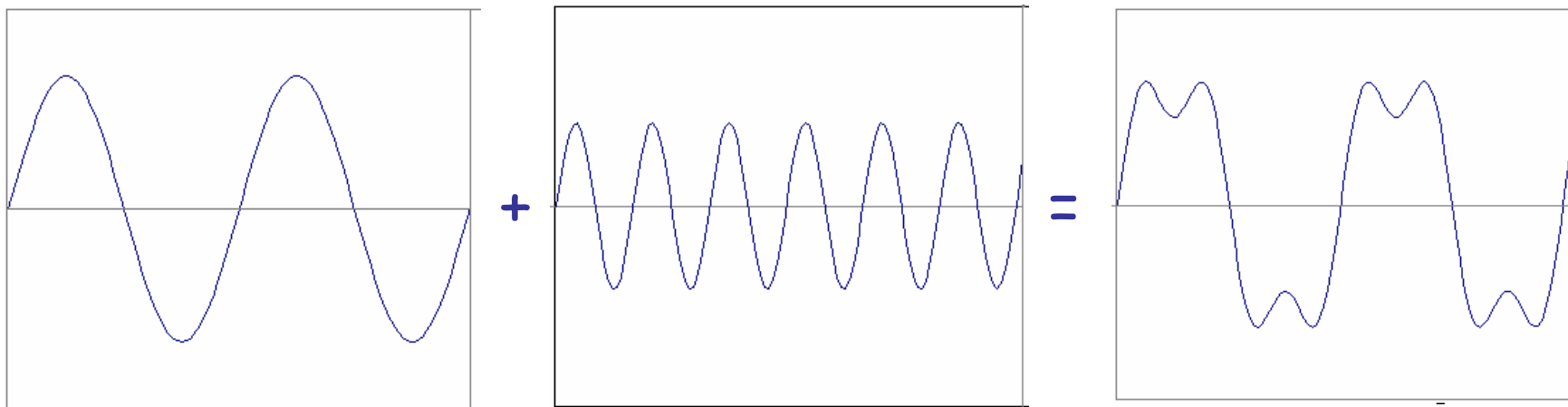
***Os slides aqui apresentados não refletem todo o conteúdo abordado em sala de aula. Muitos exercícios e detalhamento da teoria, expostos na aula presencial, não estão contemplados nestes slides. Portanto, considere-o apenas como material de referência parcial a ser complementado com o auxílio de livros, apostilas e literaturas afins. Material auxiliar está disponibilizado no site da disciplina através de textos e links.***

# ELETRÔNICA

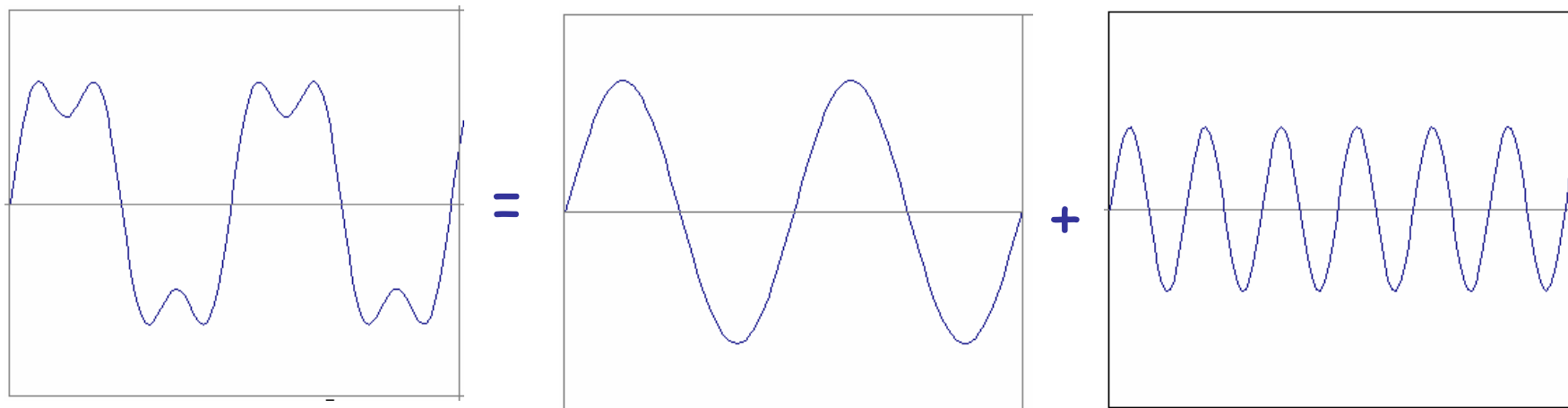
**Filtros Elétricos**  
*Ativos e Passivos*

AOC

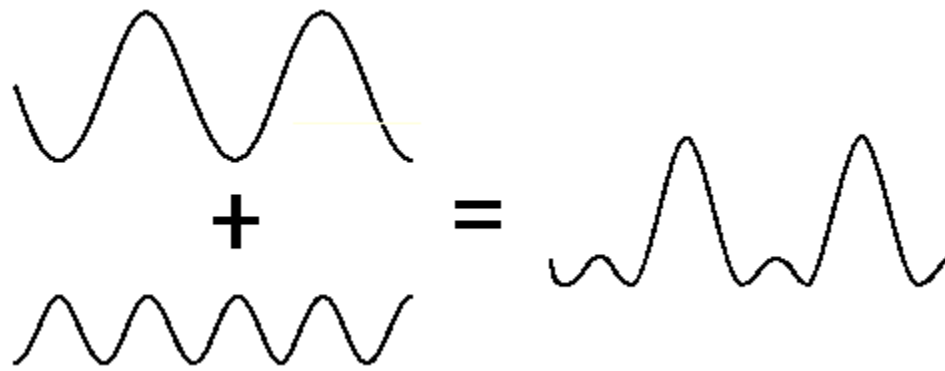
*Síntese* → *Composição*

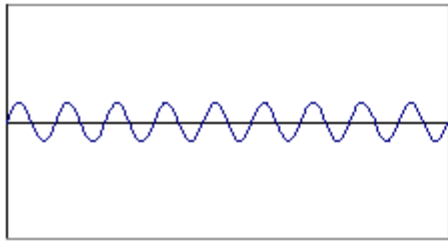
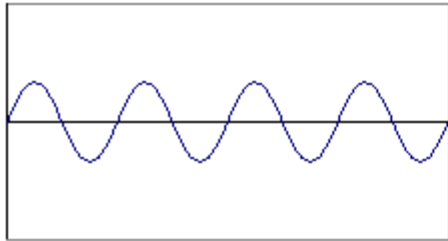
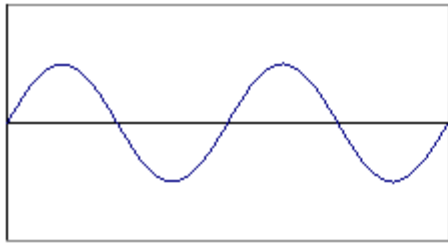


*Análise* → *Decomposição*

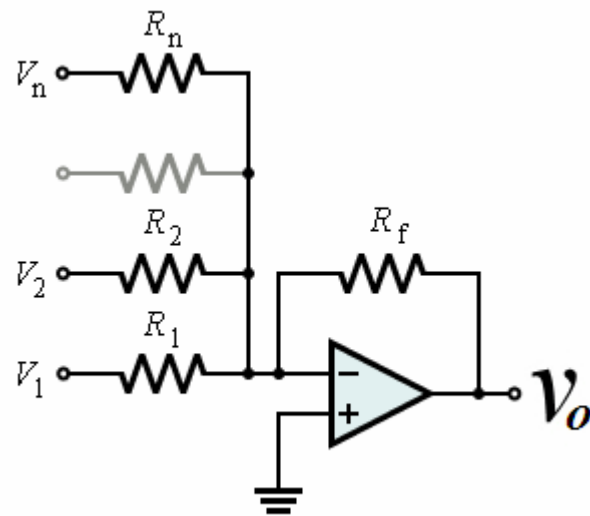
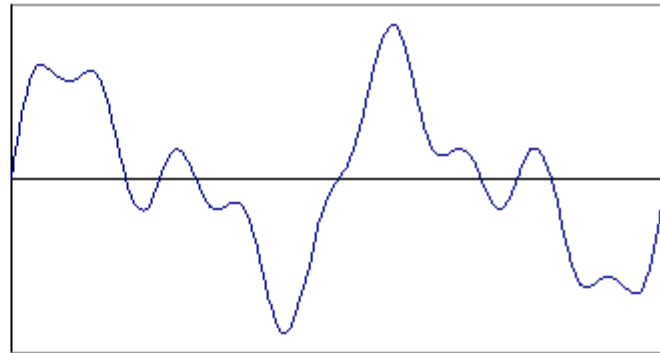


## *Combinando ondas de frequências diferentes*





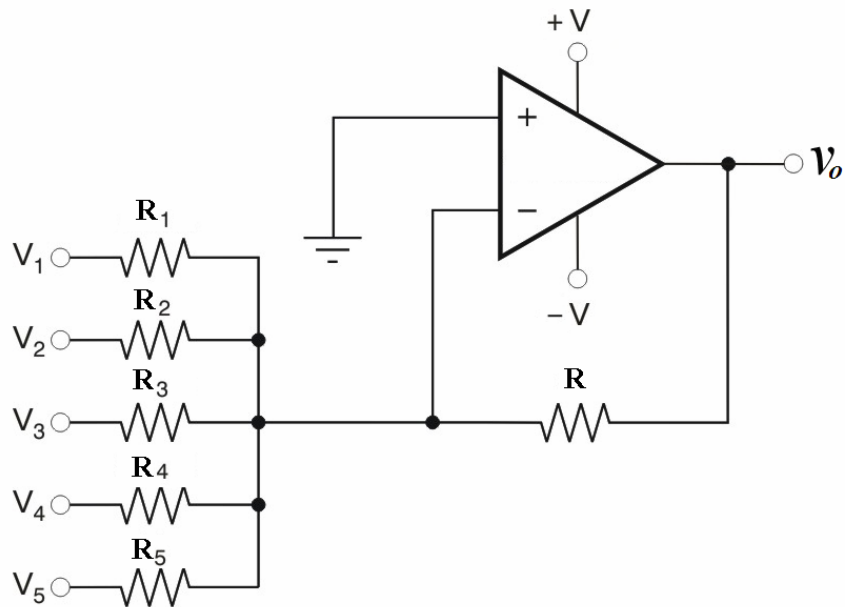
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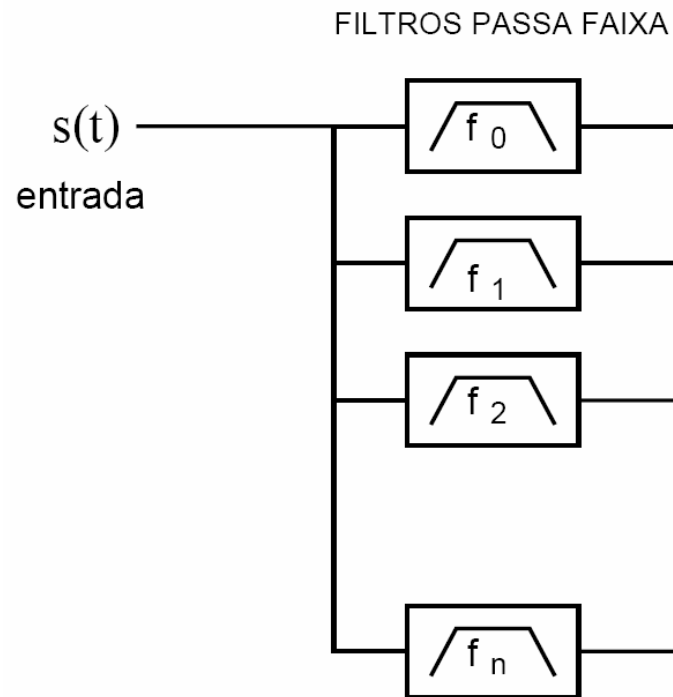


*Síntese Espectral*  
*x*  
*Análise Espectral*

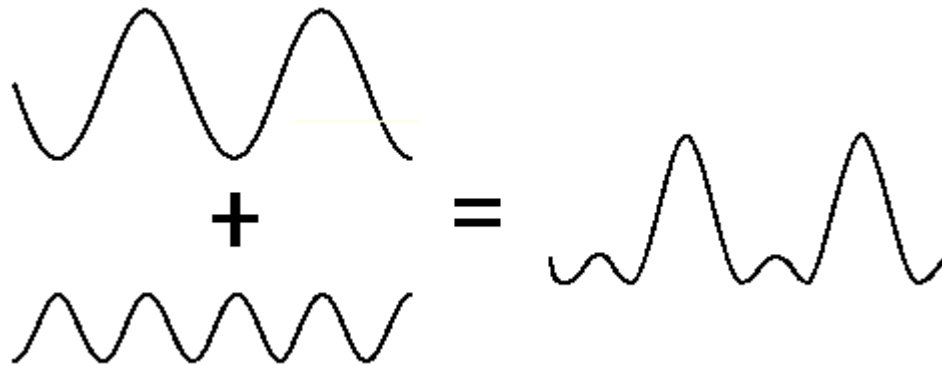
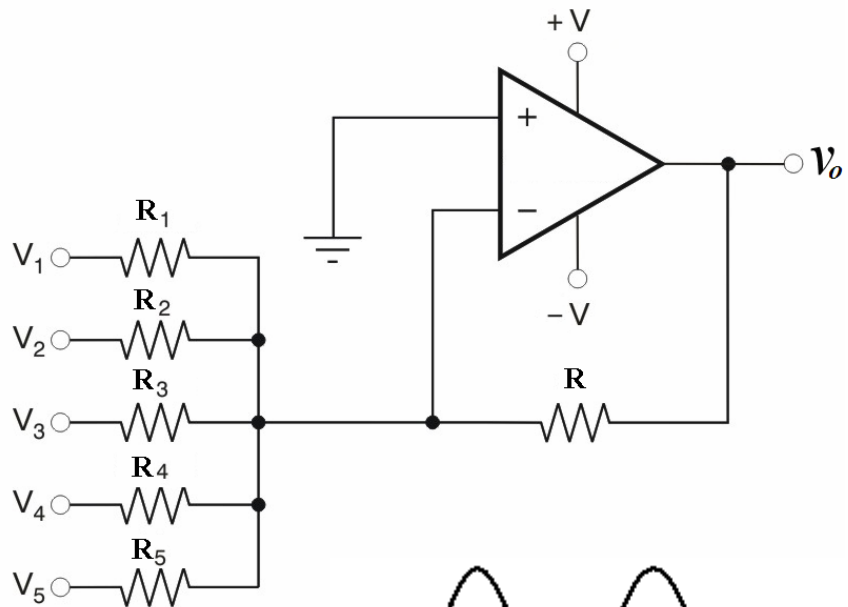
# *Síntese Espectral*



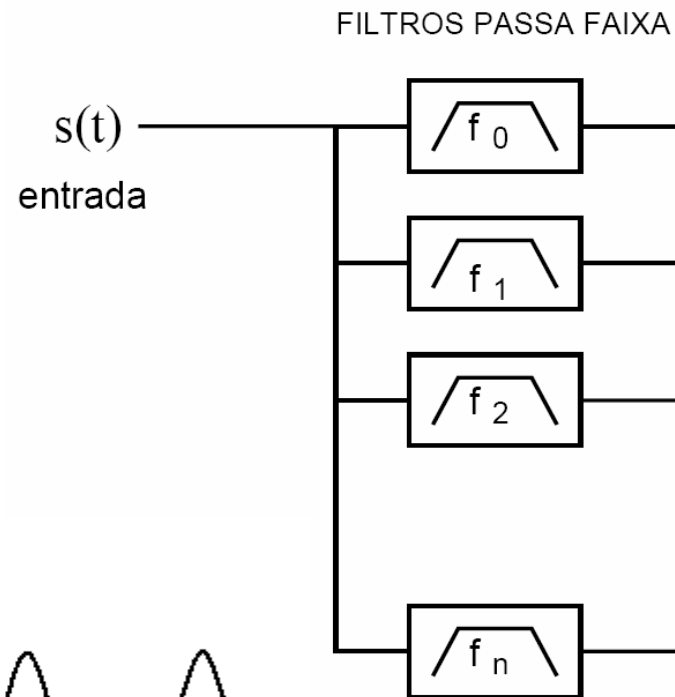
# *Análise Espectral*



# Síntese Espectral

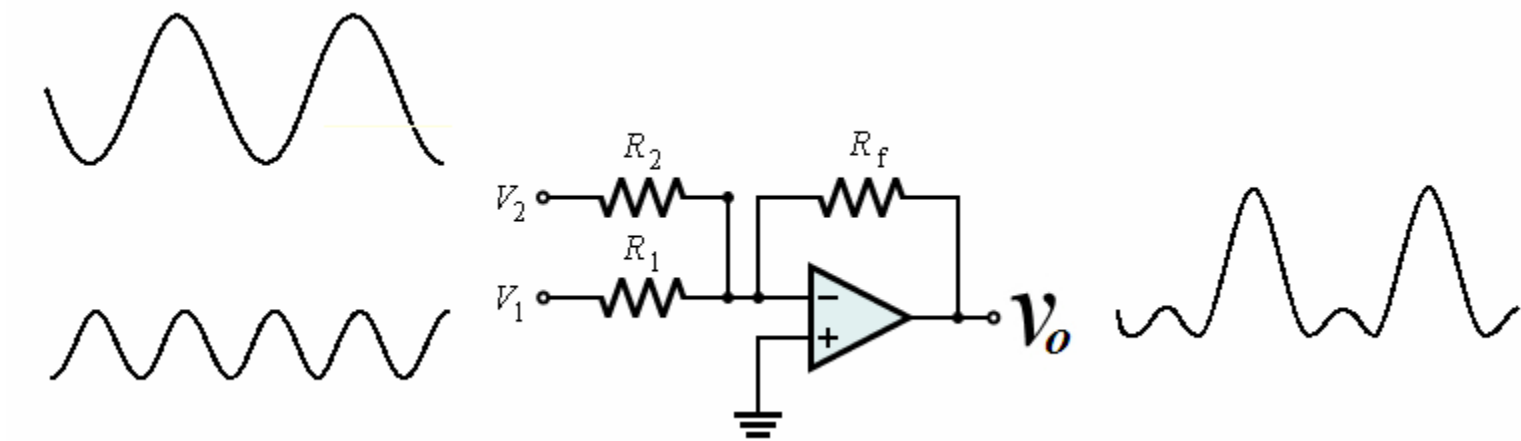


# Análise Espectral



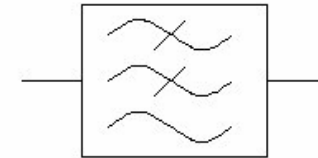
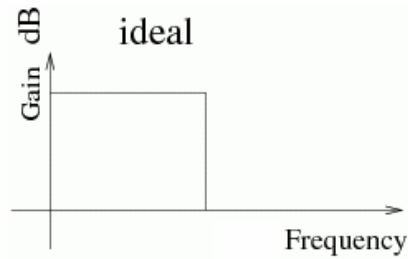


# CIRCUITO SOMADOR

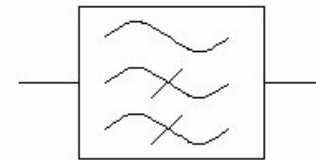
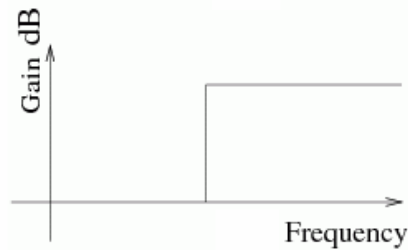


# *Classificação e Simbologia*

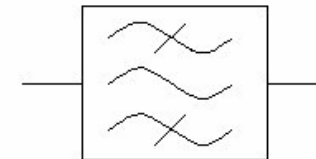
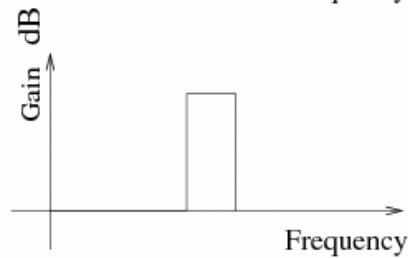
*Passa-baixa*



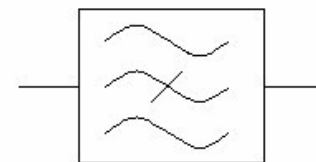
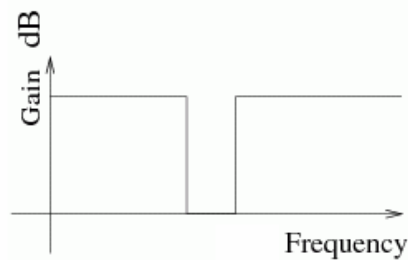
*Passa-alta*



*Passa-faixa*



*Rejeita-faixa*



# Filtros Elétricos

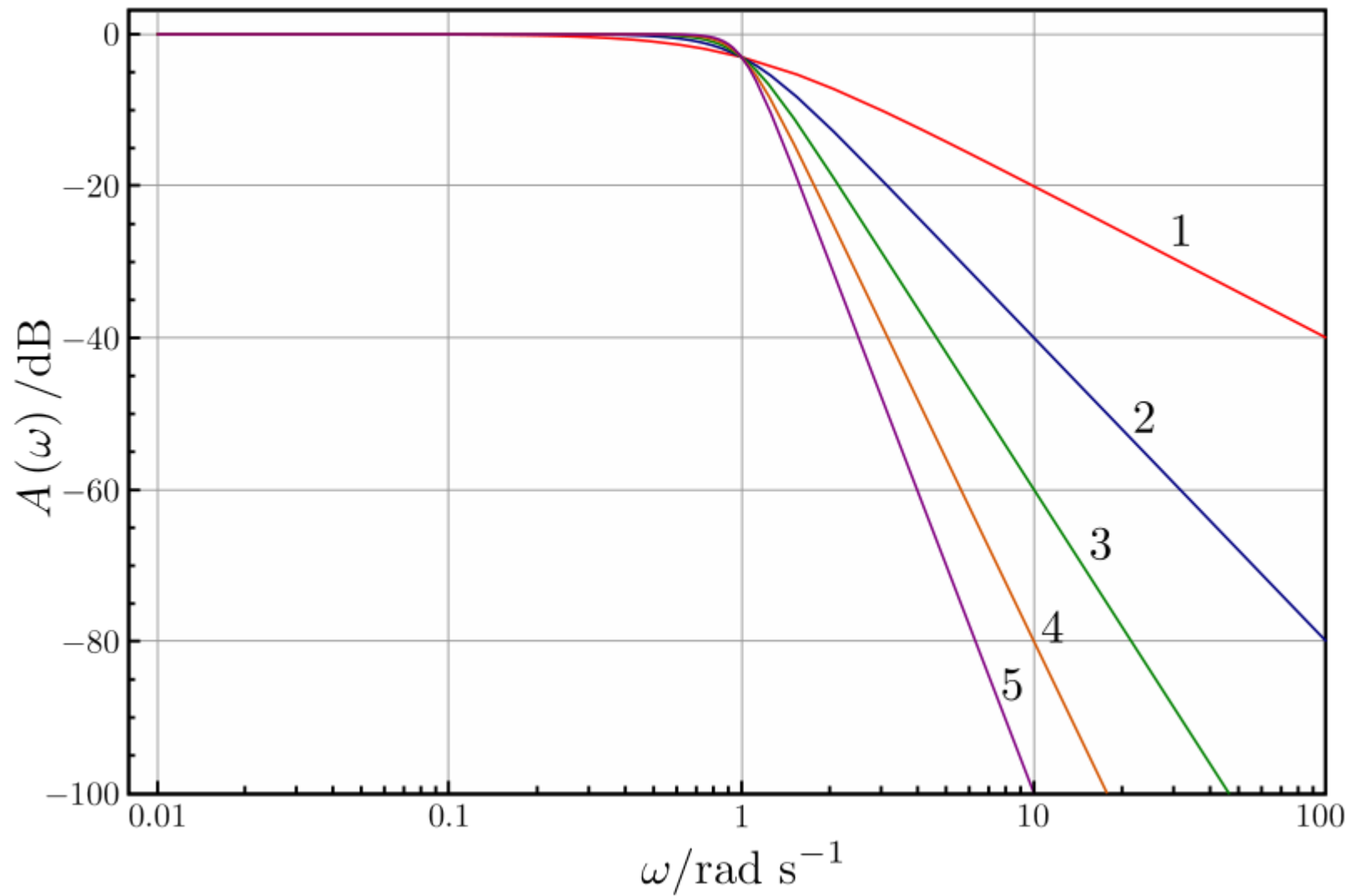
*Malha seletiva de frequência.*

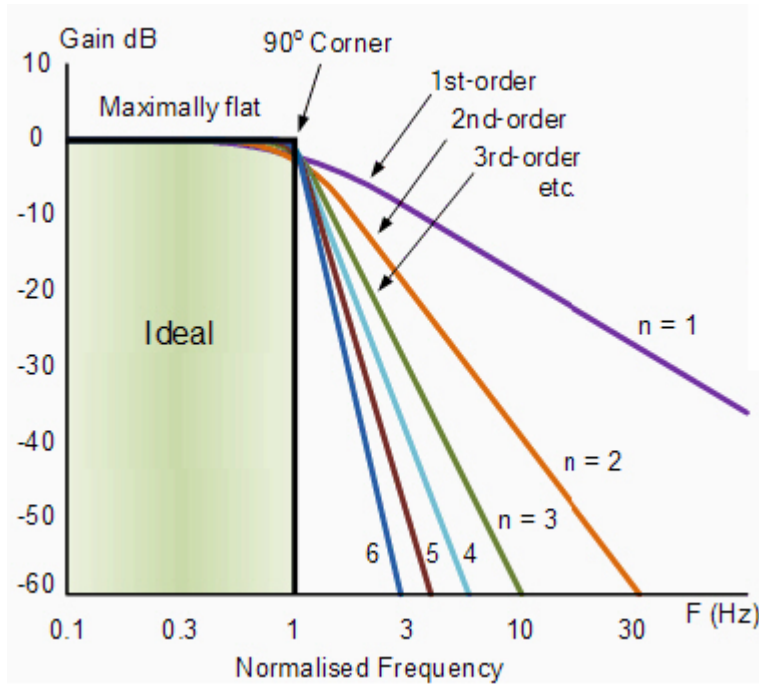
*Um filtro atenua a quantidade de energia presente em certas frequências ou faixas de frequências*

*Deixam passar ou amplificam as frequências desejadas e atenuam as indesejáveis.*

*A quantidade de atenuação para cada frequência varia de filtro para filtro.*

# Ordem do Filtro





# **ELETRÔNICA**

**Filtros Eletrônicos**  
**Resposta em Frequência**

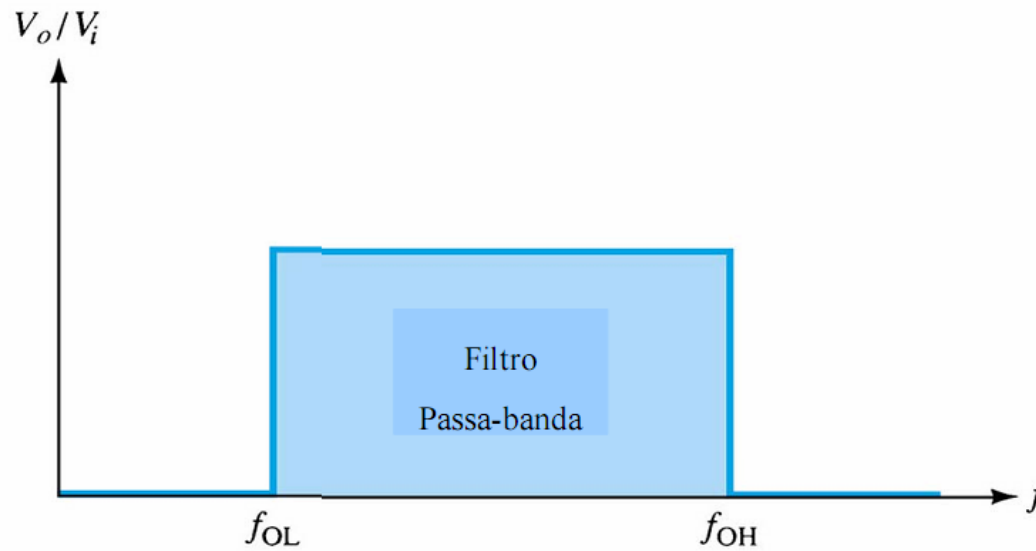
AOC



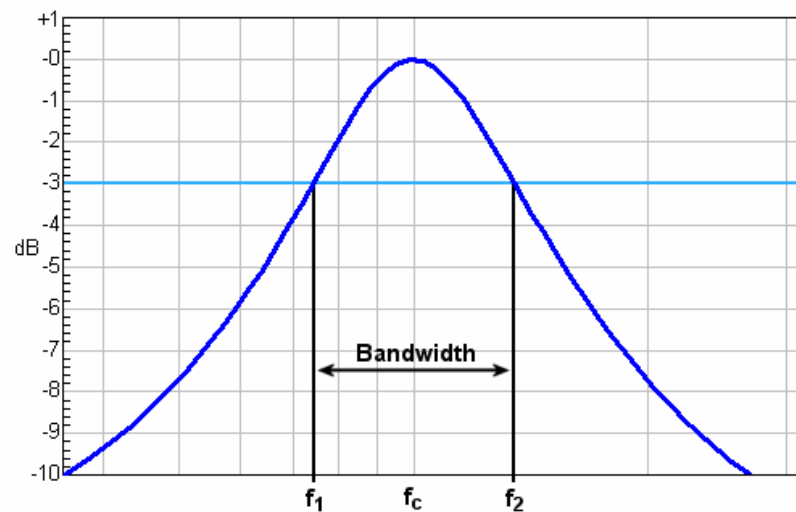
# *Filtro Passa-Faixa*

## *Ideal x Real*

**IDEAL**

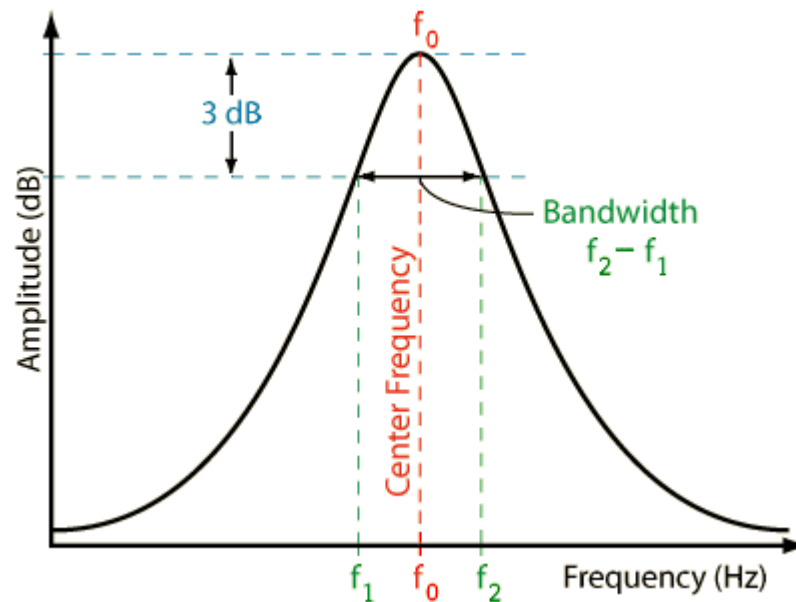


**REAL**

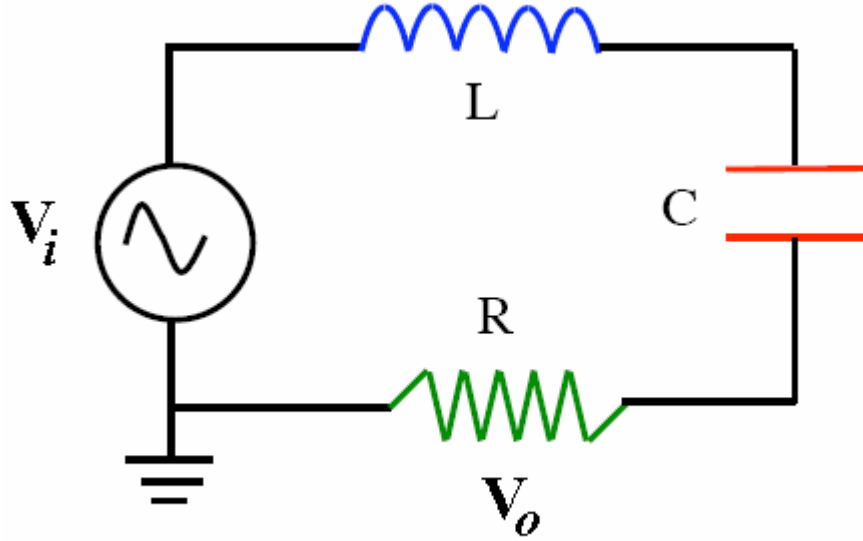


# Seletividade

- *Seletividade* é a propriedade que o circuito possui em distinguir, num dado espectro de frequências, uma determinada frequência em relação às demais.







$$H(s) = \frac{V_o(s)}{V_i(s)} = \frac{s \frac{R}{L}}{s^2 + s \frac{R}{L} + \frac{1}{LC}}$$

$$Q = \frac{\omega_o L}{R} \longrightarrow \frac{\omega_0}{Q} = \frac{R}{L}$$

$$\omega_0 = \sqrt{\frac{1}{LC}} \longrightarrow \omega_0^2 = \frac{1}{LC}$$

$$H(s) = \frac{V_o(s)}{V_i(s)} = \frac{s \frac{\omega_0}{Q}}{s^2 + s \frac{\omega_0}{Q} + \omega_0^2}$$

## Parâmetros do filtro passa faixa

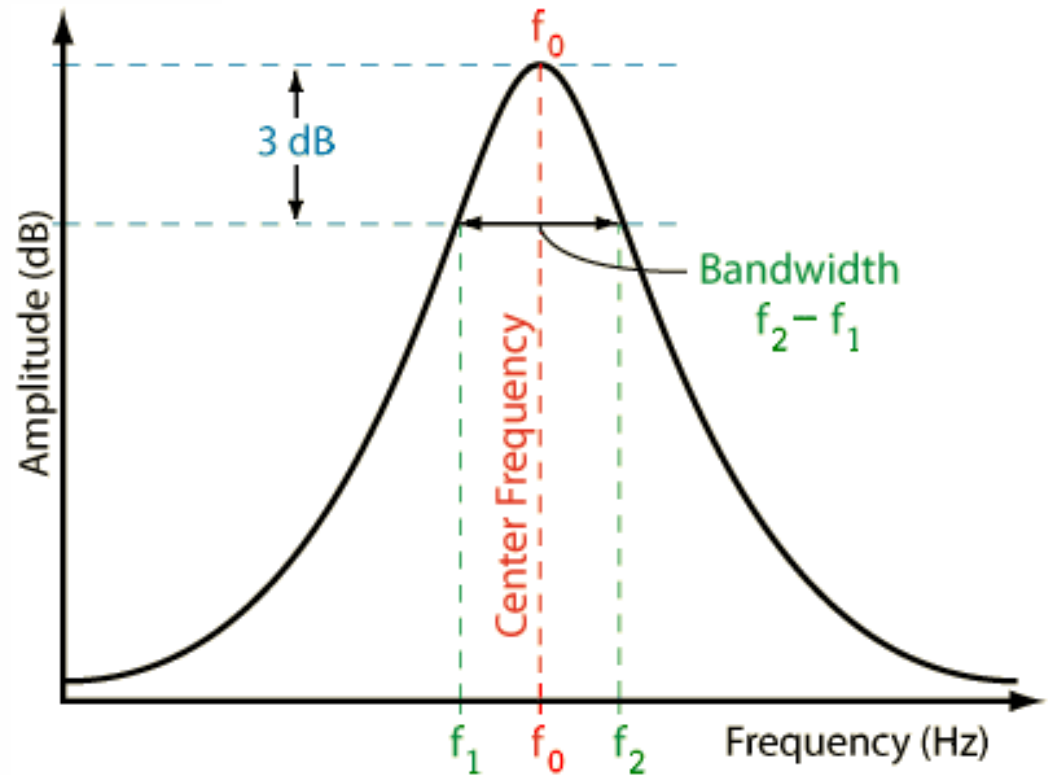
$$\Delta f = f_2 - f_1$$

$$f_0 = \sqrt{f_1 \cdot f_2} \quad (\text{média geométrica})$$

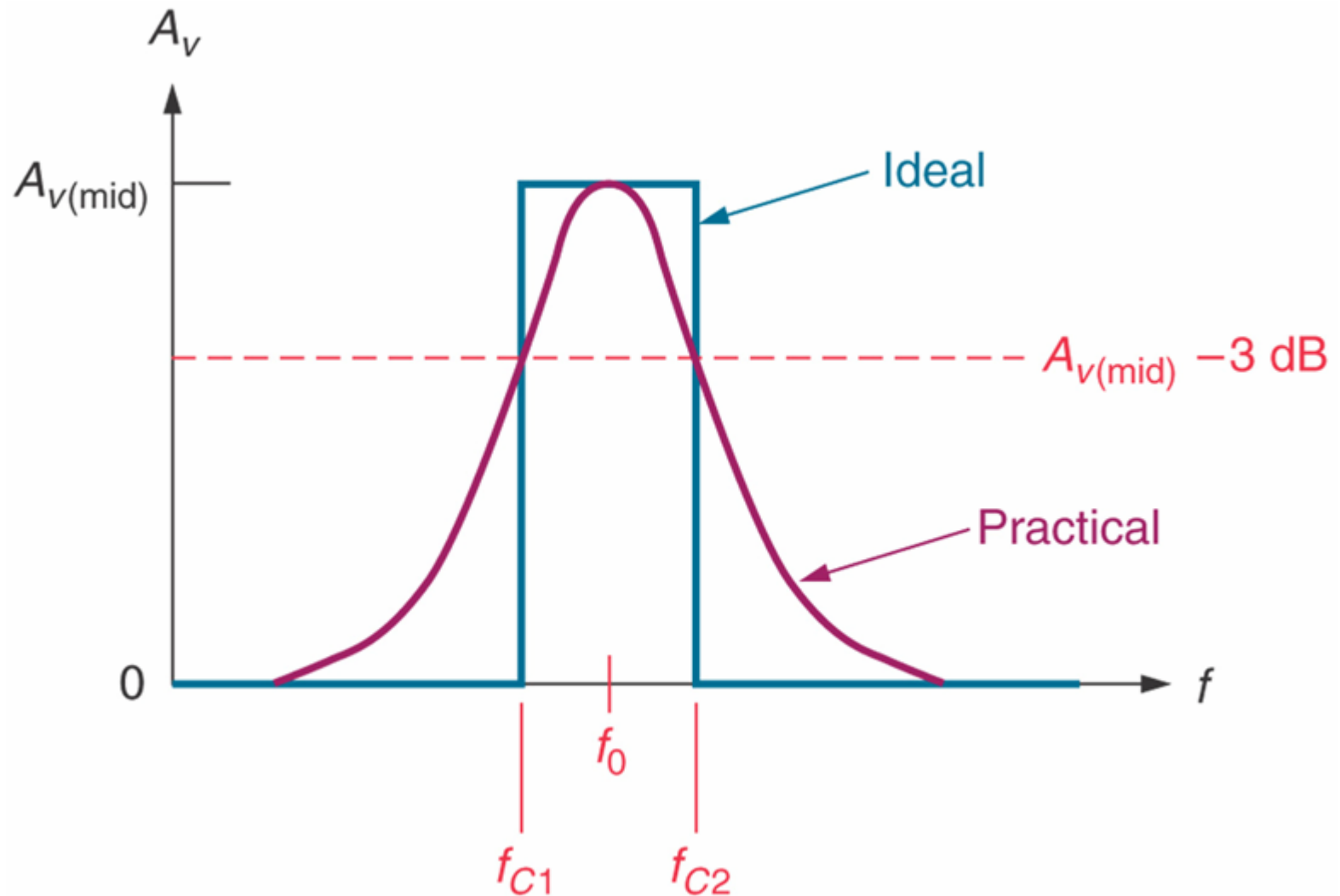
$$Q = \frac{f_0}{\Delta f}$$

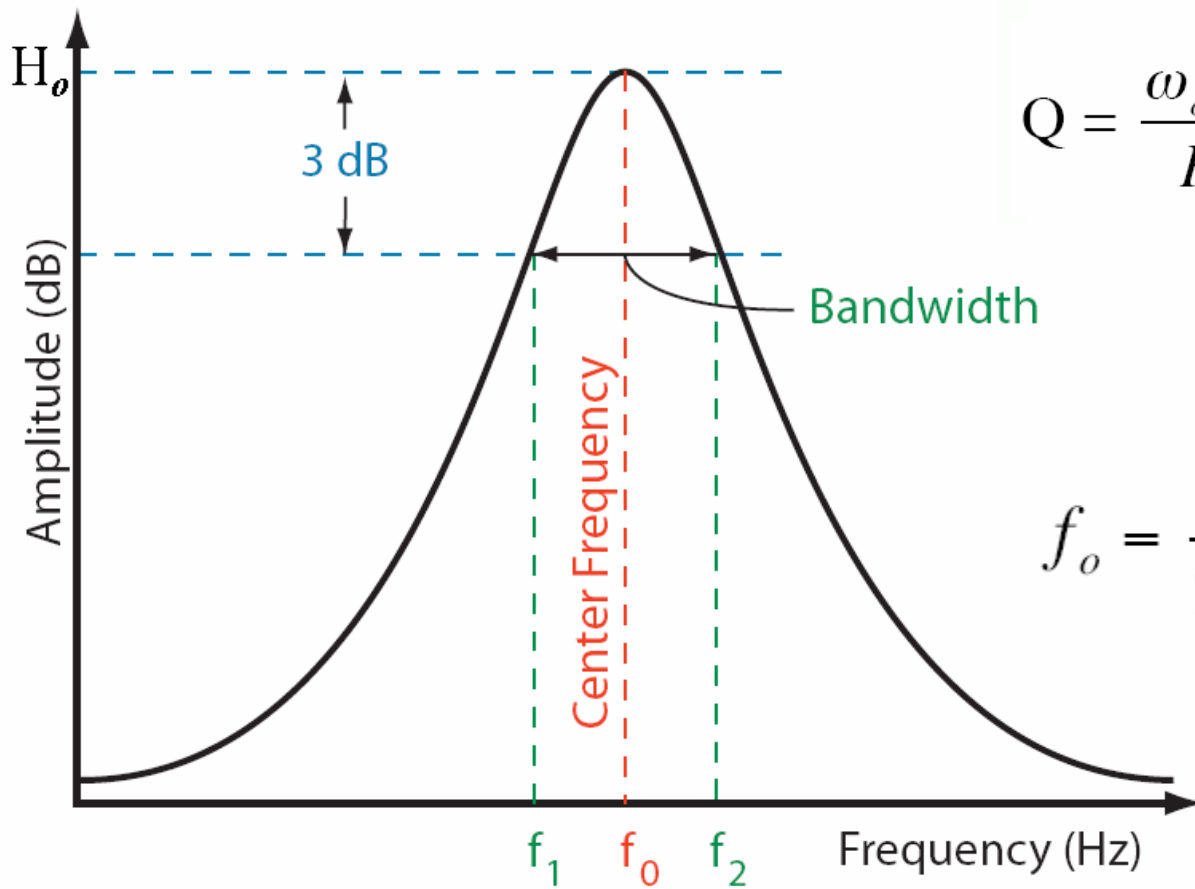
$$f_2 = \frac{f_0}{2Q} \left( 1 + \sqrt{1 + 4Q^2} \right)$$

$$f_1 = \frac{f_0}{2Q} \left( \sqrt{1 + 4Q^2} - 1 \right)$$



## Resposta em frequência do filtro passa faixa

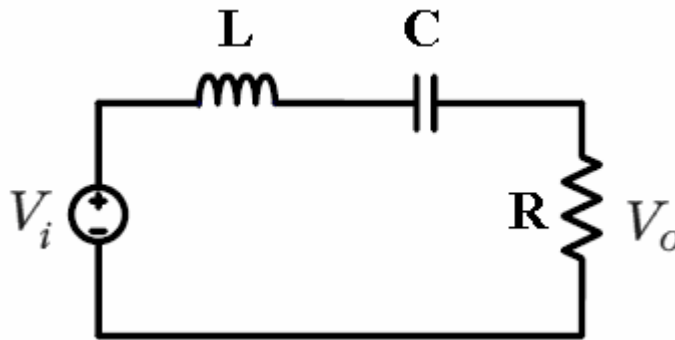




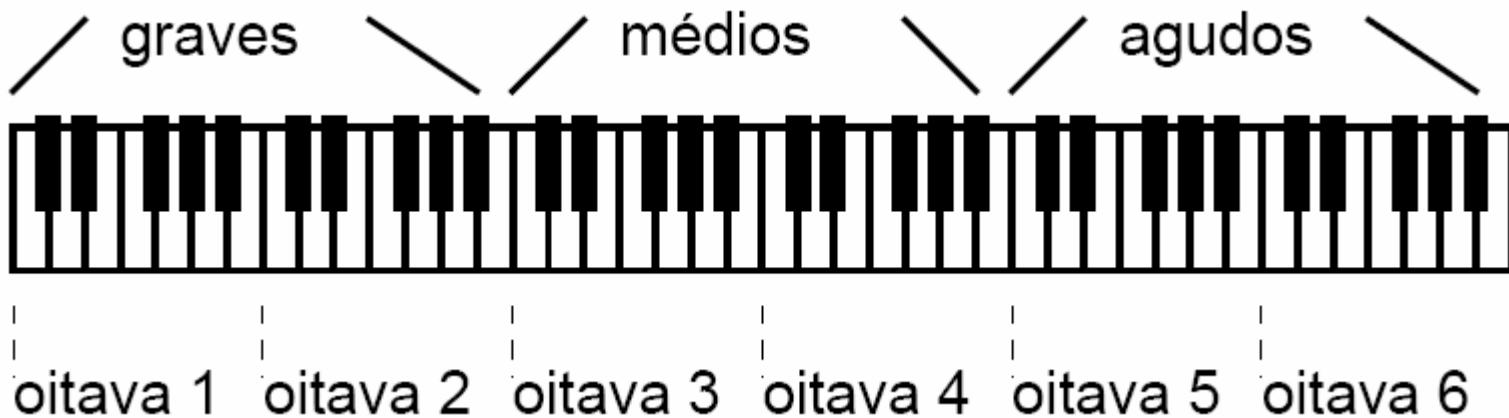
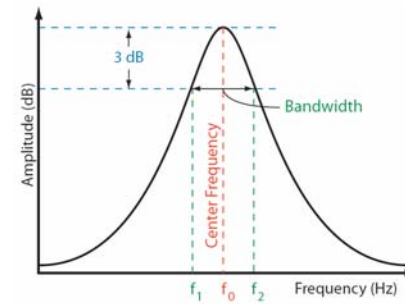
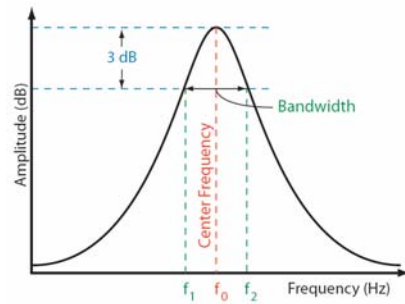
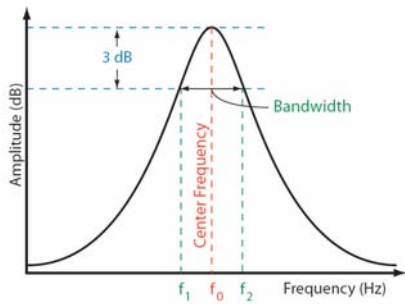
$$Q = \frac{\omega_o L}{R}$$

$$f_o = \frac{1}{2\pi} \sqrt{\frac{1}{LC}}$$

$$\frac{V_o}{V_i} = H_o \frac{\left(\frac{R}{L}\right)s}{s^2 + \left(\frac{R}{L}\right)s + \frac{1}{LC}}$$

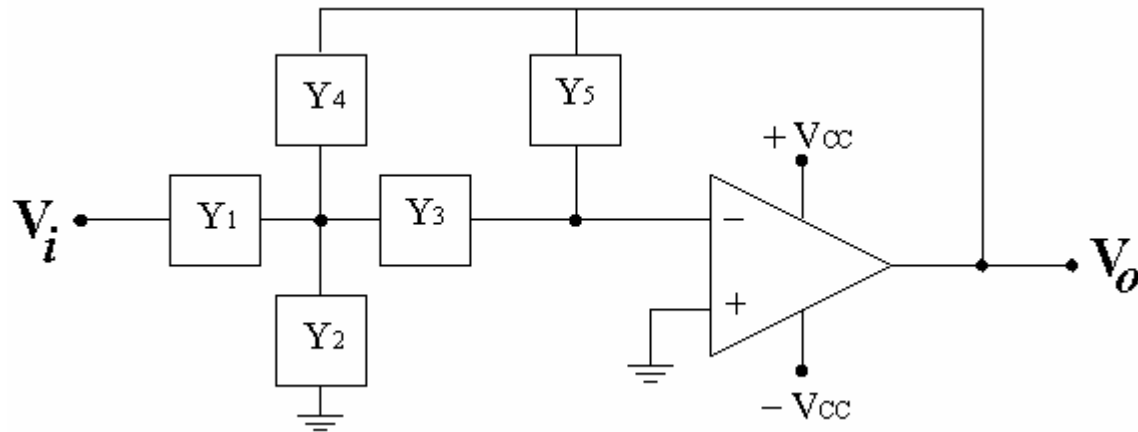


$$\frac{V_o}{V_i} = H_o \frac{\left(\frac{\omega_o}{Q}\right)s}{s^2 + \left(\frac{\omega_o}{Q}\right)s + \omega_o^2}$$



## *Filtro Ativo*

### *Configuração Realimentação Múltipla Multiple-Feedback Filter*



$$\frac{V_o}{V_i} = - \frac{Y_1 Y_3}{Y_5 (Y_1 + Y_2 + Y_3 + Y_4) + Y_3 Y_4}$$

### **Filtro Passa Baixa**

$$\frac{V_O(s)}{V_I(s)} = \frac{A_{LP} \omega_n^2}{s^2 + s \omega_n / Q + \omega_n^2}$$

### **Filtro Passa Alta**

$$\frac{V_{HP}(s)}{V_I(s)} = \frac{A_{HP} s^2}{s^2 + s \omega_n / Q + \omega_n^2}$$

### **Filtro Passa Faixa**

$$\frac{V_{BP}(s)}{V_I(s)} = \frac{A_{BP} (\omega_n / Q) s}{s^2 + s \omega_n / Q + \omega_n^2}$$

### **Filtro Rejeita Faixa**

$$\frac{V_{BR}(s)}{V_I(s)} = \frac{A_{BR} (s^2 + \omega_n^2)}{s^2 + s \omega_n / Q + \omega_n^2}$$

$$\frac{V_0}{V_i} = \frac{Y_1 Y_3}{Y_5 (Y_1 + Y_2 + Y_3 + Y_4) + Y_3 Y_4}$$

### Filtro Passa Baixa

$$\frac{V_O(s)}{V_I(s)} = \frac{A_{LP} \omega_n^2}{s^2 + s \omega_n / Q + \omega_n^2}$$

### Filtro Passa Alta

$$\frac{V_{HP}(s)}{V_I(s)} = \frac{A_{HP} s^2}{s^2 + s \omega_n / Q + \omega_n^2}$$

### Filtro Passa Faixa

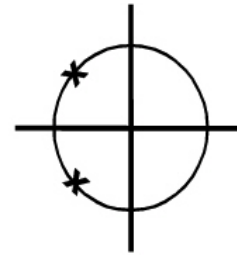
$$\frac{V_{BP}(s)}{V_I(s)} = \frac{A_{BP} (\omega_n / Q) s}{s^2 + s \omega_n / Q + \omega_n^2}$$

### Filtro Rejeita Faixa

$$\frac{V_{BR}(s)}{V_I(s)} = \frac{A_{BR} (s^2 + \omega_n^2)}{s^2 + s \omega_n / Q + \omega_n^2}$$

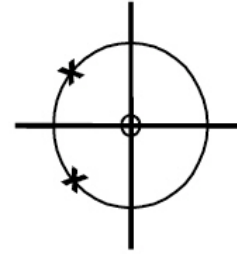
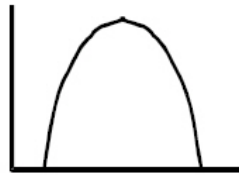


**PASSA-BAIXA**



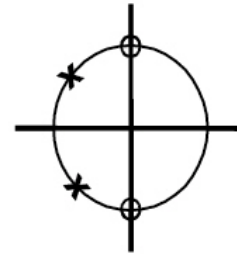
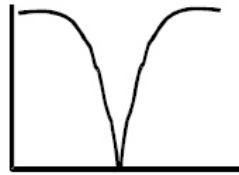
$$\frac{\omega_0^2}{s^2 + \frac{\omega_0}{Q}s + \omega_0^2}$$

**PASSA-FAIXA**



$$\frac{\omega_0^2 Q s}{s^2 + \frac{\omega_0}{Q}s + \omega_0^2}$$

**REJEITA-FAIXA**  
**NOTCH**



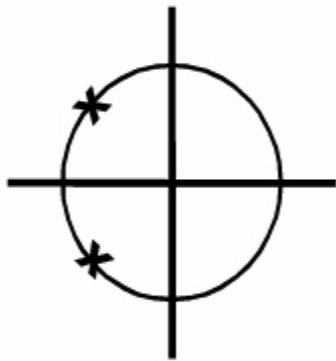
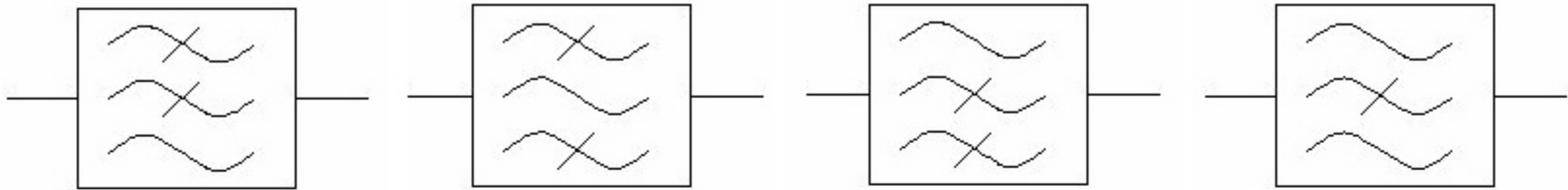
$$\frac{s^2 + \omega_z^2}{s^2 + \frac{\omega_0}{Q}s + \omega_0^2}$$

**PASSA-ALTA**

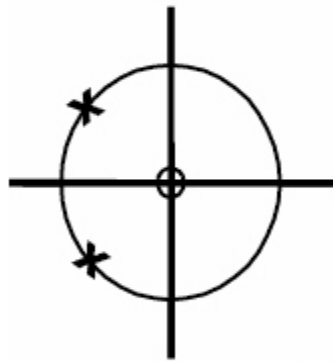


$$\frac{s^2}{s^2 + \frac{\omega_0}{Q}s + \omega_0^2}$$

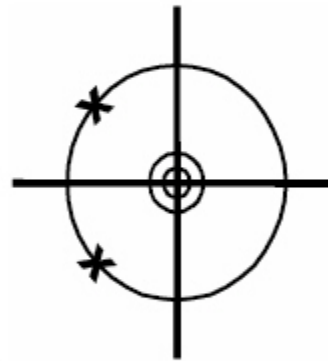
# Filtros Elétricos - Configuração de pólos e zeros



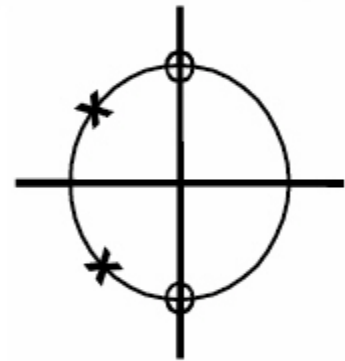
$$\frac{\omega_0^2}{s^2 + \frac{\omega_0}{Q}s + \omega_0^2}$$



$$\frac{\frac{\omega_0}{Q}s}{s^2 + \frac{\omega_0}{Q}s + \omega_0^2}$$

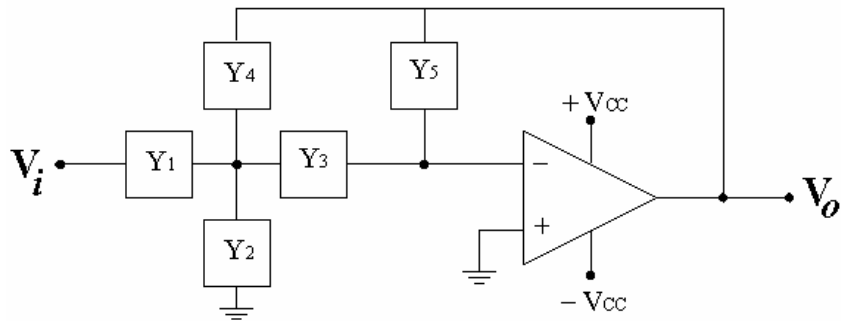


$$\frac{s^2}{s^2 + \frac{\omega_0}{Q}s + \omega_0^2}$$



$$\frac{s^2 + \omega_z^2}{s^2 + \frac{\omega_0}{Q}s + \omega_0^2}$$

$$\frac{V_o}{V_i} = - \frac{Y_1 Y_3}{Y_5 (Y_1 + Y_2 + Y_3 + Y_4) + Y_3 Y_4}$$



### Filtro Passa Baixa

$$\frac{V_o(s)}{V_i(s)} = \frac{A_{LP} \omega_n^2}{s^2 + s \omega_n / Q + \omega_n^2}$$

### Filtro Passa Alta

$$\frac{V_{HP}(s)}{V_i(s)} = \frac{A_{HP} s^2}{s^2 + s \omega_n / Q + \omega_n^2}$$

### Filtro Passa Faixa

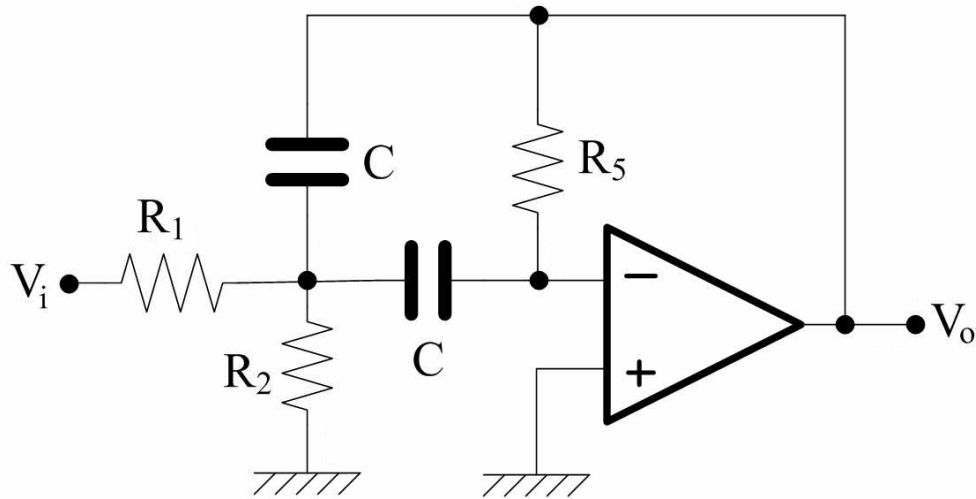
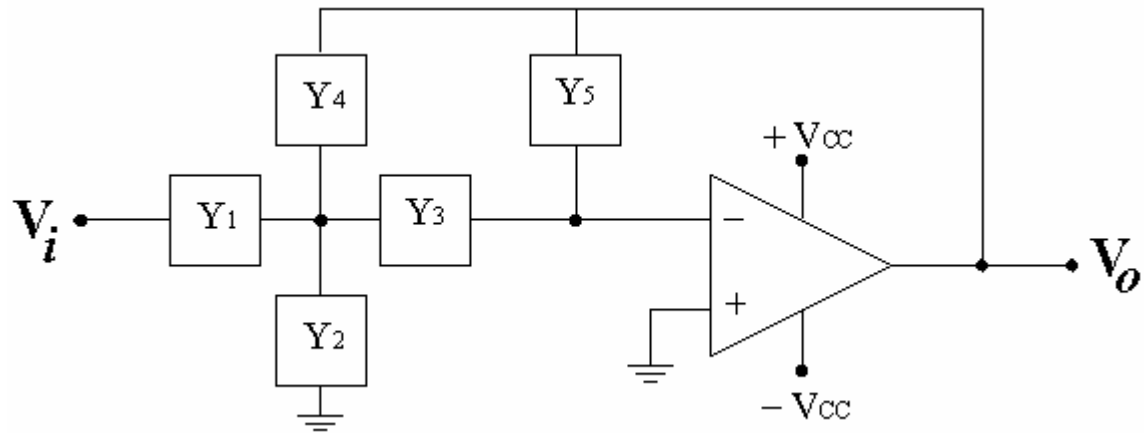
$$\frac{V_{BP}(s)}{V_i(s)} = \frac{A_{BP} (\omega_n / Q) s}{s^2 + s \omega_n / Q + \omega_n^2}$$

### Filtro Rejeita Faixa

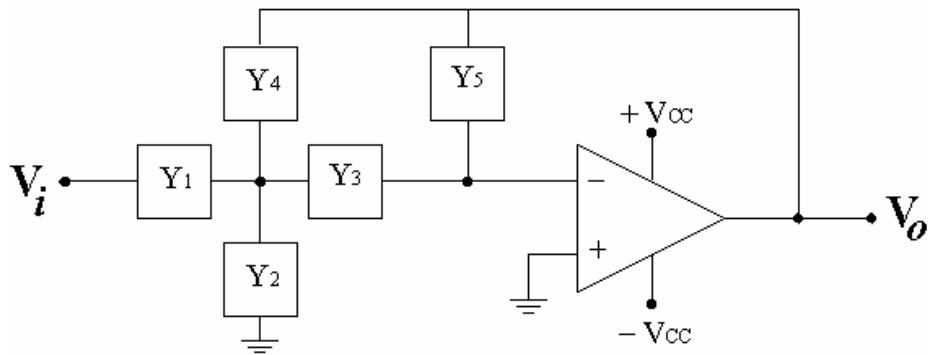
$$\frac{V_{BR}(s)}{V_i(s)} = \frac{A_{BR} (s^2 + \omega_n^2)}{s^2 + s \omega_n / Q + \omega_n^2}$$

# *Filtro Passa Faixa*

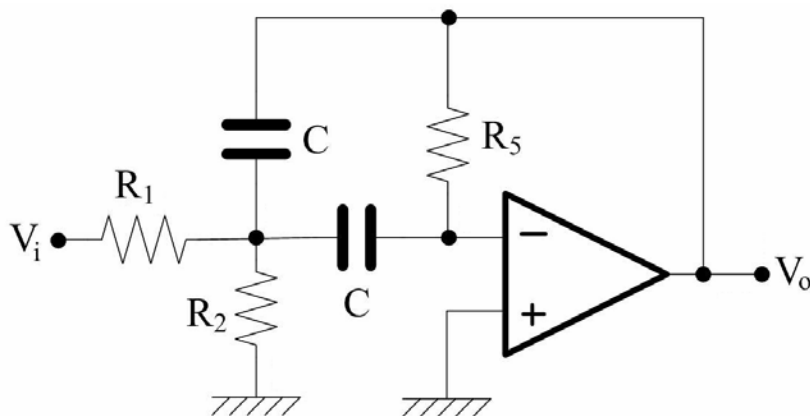
## *Multiple-Feedback Band-Pass Filter*



# *Filtro Passa Faixa*

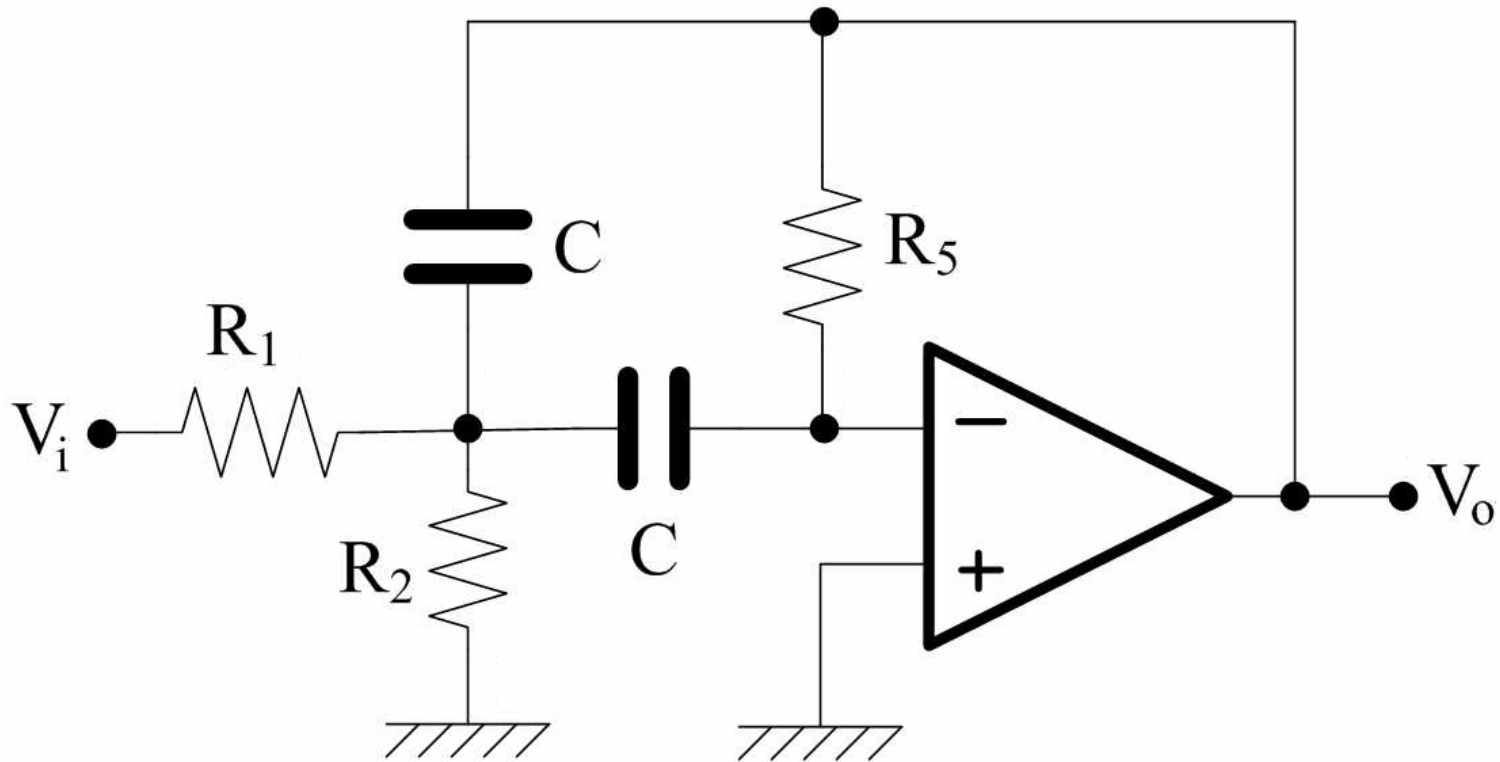


$$\frac{V_o}{V_i} = - \frac{Y_1 Y_3}{Y_5 (Y_1 + Y_2 + Y_3 + Y_4) + Y_3 Y_4}$$



$$\frac{V_o}{V_i} = \frac{\frac{1}{R_1 C} s}{s^2 + \frac{2}{R_5 C} s + \frac{(R_1 + R_2)}{R_1 R_2 R_5 C^2}}$$

## *Filtro Passa Faixa*

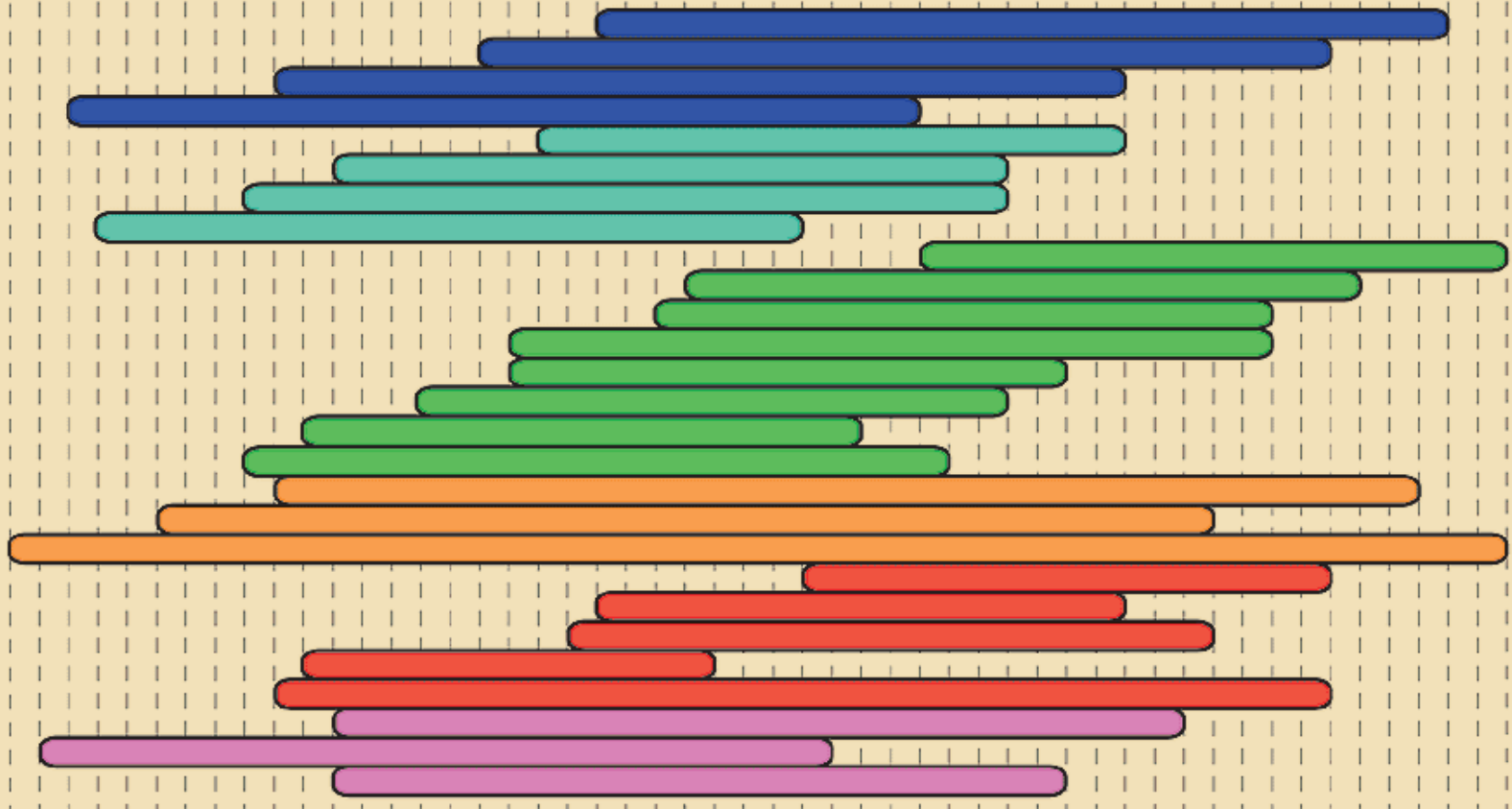


$$\frac{V_o}{V_i} = - \frac{\frac{1}{R_1 C} s}{s^2 + \frac{2}{R_5 C} s + \frac{(R_1 + R_2)}{R_1 R_2 R_5 C^2}}$$

## *Filtro Passa Faixa*

$$\frac{V_o}{V_i} = H_o \frac{\frac{\omega_0}{Q} s}{s^2 + \frac{\omega_0}{Q} s + \omega_0^2}$$

$$\frac{V_o}{V_i} = - \frac{\frac{1}{R_1 C} s}{s^2 + \frac{2}{R_5 C} s + \frac{(R_1 + R_2)}{R_1 R_2 R_5 C^2}}$$



- Violin
- Viola
- Cello
- Bass
- Trumpet
- Trombone
- French Horn
- Tuba
- Piccolo
- Flute
- Oboe
- Clarinet
- Alto Sax
- Tenor Sax
- Baritone Sax
- Bassoon
- Harp
- Harpsichord
- Piano
- Xylophone
- Glockenspiel
- Vibraphone
- Timpani
- Marimba
- Guitar
- Bass Guitar
- Voice



# *Faixa Espectral da voz humana*

**Speech**

men 110–165 Hz      women and children 220–330 Hz



**Singing**

bass 82–330 Hz

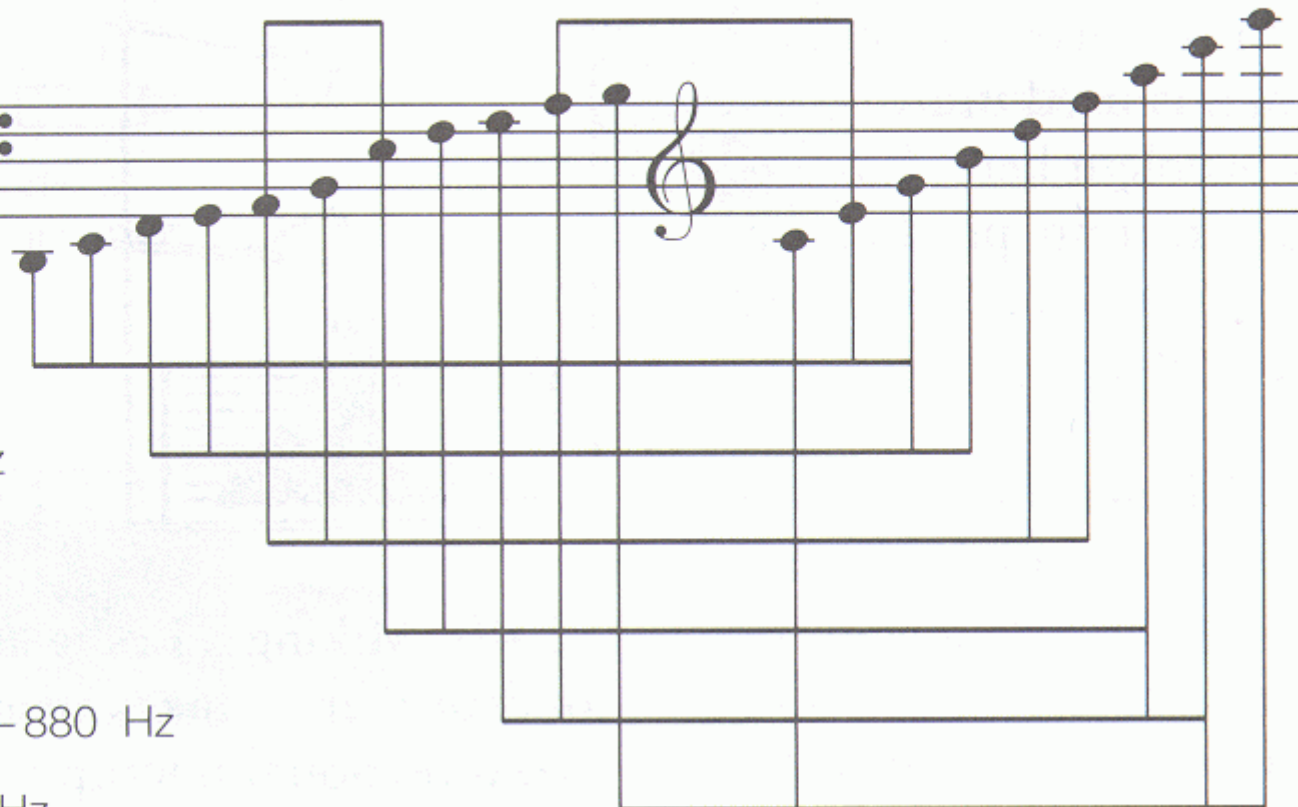
baritone 98–392 Hz

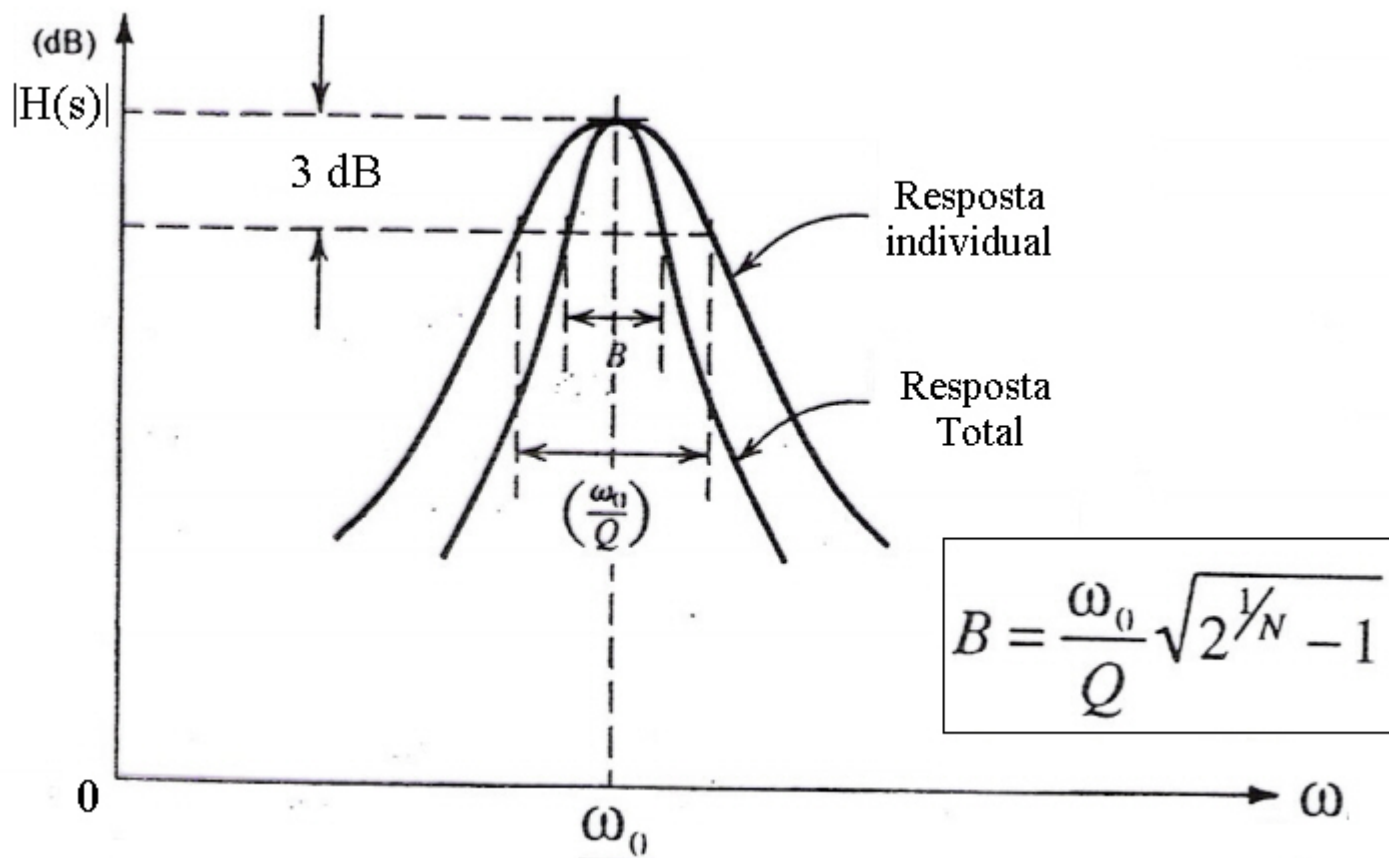
tenor 124–494 Hz

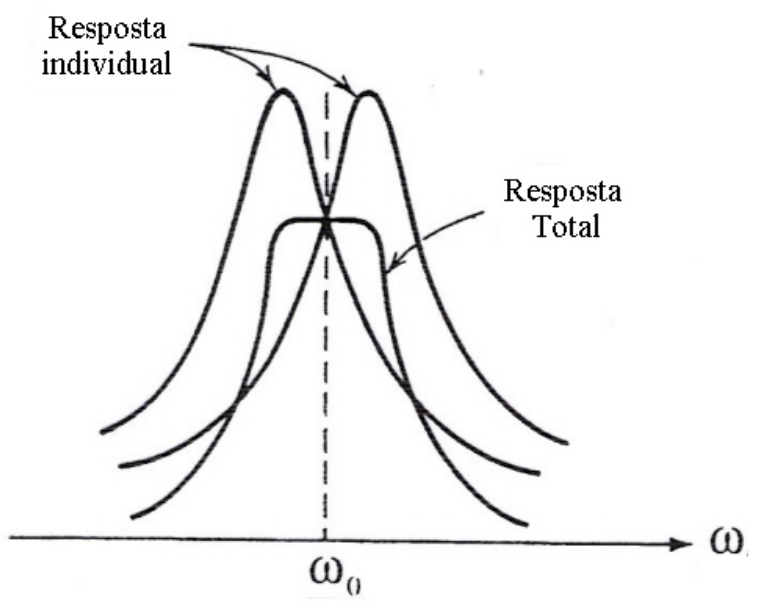
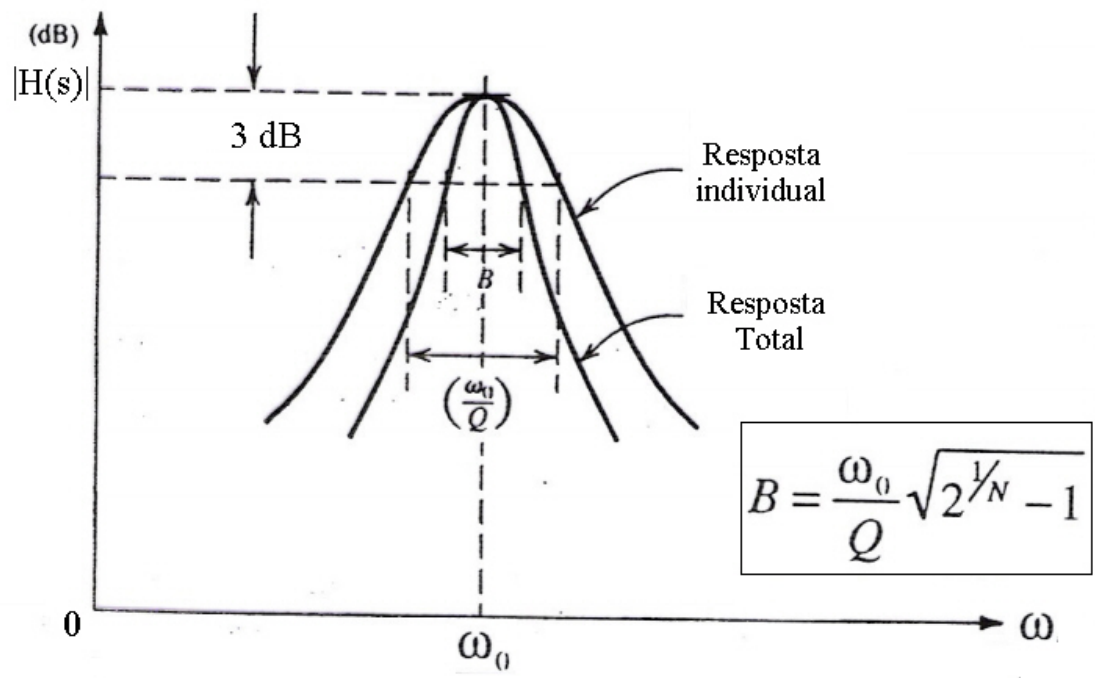
alto 175–699 Hz

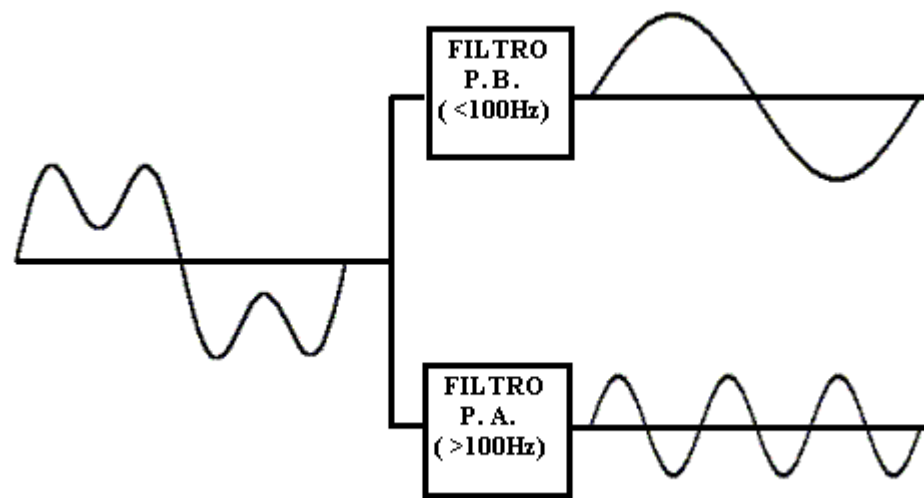
mezzo-soprano 220–880 Hz

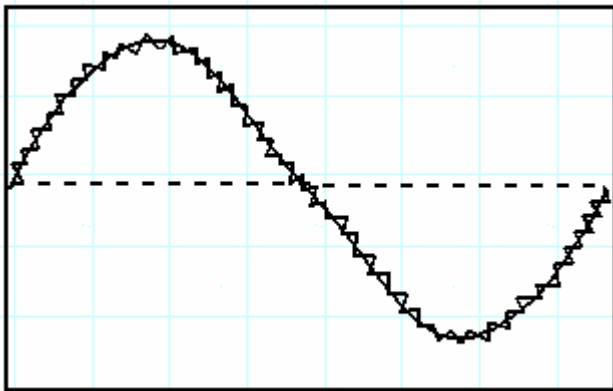
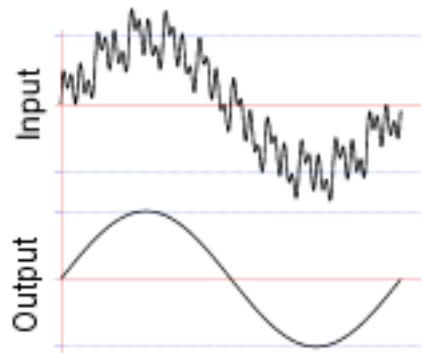
soprano 262–1047 Hz



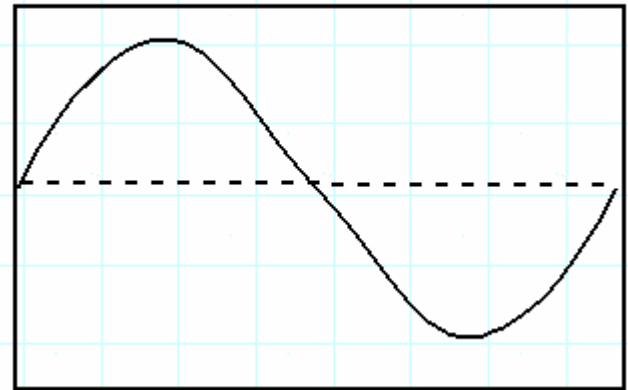


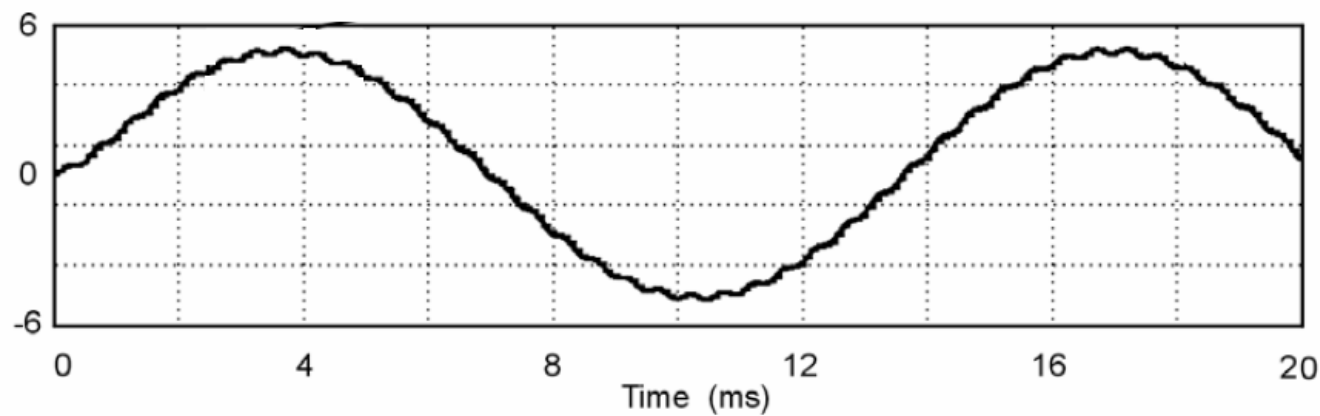
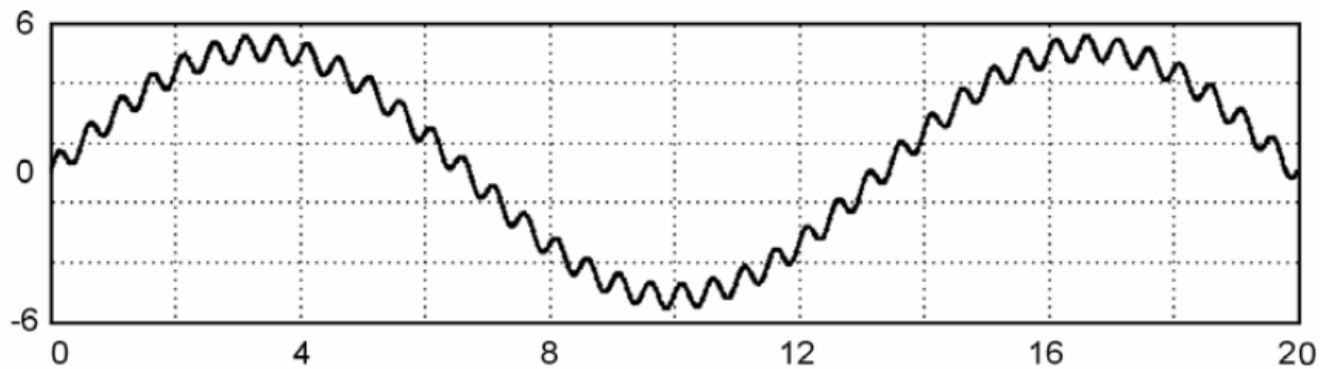
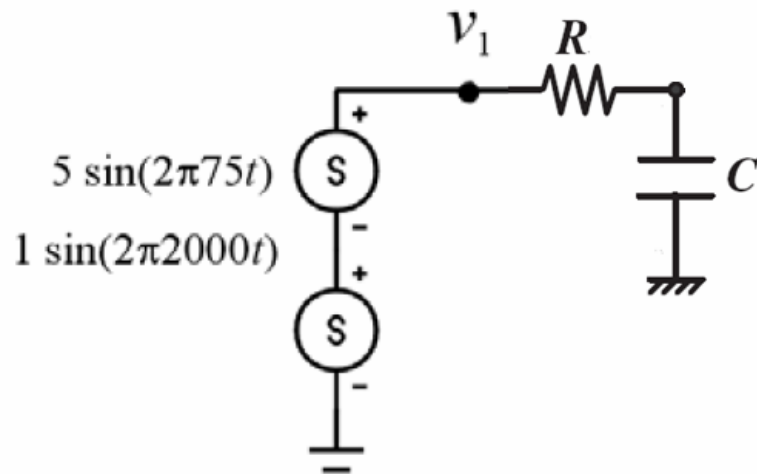




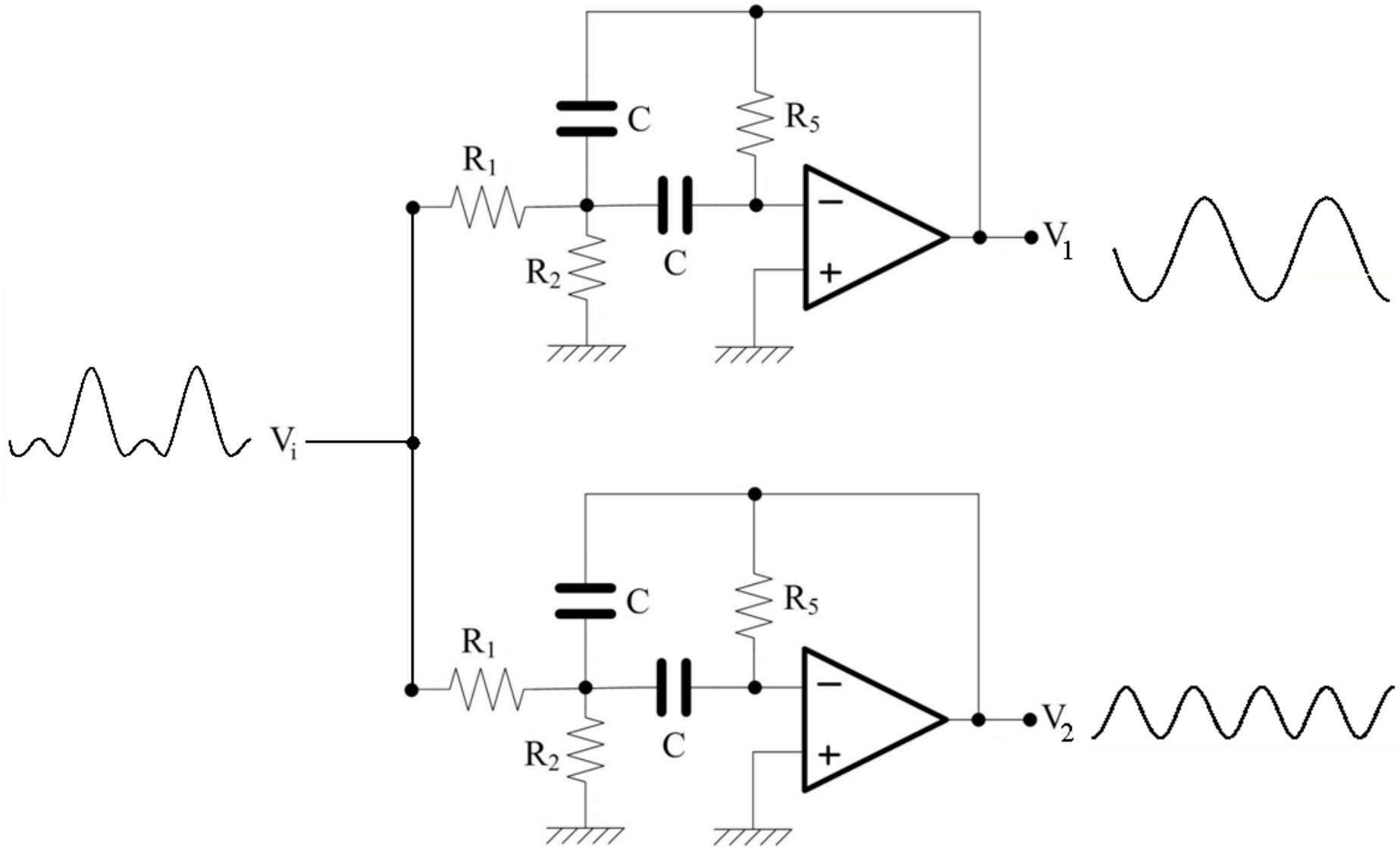


*filtragem*

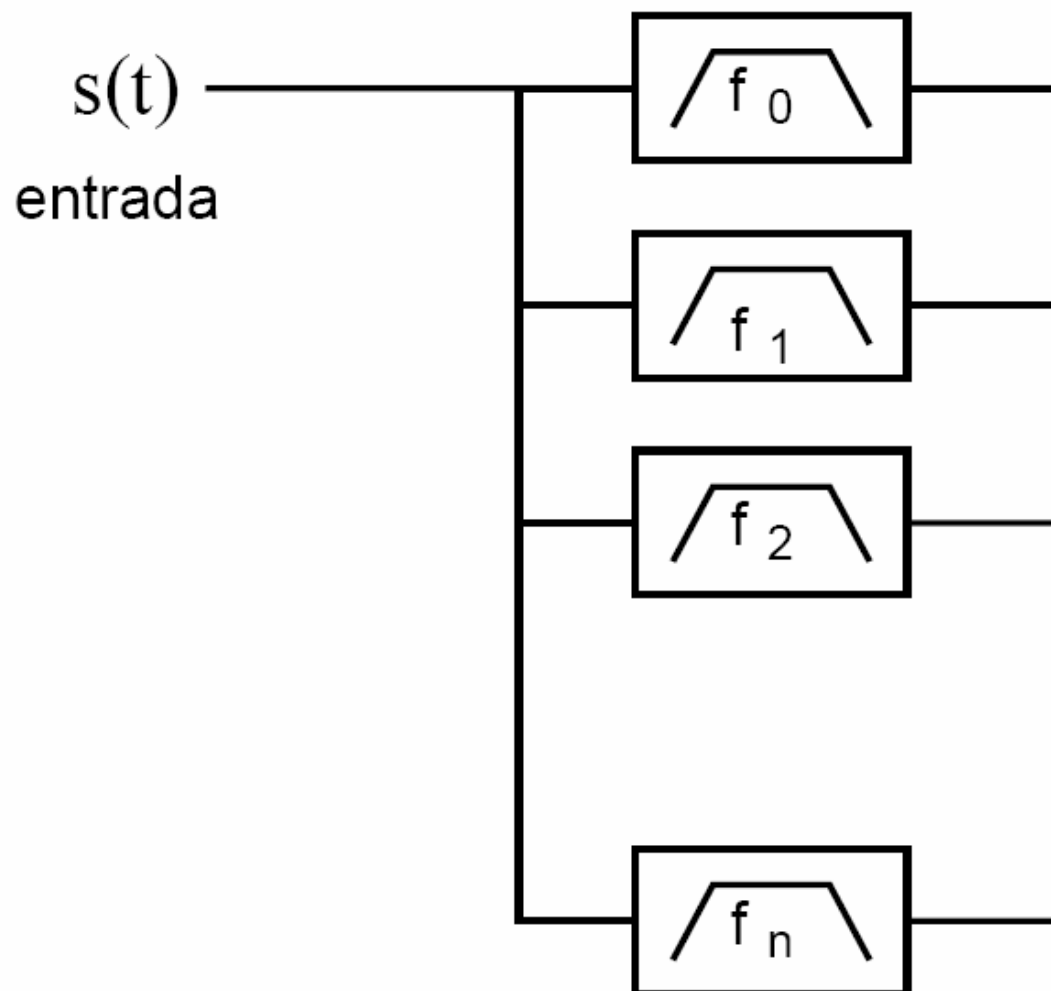




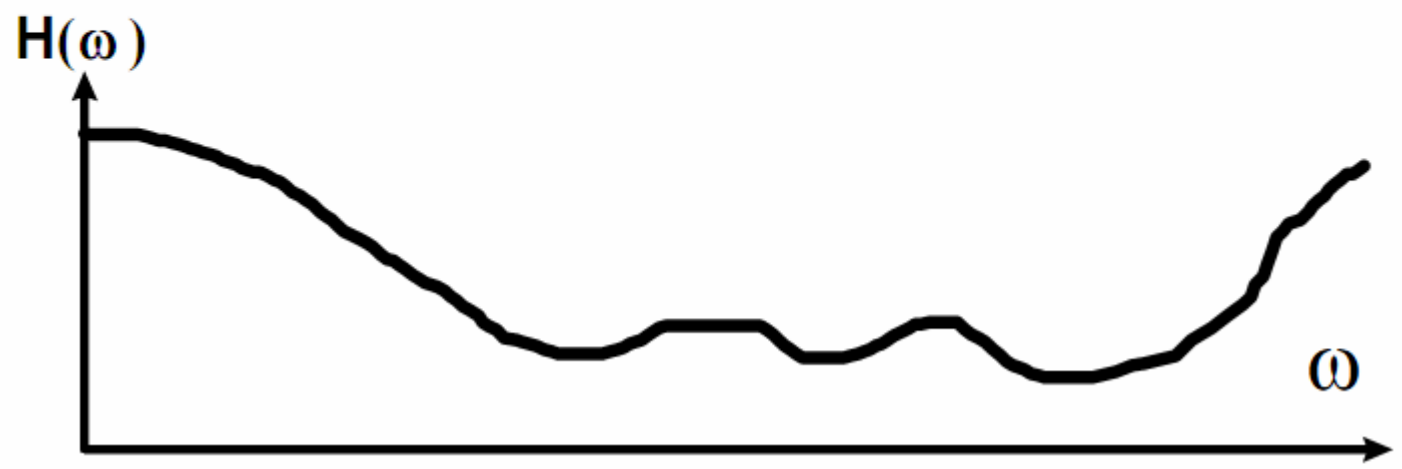
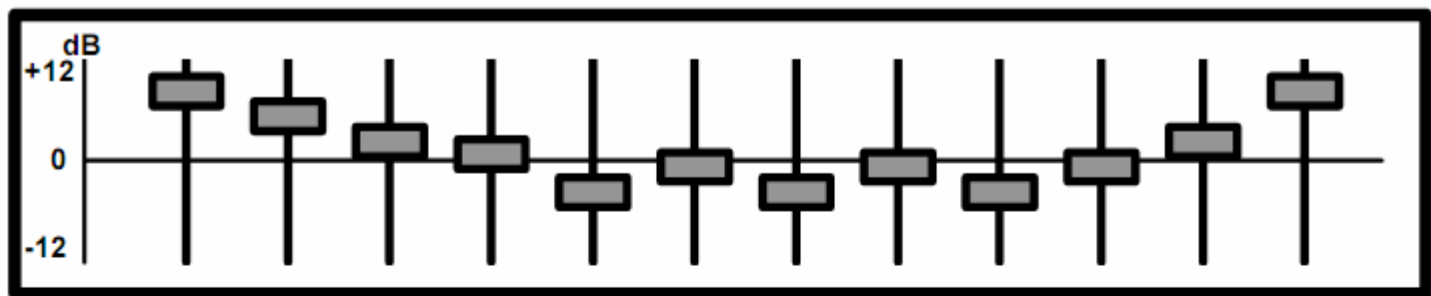
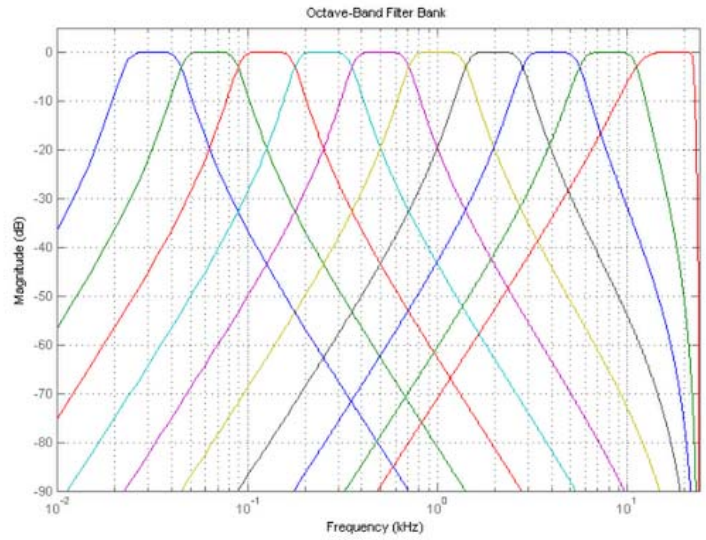
# Análise Espectral



# FILTROS PASSA FAIXA

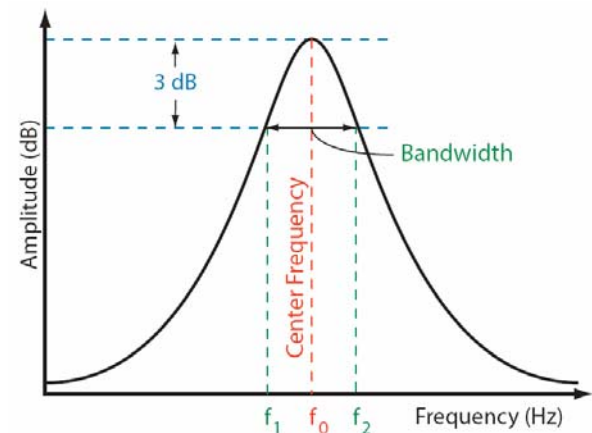
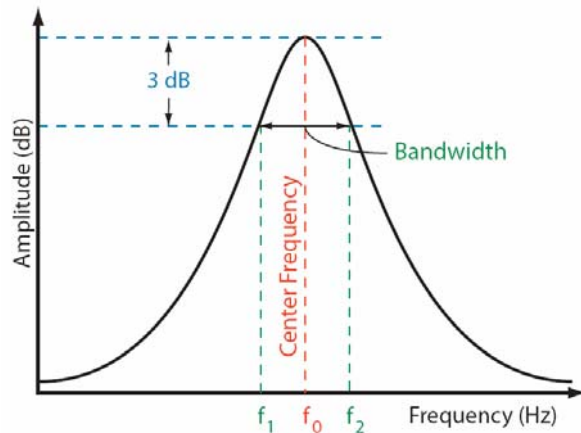
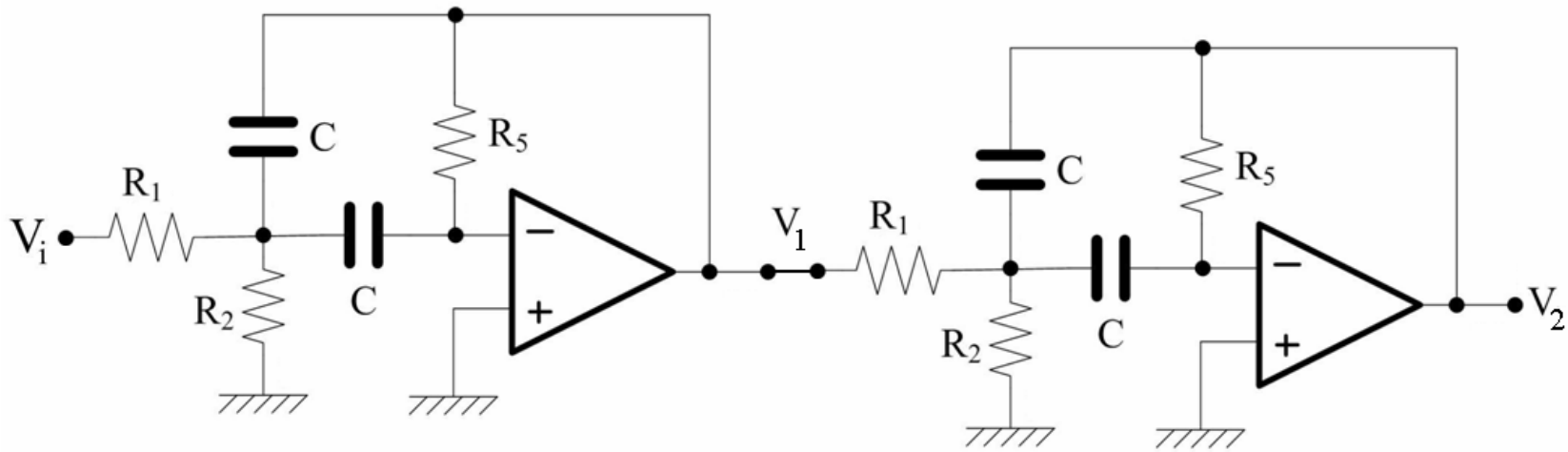






# Filtros Passa-Faixas em Série

## Filtro Passa-Baixa de Quarta Ordem

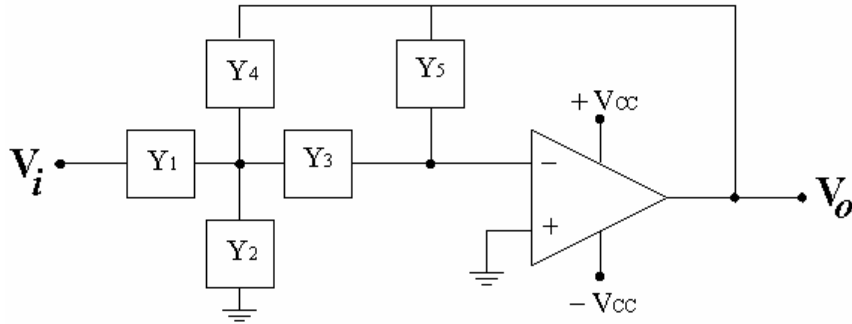


## *Fator de Qualidade Equivalente*

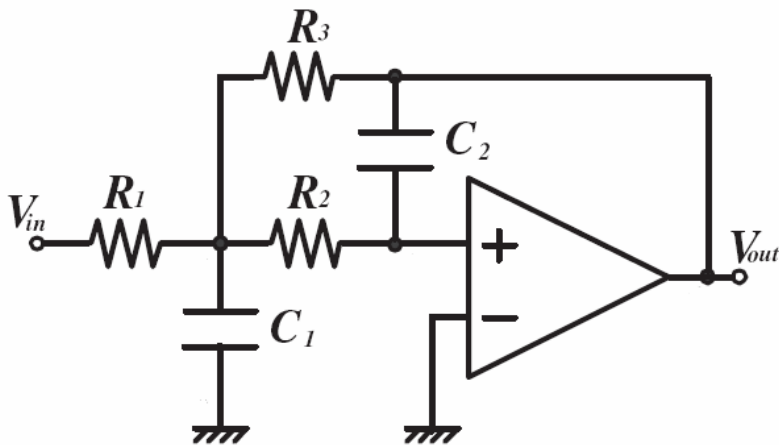
*$N \rightarrow$  número de filtros em série de mesma frequência central e com fatores de qualidades iguais a  $Q_0$*

$$Q_T = \frac{Q_0}{\sqrt{\sqrt[N]{2} - 1}}$$

# Filtro Passa Baixa

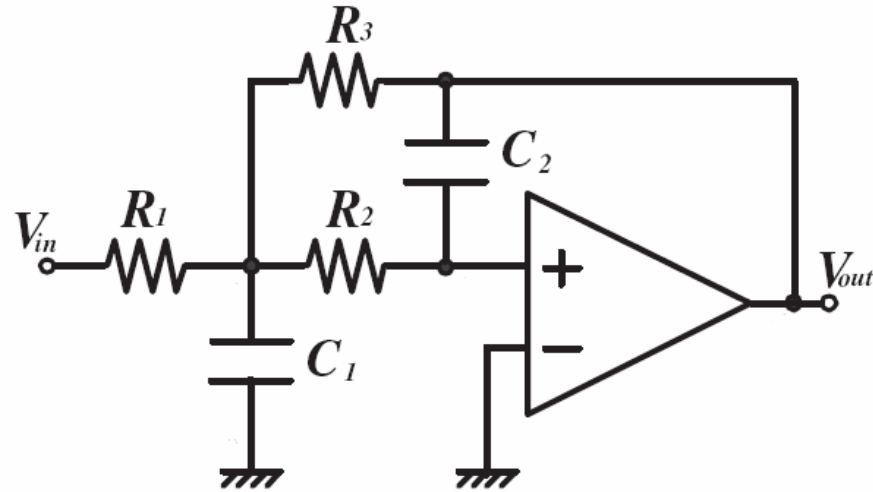


$$\frac{V_0}{V_i} = - \frac{Y_1 Y_3}{Y_5 (Y_1 + Y_2 + Y_3 + Y_4) + Y_3 Y_4}$$



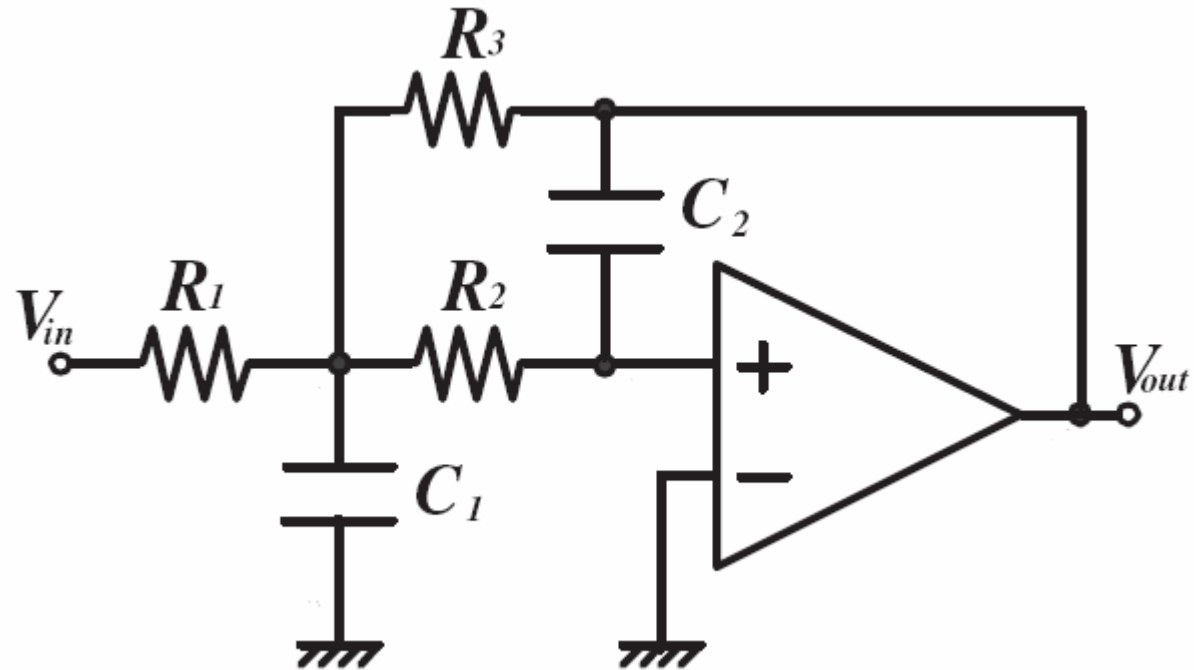
$$\frac{V_0}{V_i} = - \frac{\frac{1}{R_1 R_2 C_1 C_2}}{s^2 + s \frac{1}{C_1} \left( \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} \right) + \frac{1}{R_2 R_3 C_1 C_2}}$$

## *Filtro Passa Baixa*



$$\frac{V_0}{V_i} = - \frac{1}{R_1 R_2 C_1 C_2} \frac{1}{s^2 + s \frac{1}{C_1} \left( \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} \right) + \frac{1}{R_2 R_3 C_1 C_2}}$$

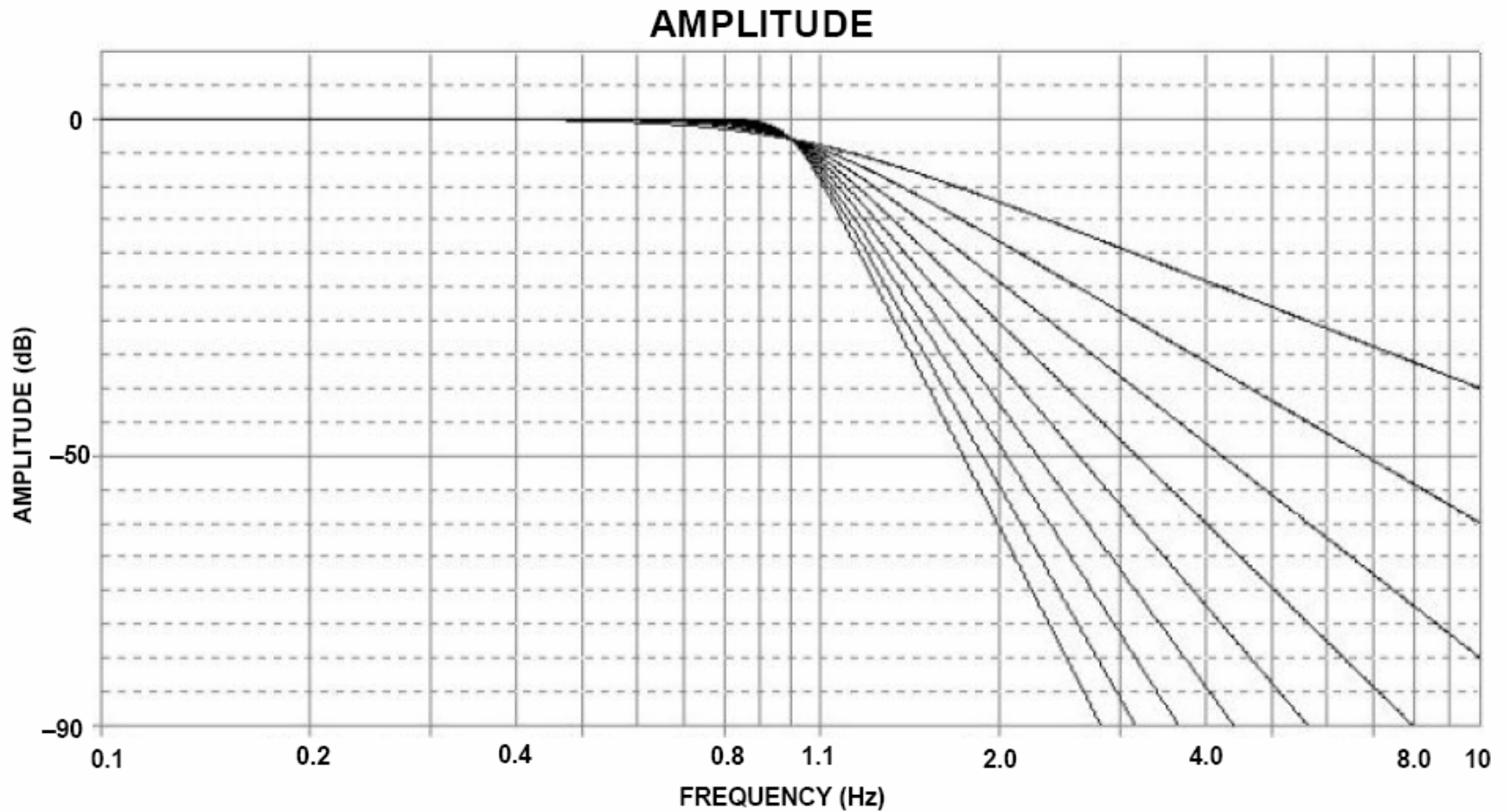
## *Filtro Passa Baixa*



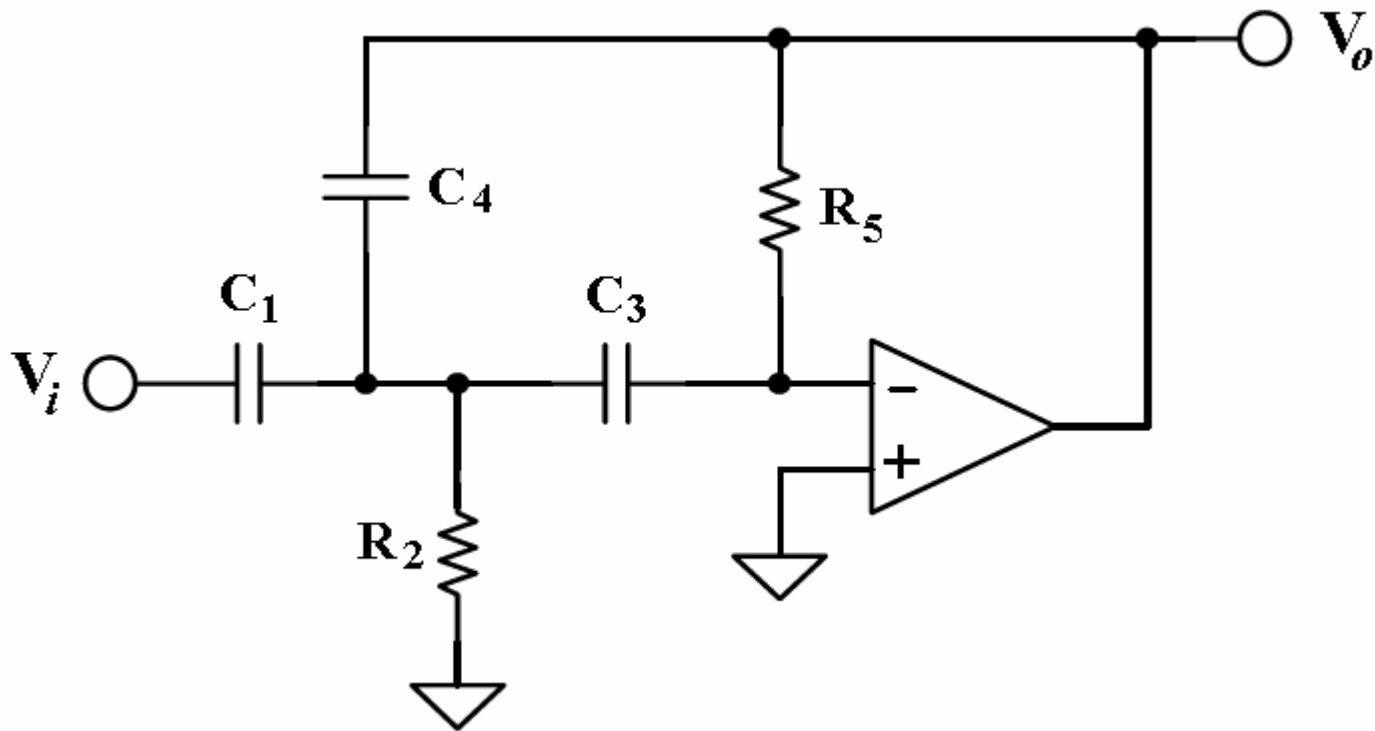
$$H(s) = \frac{H_0}{s^2 + \frac{\omega_0}{Q} s + \omega_0^2}$$

$$\omega_0 = \frac{1}{\sqrt{R_2 R_3 C_1 C_2}}$$

# *Ordem do Filtro Passa Baixa*



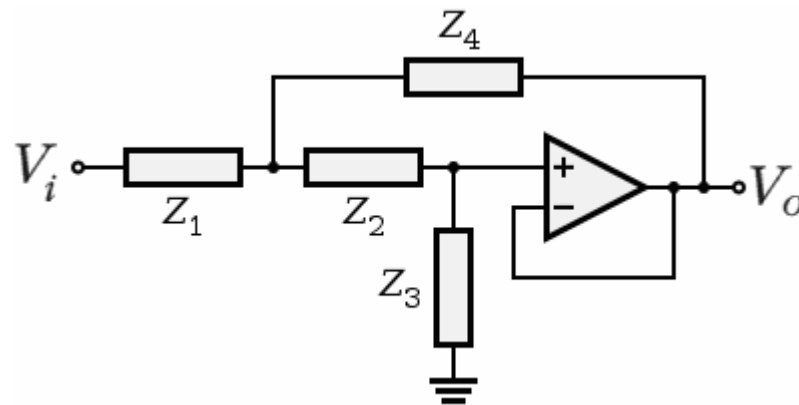
## *Filtro Passa Alta*



$$H(s) = \frac{H_0 s^2}{s^2 + \frac{\omega_0}{Q} s + \omega_0^2}$$

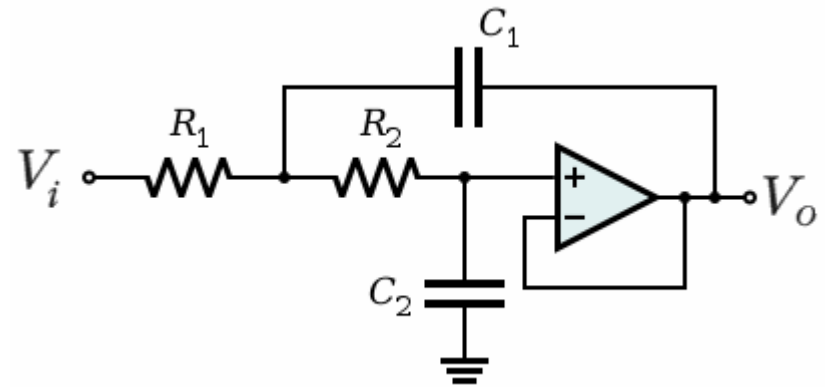
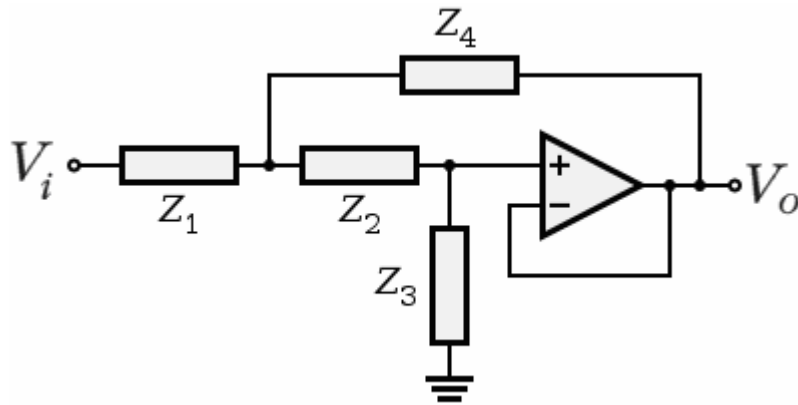


## *Topologia Sallen–Key*



$$\frac{v_o}{v_i} = \frac{Z_3 Z_4}{Z_1 Z_2 + Z_4(Z_1 + Z_2) + Z_3 Z_4}$$

# *Filtro Passa Baixa Sallen-Key*

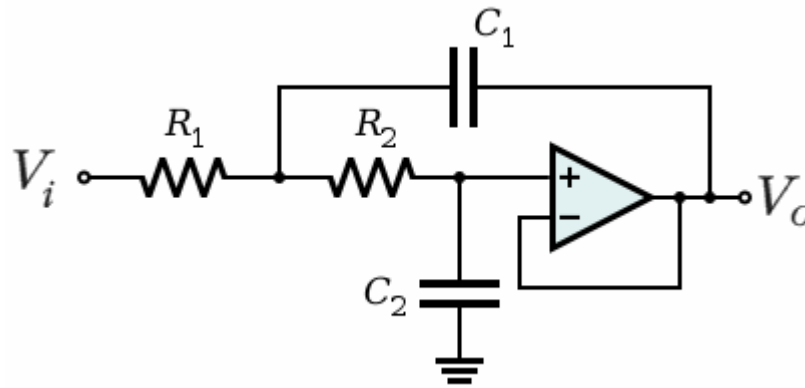


$$Z_1 = R_1, \quad Z_2 = R_2, \quad Z_3 = \frac{1}{sC_2}, \quad Z_4 = \frac{1}{sC_1}.$$

$$\frac{v_o}{v_i} = \frac{Z_3 Z_4}{Z_1 Z_2 + Z_4(Z_1 + Z_2) + Z_3 Z_4}$$

$$\frac{V_o(s)}{V_i(s)} = \frac{\frac{1}{R_1 C_1 R_2 C_2}}{s^2 + s \left( \frac{1}{R_2 C_1} + \frac{1}{R_1 C_1} \right) + \frac{1}{R_1 C_1 R_2 C_2}}$$

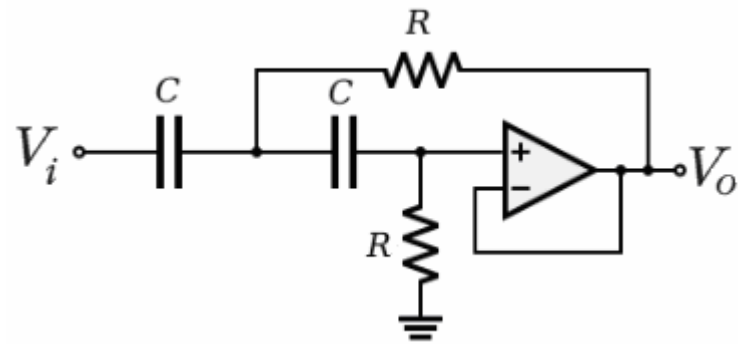
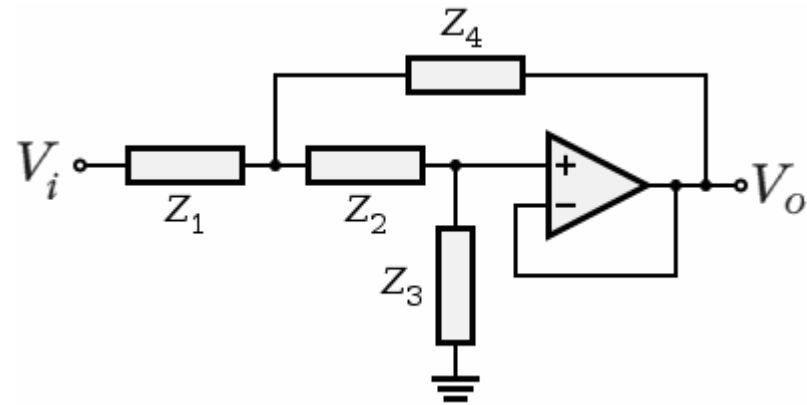
## *Filtro Passa Baixa Sallen-Key*



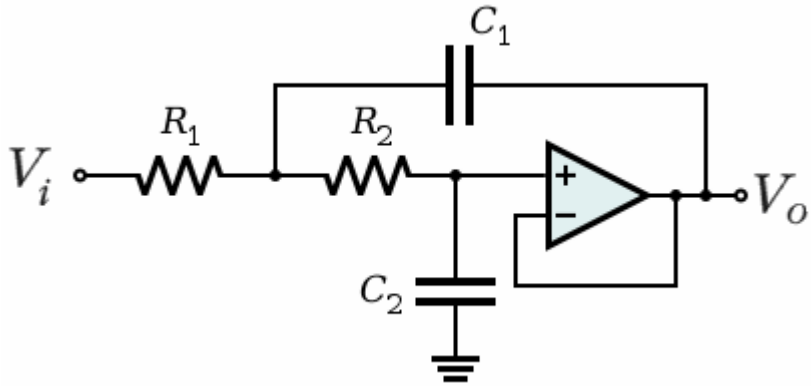
$$H(s) = \frac{1}{R_1 R_2 C_1 C_2} \frac{1}{s^2 + s \frac{1}{C_1} \left( \frac{1}{R_1} + \frac{1}{R_2} \right) + \frac{1}{R_1 R_2 C_1 C_2}}$$

$$H(s) = \frac{\omega_0^2}{s^2 + s \frac{\omega_0}{Q} + \omega_0^2}$$

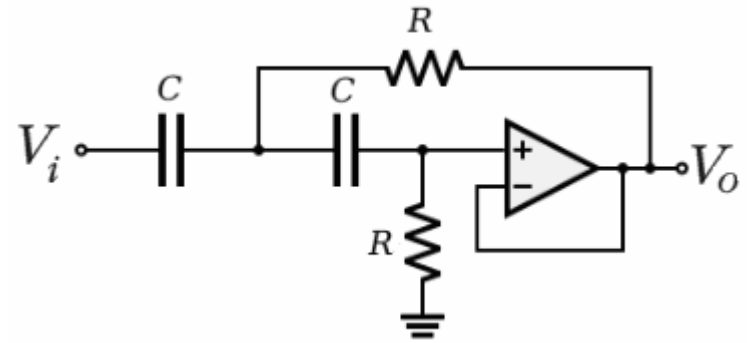
## *Filtro Passa Alta Sallen-Key*



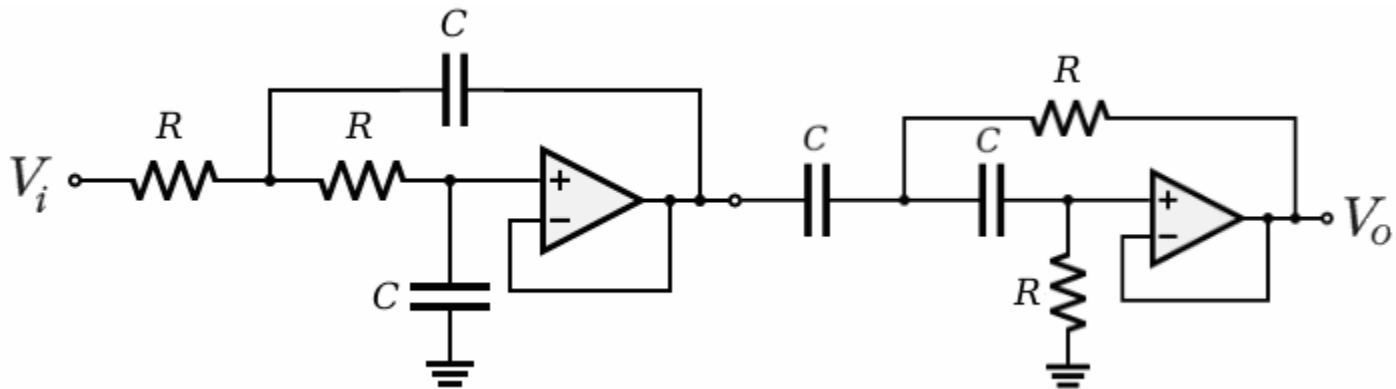
## *PASSA BAIXA*



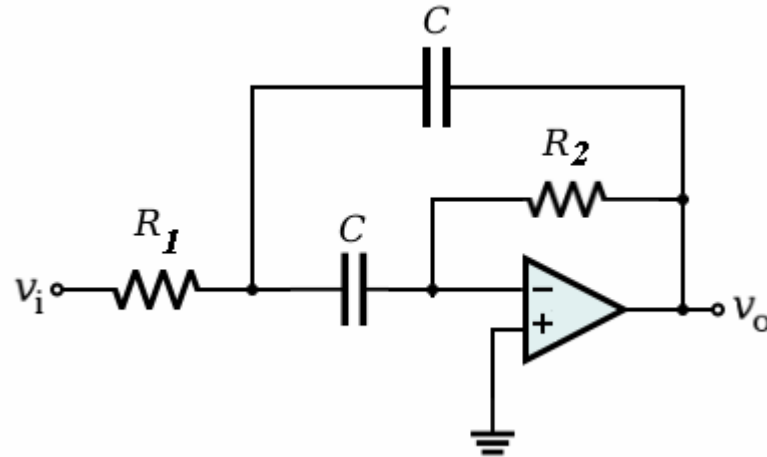
## *PASSA ALTA*



## *PASSA FAIXA*



# *Filtro Passa Faixa*

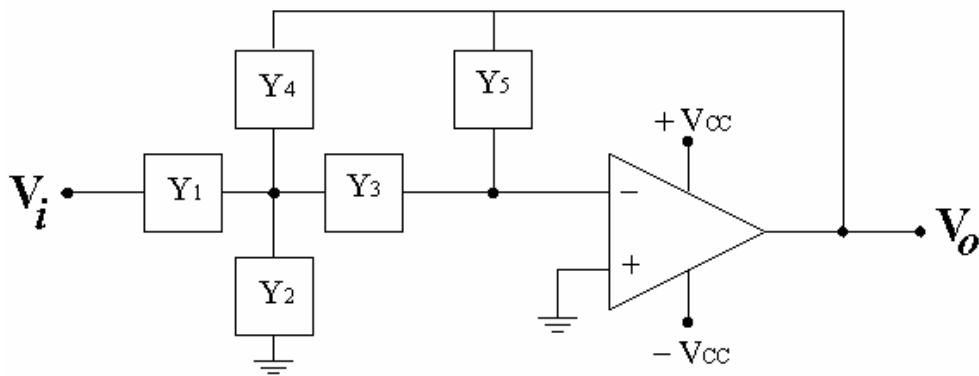


$$H(s) = -\frac{R_2}{2R_1} \frac{s \frac{2}{R_2 C}}{s^2 + \frac{2}{R_2 C} s + \frac{1}{R_1 R_2 C^2}}$$

$$\frac{V_o}{V_i} = H_0 \frac{\frac{\omega_0}{Q} s}{s^2 + \frac{\omega_0}{Q} s + \omega_0^2}$$

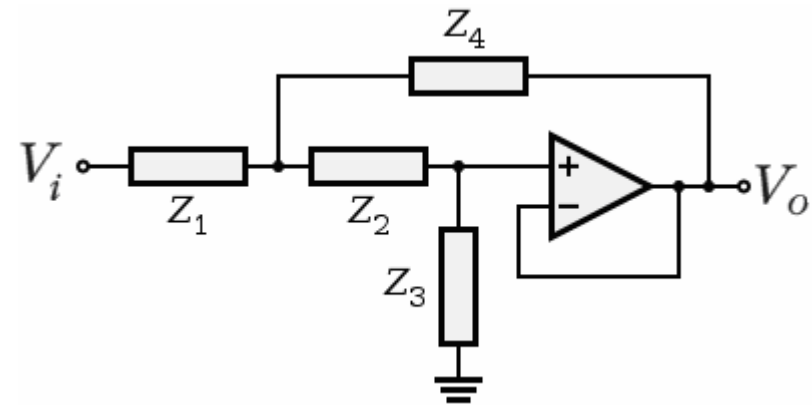
# Filtros Ativos

## Multipla Realimentação



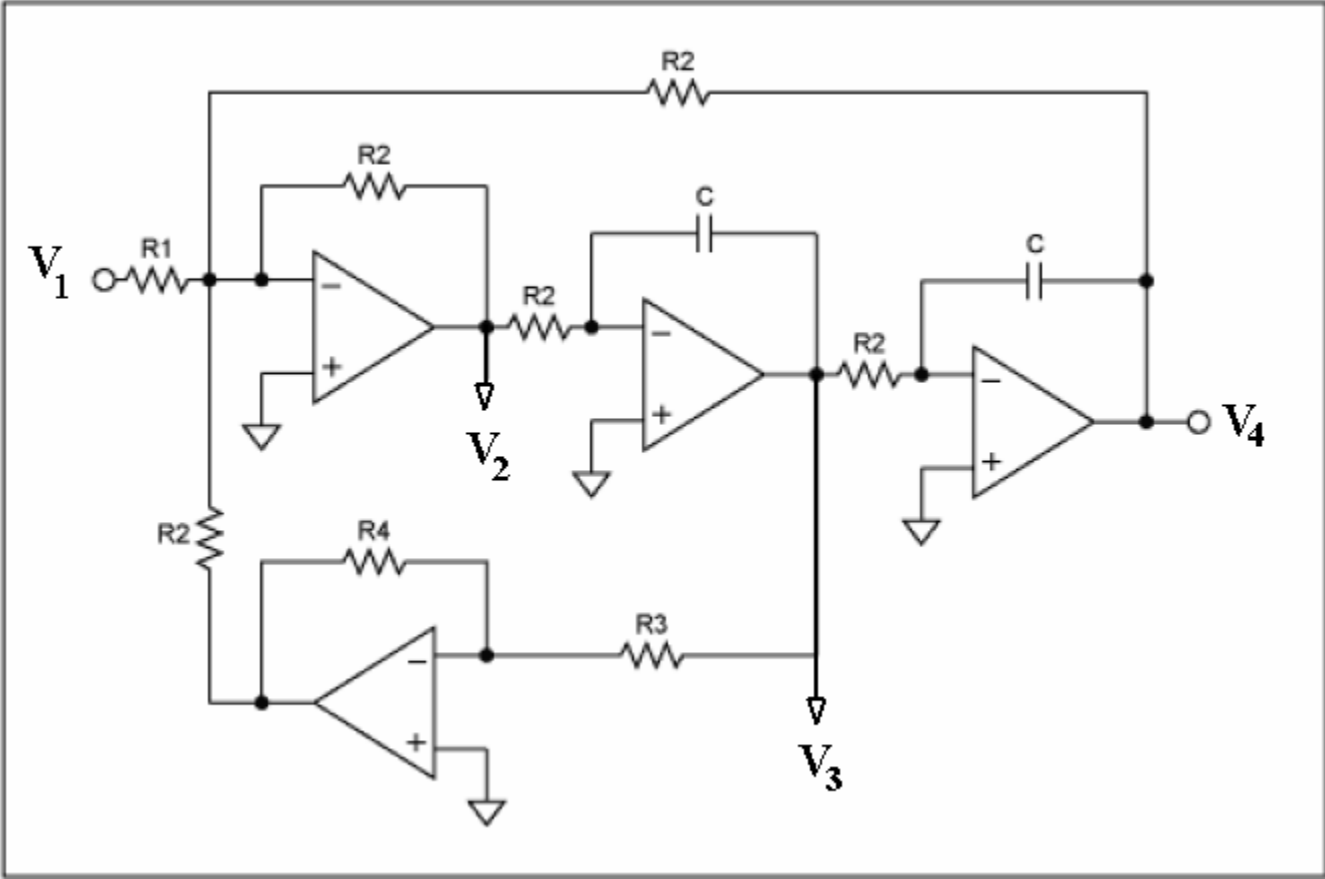
$$\frac{V_o}{V_i} = -\frac{Y_1 Y_3}{Y_5(Y_1 + Y_2 + Y_3 + Y_4) + Y_3 Y_4}$$

## Sallen-Key



$$\frac{v_o}{v_i} = \frac{Z_3 Z_4}{Z_1 Z_2 + Z_4(Z_1 + Z_2) + Z_3 Z_4}$$

# Filtro Ativo Universal

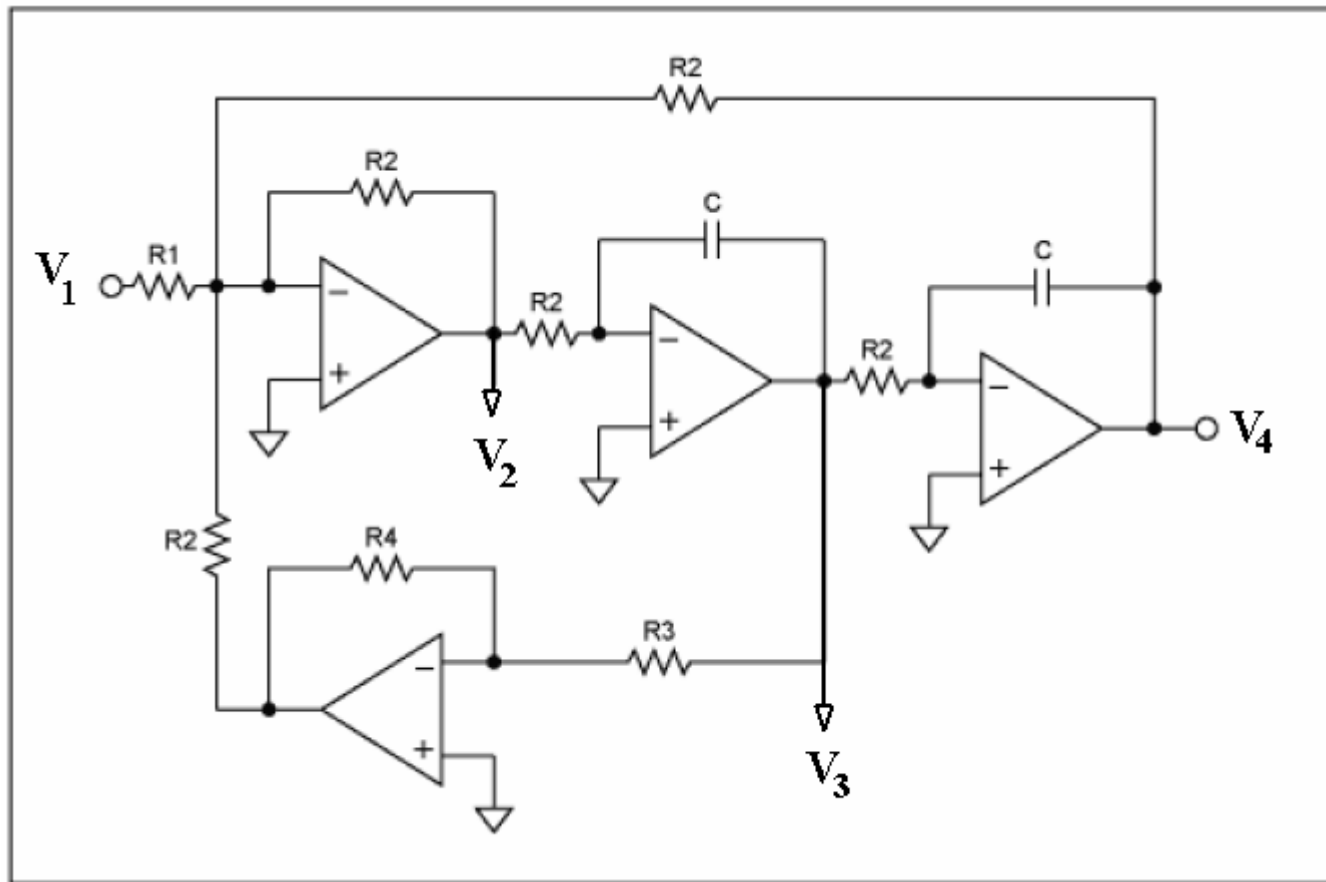




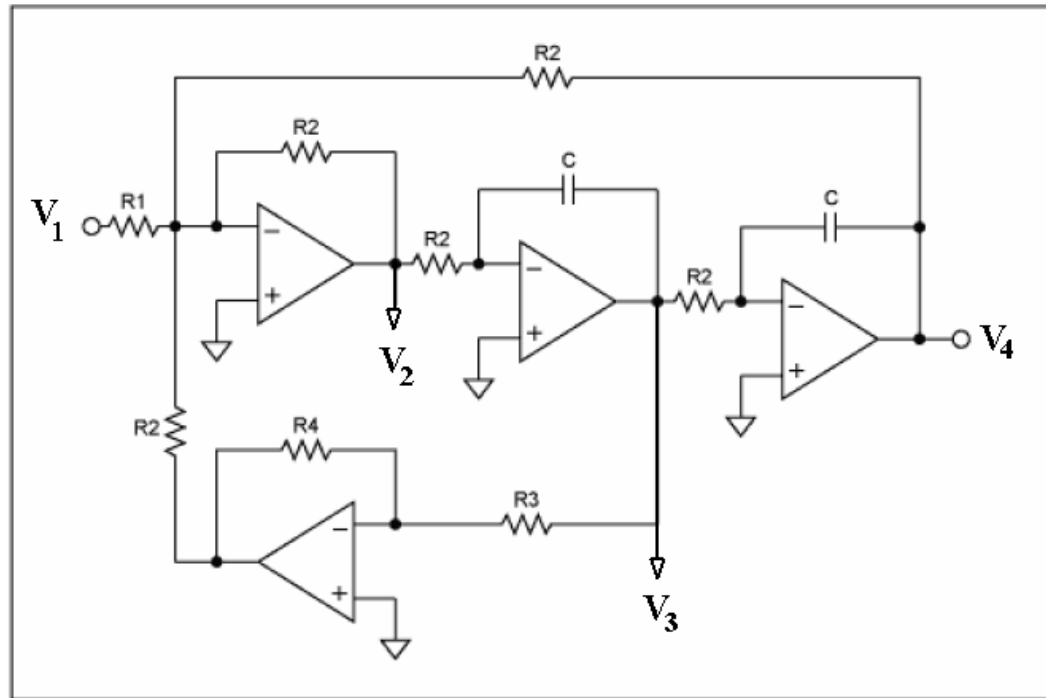
# *Filtro Ativo Universal*

## *State Variable Filter*

*Kerwin-Huelsman-Newcomb (KHN)*



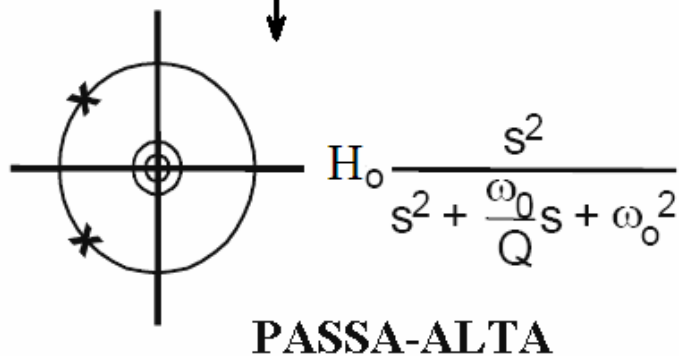
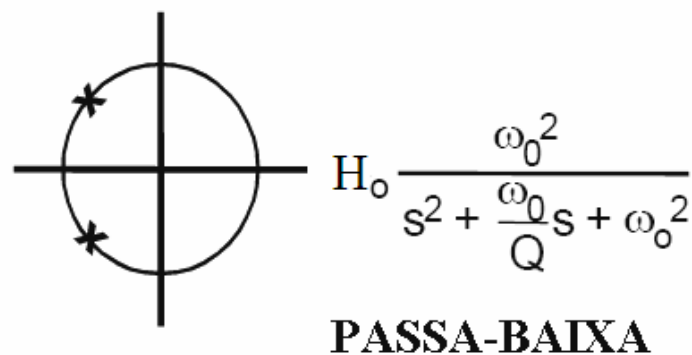
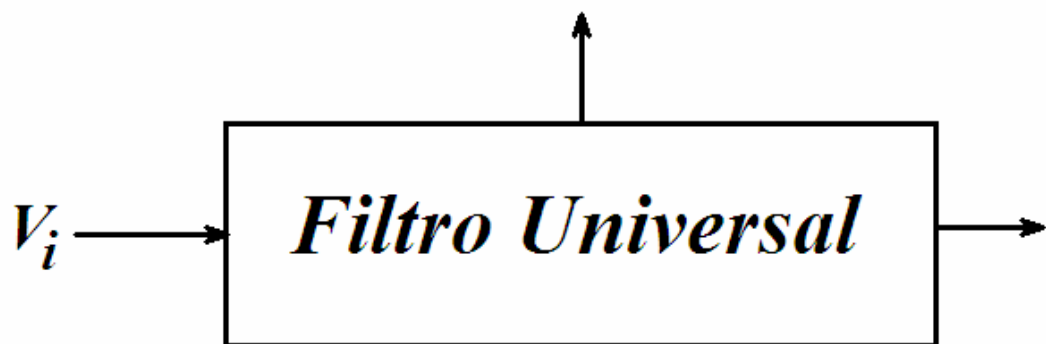
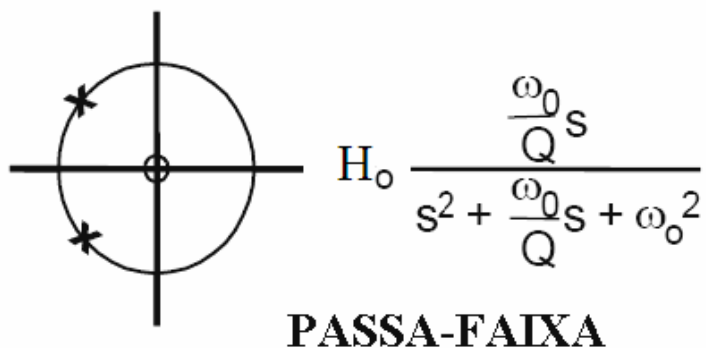
# *Filtro Ativo Universal Segunda Ordem*



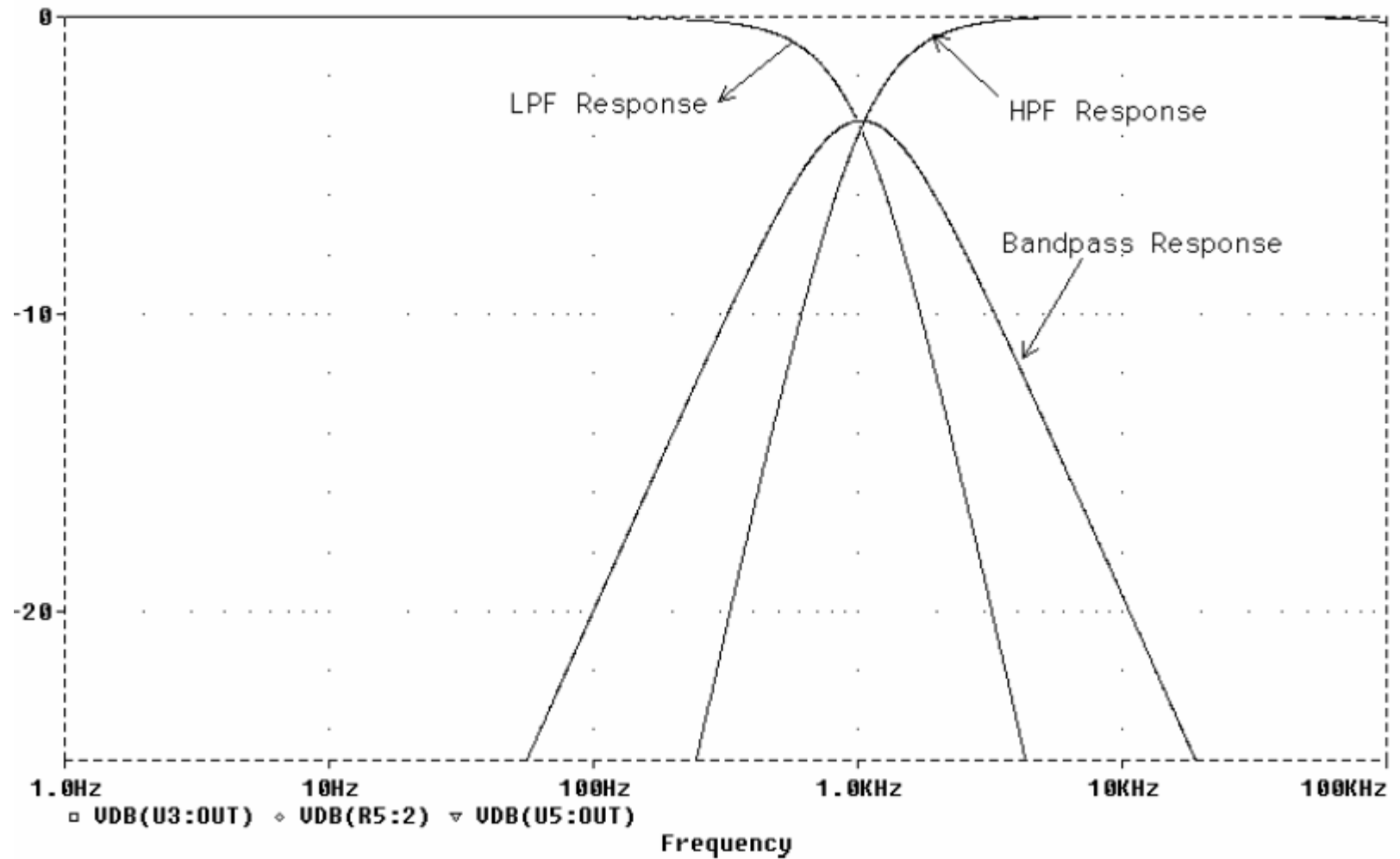
$V_2 \rightarrow$  *Filtro Passa Alta*

$V_3 \rightarrow$  *Filtro Passa Faixa*

$V_4 \rightarrow$  *Filtro Passa Baixa*



$$H(s) = \frac{as}{s^2 + bs + c}$$



*Frequency response of a state variable filter.*

# **ELETRÔNICA**

**Filtros Ativos  
Butterworth e Chebyshev**

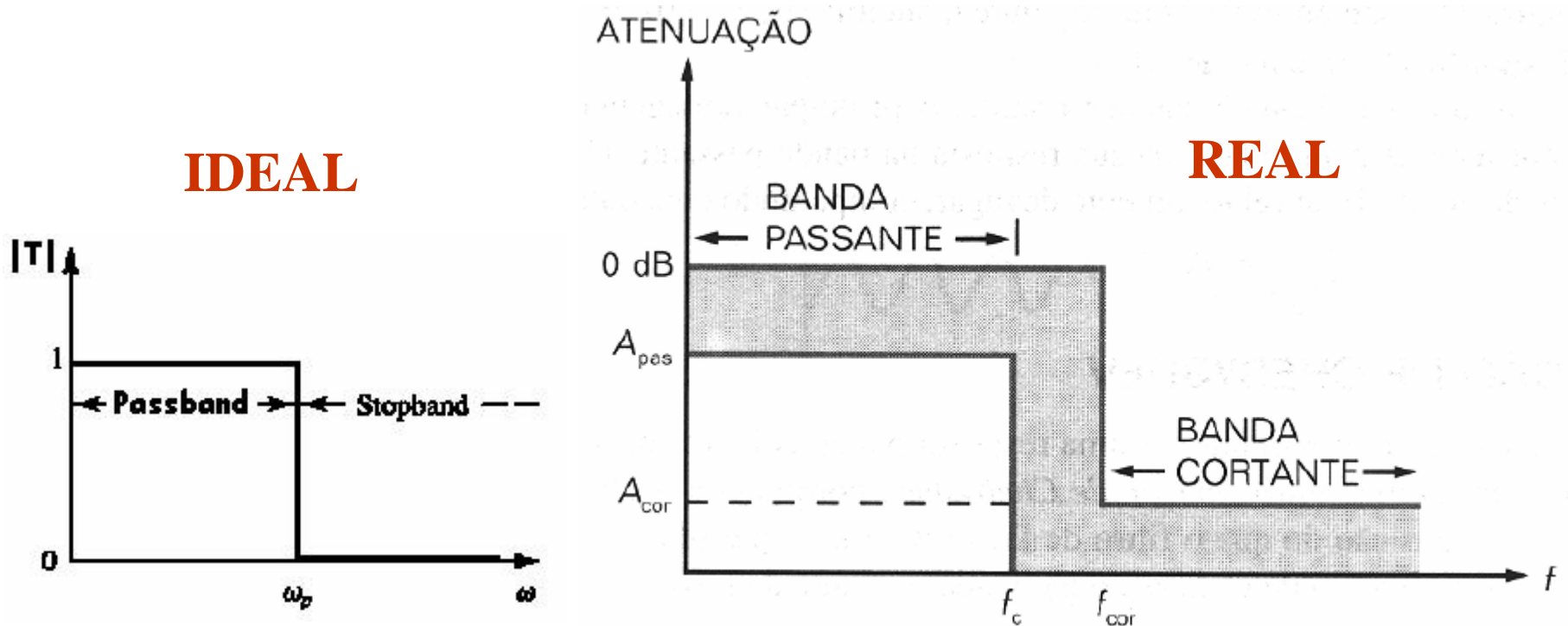
AOC

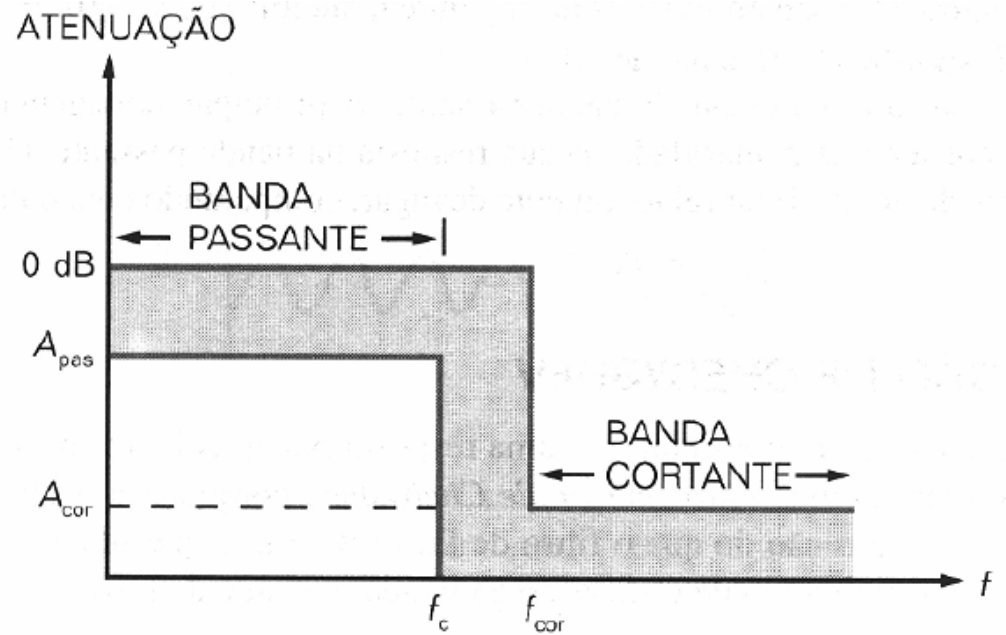
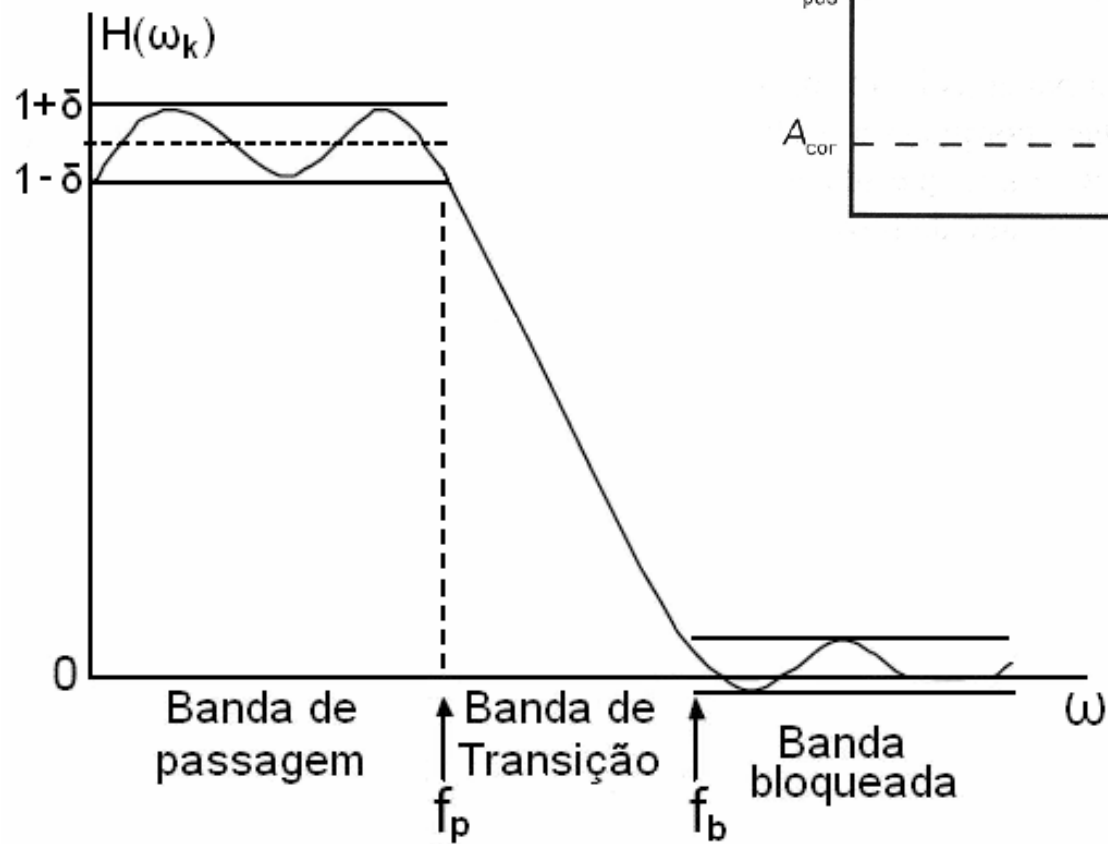


# Aproximações de Filtros

# Aproximações de Filtros

- *Filtro Butterworth*
- *Filtro Cauer ou Elíptico*
- *Filtro Chebyshev*
- *Filtro Bessel*



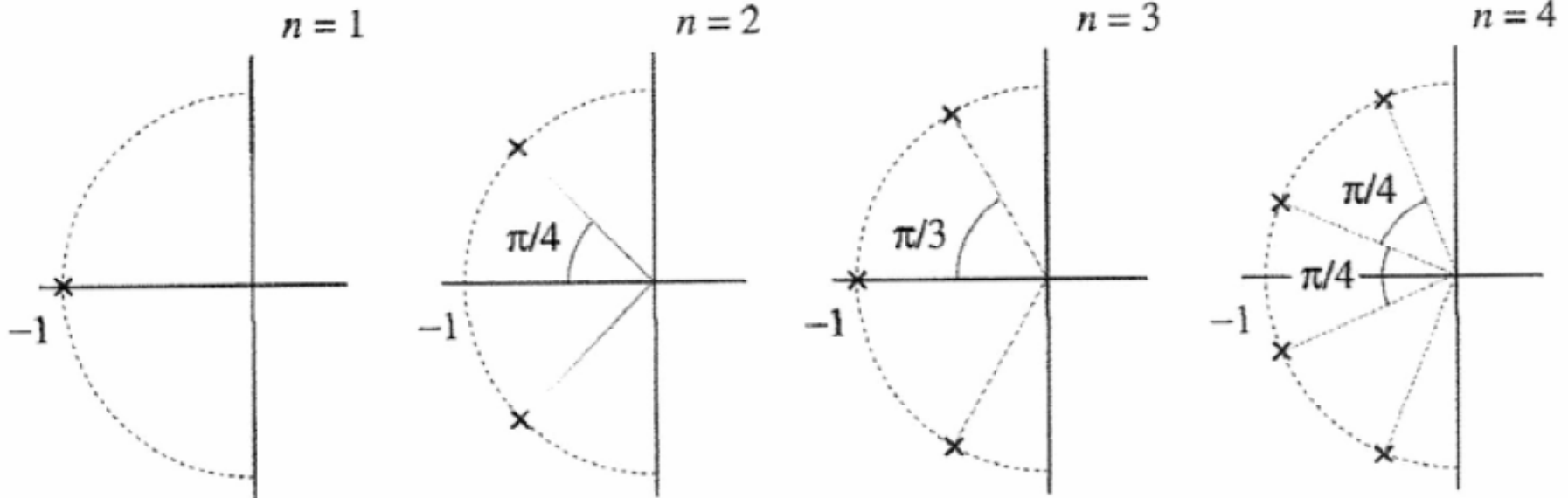




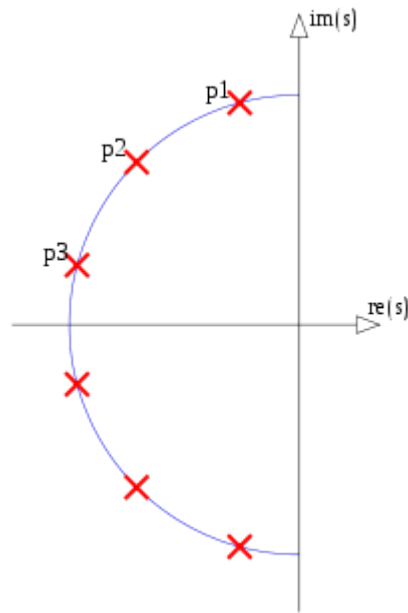
# Aproximação Butterworth

Os pólos do filtro de Butterworth são espaçados regularmente na metade esquerda de um círculo centrado na origem do plano complexo.

$$s_k = e^{\frac{j\pi}{2n}(2k+n-1)} \quad k = 1, 2, 3, \dots, n \quad (\omega_c = 1)$$

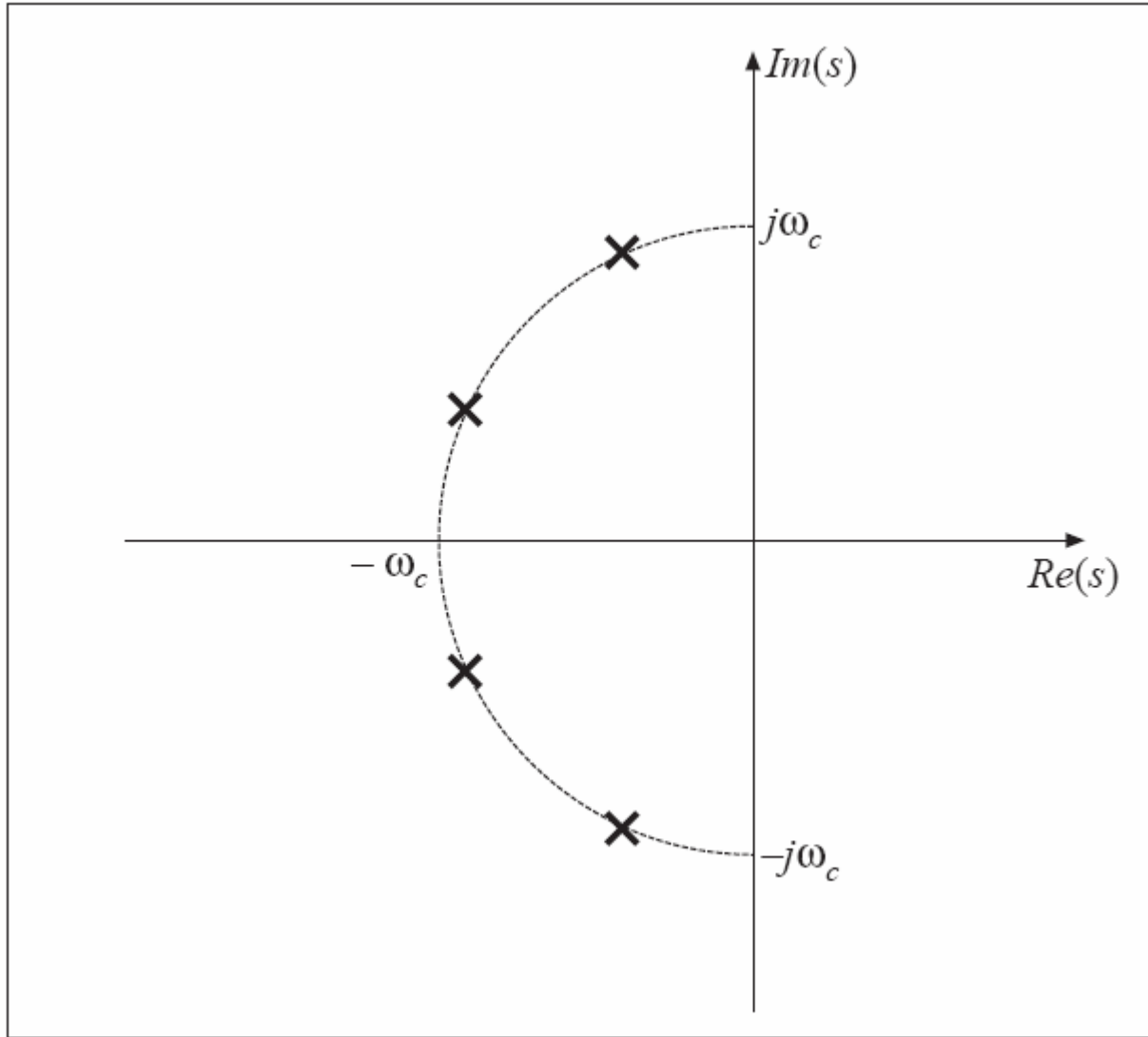


Os pólos do filtro de Butterworth são espaçados regularmente na metade esquerda de um círculo centrado na origem do plano complexo.

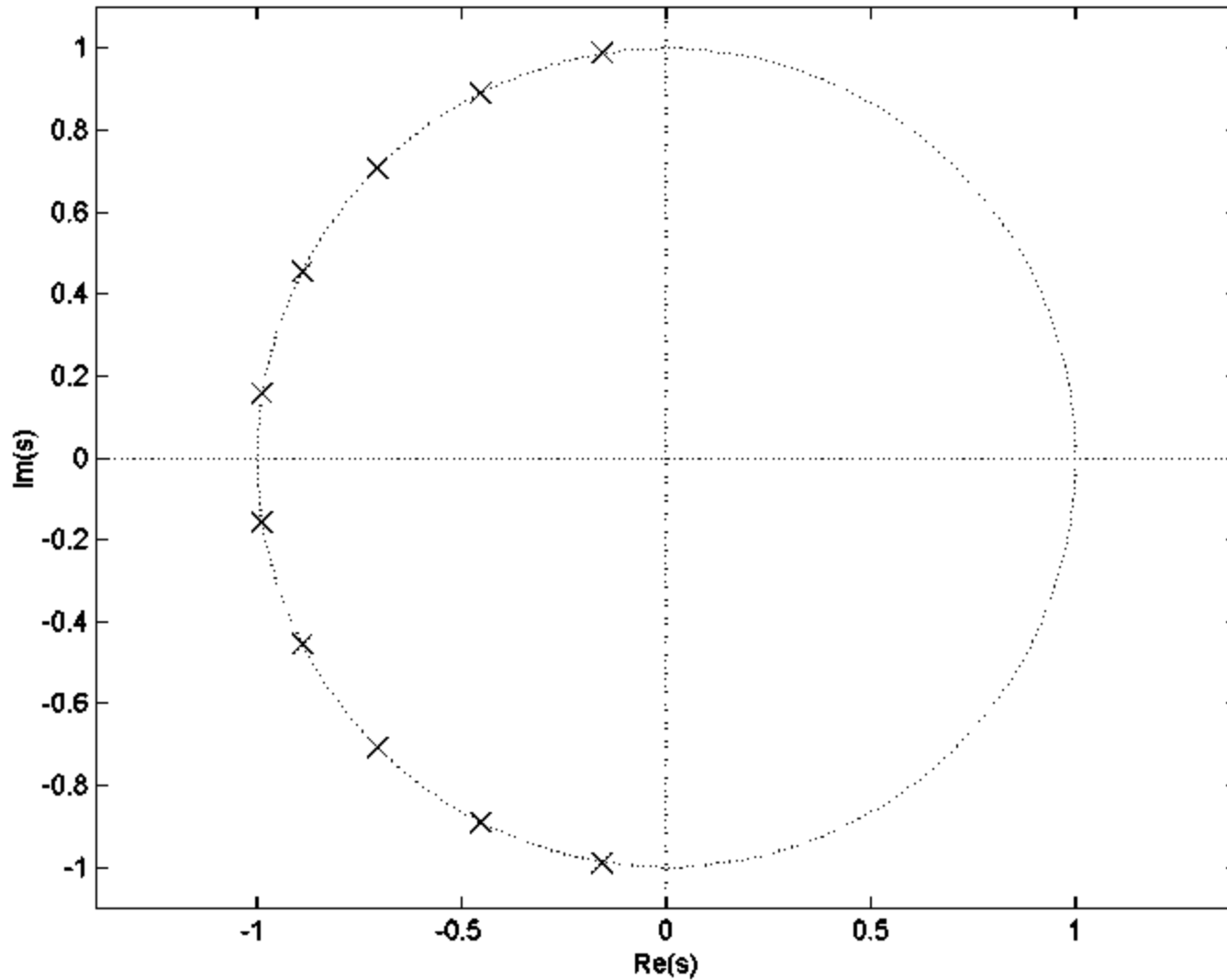


# *Polinômio de Butterworth*

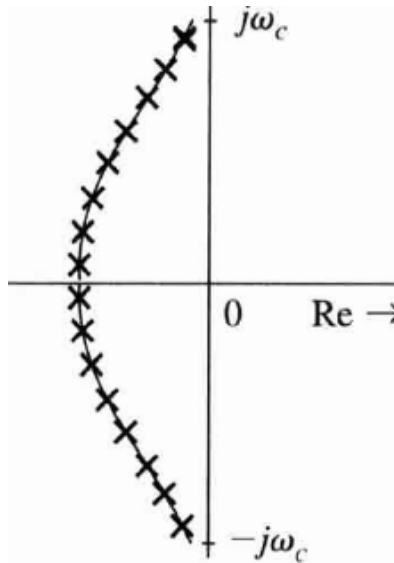
## *Quarta Ordem*



# *Aproximação Butterworth*



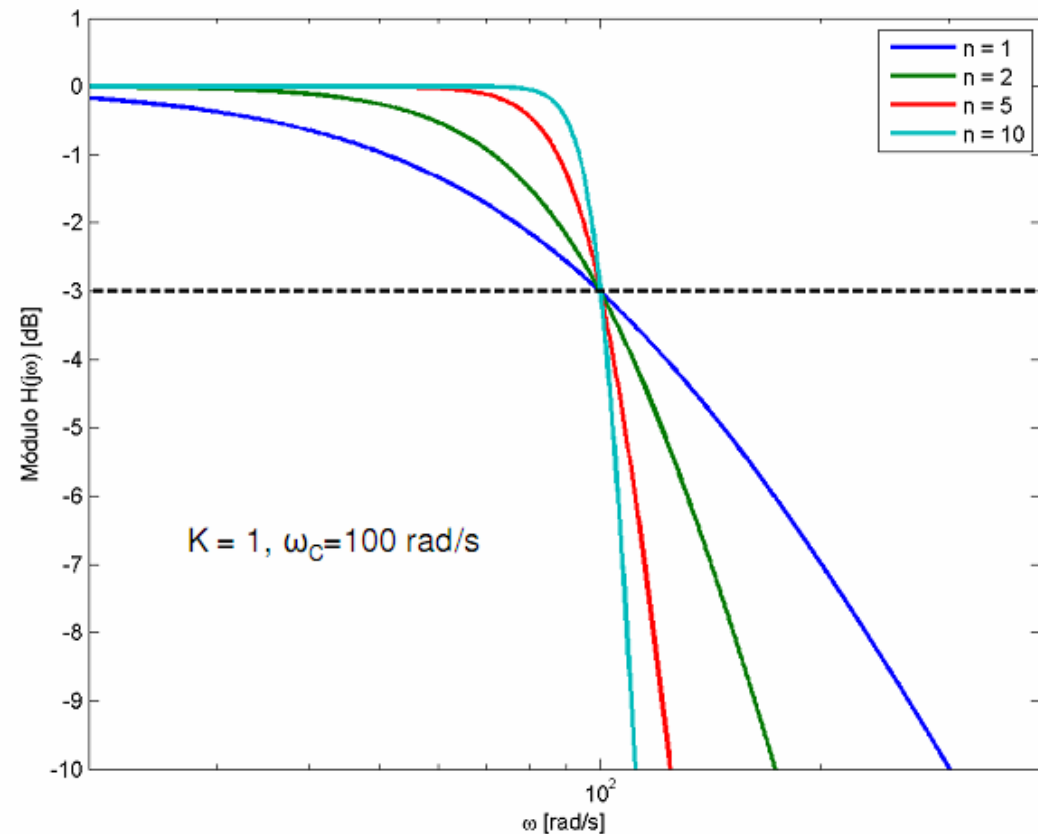
# *Aproximação Butterworth*



# Filtros ativos

- Filtros de segunda ordem
  - Filtro Passa Baixas de Butterworth
    - Fórmula do módulo para qualquer ordem n:

$$|H(j\omega)| = \frac{|K|}{\sqrt{1 + \left(\frac{\omega}{\omega_C}\right)^{2n}}}$$

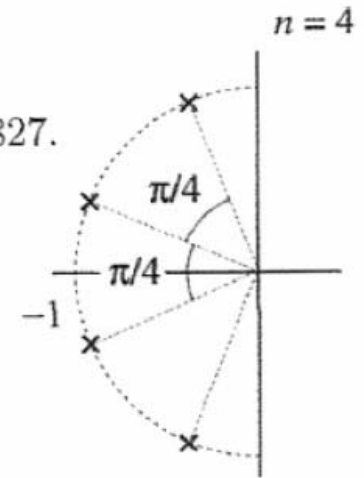


# Butterworth Filters

$$|H(\omega)| = \frac{1}{\sqrt{1 + (\omega/\omega_c)^{2n}}}$$

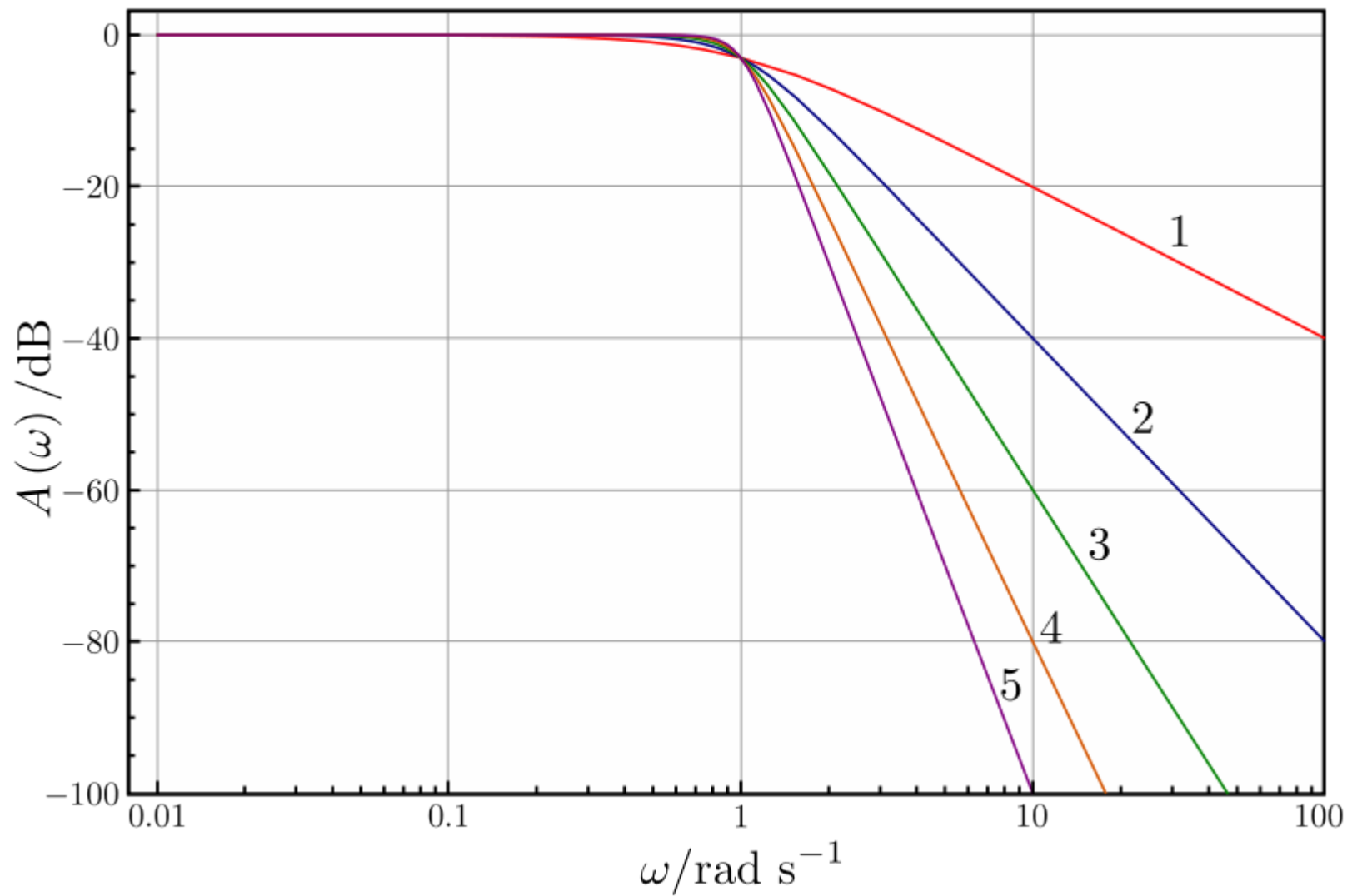
- ◆ Consider a fourth-order Butterworth filter (i.e.  $n=4$ ).
- ◆ The poles are at angles  $5\pi/8$ ,  $7\pi/8$ ,  $9\pi/8$  and  $11\pi/8$ .
- ◆ Therefore, the pole locations are:  $-0.3827 \pm j0.9239$ ,  $-0.9239 \pm j0.3827$ .
- ◆ Therefore:

$$\begin{aligned} \mathcal{H}(s) &= \frac{1}{(s^2 + 0.7654s + 1)(s^2 + 1.8478s + 1)} \\ &= \frac{1}{s^4 + 2.6131s^3 + 3.4142s^2 + 2.6131s + 1} \end{aligned}$$



Coefficients of Butterworth Polynomial  $B_n(s) = s^n + a_{n-1}s^{n-1} + \dots + a_1s + 1$

| $n$ | $a_1$      | $a_2$       | $a_3$       | $a_4$       | $a_5$       | $a_6$       | $a_7$       | $a_8$       | $a_9$      |
|-----|------------|-------------|-------------|-------------|-------------|-------------|-------------|-------------|------------|
| 2   | 1.41421356 |             |             |             |             |             |             |             |            |
| 3   | 2.00000000 | 2.00000000  |             |             |             |             |             |             |            |
| 4   | 2.61312593 | 3.41421356  | 2.61312593  |             |             |             |             |             |            |
| 5   | 3.23606798 | 5.23606798  | 5.23606798  | 3.23606798  |             |             |             |             |            |
| 6   | 3.86370331 | 7.46410162  | 9.14162017  | 7.46410162  | 3.86370331  |             |             |             |            |
| 7   | 4.49395921 | 10.09783468 | 14.59179389 | 14.59179389 | 10.09783468 | 4.49395921  |             |             |            |
| 8   | 5.12583090 | 13.13707118 | 21.84615097 | 25.68835593 | 21.84615097 | 13.13707118 | 5.12583090  |             |            |
| 9   | 5.75877048 | 16.58171874 | 31.16343748 | 41.98638573 | 41.98638573 | 31.16343748 | 16.58171874 | 5.75877048  |            |
| 10  | 6.39245322 | 20.43172909 | 42.80206107 | 64.88239627 | 74.23342926 | 64.88239627 | 42.80206107 | 20.43172909 | 6.39245322 |





# Aproximação Butterworth

## 4. Polinômiais Butterworth normalizados

| $n$ | Fatores de polinômiais $B_n(s)$   |
|-----|---|
| 1   | $(s + 1)$   |
| 2   | $s^2 + 1.414s + 1$  |
| 3   | $(s + 1)(s^2 + s + 1)$  |
| 4   | $(s^2 + 0.7654s + 1)(s^2 + 1.8478s + 1)$                                      |
| 5   | $(s + 1)(s^2 + 0.6180s + 1)(s^2 + 1.6180s + 1)$                               |
| 6   | $(s^2 + 0.5176s + 1)(s^2 + 1.414s + 1)(s^2 + 1.9318s + 1)$                    |
| 7   | $(s + 1)(s^2 + 0.4450s + 1)(s^2 + 1.247s + 1)(s^2 + 1.8022s + 1)$             |
| 8   | $(s^2 + 0.3986s + 1)(s^2 + 1.111s + 1)(s^2 + 1.6630s + 1)(s^2 + 1.9622s + 1)$ |

$$H(s) = \frac{1}{as^2 + bs + 1}$$

## *Filtro de Butterworth*

| <b>N</b> | <b>Fatores de Polinômios <math>B_N(s)</math> Normalizados</b>              |
|----------|--|
| 1        | $s + 1$  |
| 2        | $s^2 + 1.414s + 1$   |
| 3        | $(s+1)(s^2 + s + 1)$   |
| 4        | $(s^2 + 0.765s + 1)(s^2 + 1.848s + 1)$                                     |
| 5        | $(s + 1)(s^2 + 0.618s + 1)(s^2 + 1.618s + 1)$                              |
| 6        | $(s^2 + 0.518s + 1)(s^2 + 1.414s + 1)(s^2 + 1.932s + 1)$                   |
| 7        | $(s + 1)(s^2 + 0.445s + 1)(s^2 + 1.247s + 1)(s^2 + 1.802s + 1)$            |
| 8        | $(s^2 + 0.390s + 1)(s^2 + 1.111s + 1)(s^2 + 1.663s + 1)(s^2 + 1.962s + 1)$ |

## *Desnormalização em Frequência*

**Considerando que a tabela esta normalizada para a frequência de 1 rad/s podemos projetar filtros para qualquer outra frequência de corte substituindo  $s$  por  $s/\omega_c$ .**

**Por exemplo, para  $\omega_c = 100$  rad/s**

$$\begin{aligned} H(s) &= \frac{1}{\left(\frac{s}{100}\right)^2 + \sqrt{2}\left(\frac{s}{100}\right) + 1} \\ &= \frac{10^4}{s^2 + 100\sqrt{2}s + 10^4} \end{aligned}$$

$$\mathbf{H(s) = \frac{\omega_0^2}{s^2 + s \frac{\omega_0}{Q} + \omega_0^2}}$$

#### 4. Polinômiais Butterworth normalizados

**n** Fatores de polinômiais  $B_n(s)$

1  $(s + 1)$

2  $s^2 + 1.414s + 1$

3  $(s + 1)(s^2 + s + 1)$

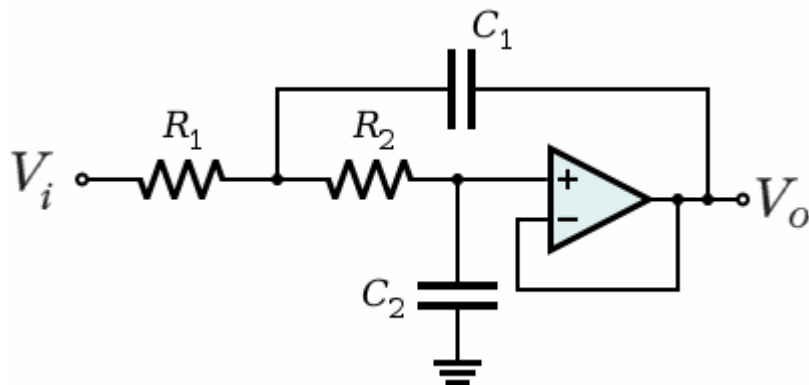
4  $(s^2 + 0.7654s + 1)(s^2 + 1.8478s + 1)$

5  $(s + 1)(s^2 + 0.6180s + 1)(s^2 + 1.6180s + 1)$

6  $(s^2 + 0.5176s + 1)(s^2 + 1.414s + 1)(s^2 + 1.9318s + 1)$

7  $(s + 1)(s^2 + 0.4450s + 1)(s^2 + 1.247s + 1)(s^2 + 1.8022s + 1)$

8  $(s^2 + 0.3986s + 1)(s^2 + 1.111s + 1)(s^2 + 1.6630s + 1)(s^2 + 1.9622s + 1)$



$$H(s) = - \frac{1}{R_1 R_2 C_1 C_2} \frac{1}{s^2 + \frac{1}{C_1} \left( \frac{1}{R_1} + \frac{1}{R_2} \right) s + \frac{1}{R_1 R_2 C_1 C_2}}$$

$$H(s) = H_0 \frac{b_0}{s^2 + a_1 s + a_0}$$

## 4. Polinômiais Butterworth normalizados

<sup>n</sup> Fatores de polinômiais  $B_n(s)$

1  $(s + 1)$

2  $s^2 + 1.414s + 1$

3  $(s + 1)(s^2 + s + 1)$

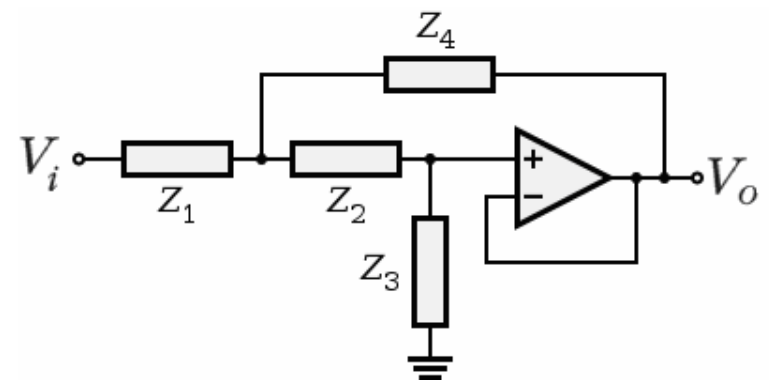
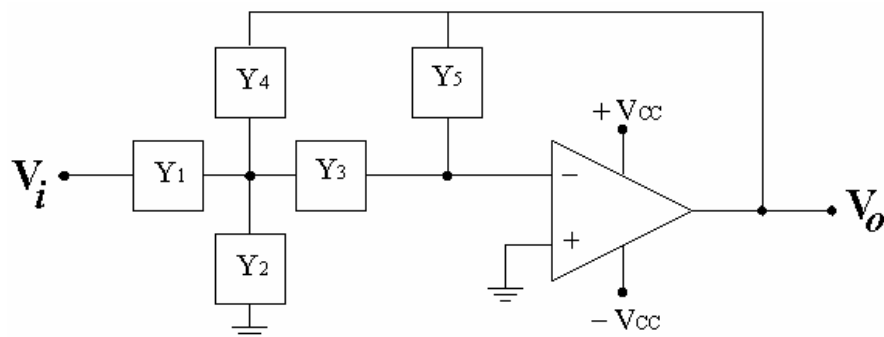
4  $(s^2 + 0.7654s + 1)(s^2 + 1.8478s + 1)$

5  $(s + 1)(s^2 + 0.6180s + 1)(s^2 + 1.6180s + 1)$

6  $(s^2 + 0.5176s + 1)(s^2 + 1.414s + 1)(s^2 + 1.9318s + 1)$

7  $(s + 1)(s^2 + 0.4450s + 1)(s^2 + 1.247s + 1)(s^2 + 1.8022s + 1)$

8  $(s^2 + 0.3986s + 1)(s^2 + 1.111s + 1)(s^2 + 1.6630s + 1)(s^2 + 1.9622s + 1)$



# *Coefficientes do Filtro Butterworth*

## **Butterworth filter**

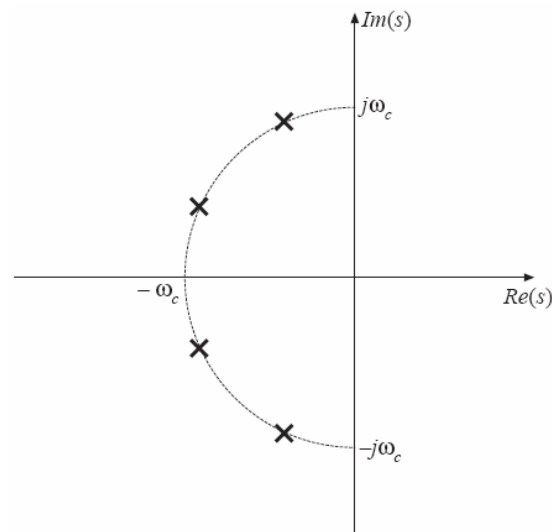
$$G(s) = \frac{1}{B(s)}$$

$B(s) = \text{Butterworth polynomial}$

---

| $n$ | $B(s)$  |
|-----|---|
| 1   | $\left(1 + \frac{s}{\omega_c}\right)$   |
| 2   | $\left(1 + \sqrt{2} \frac{s}{\omega_c} + \left(\frac{s}{\omega_c}\right)^2\right)$  |
| 3   | $\left(1 + \frac{s}{\omega_c}\right)\left(1 + \frac{s}{\omega_c} + \left(\frac{s}{\omega_c}\right)^2\right)$  |
| 4   | $\left(1 + 0.7654 \frac{s}{\omega_c} + \left(\frac{s}{\omega_c}\right)^2\right)\left(1 + 1.848 \frac{s}{\omega_c} + \left(\frac{s}{\omega_c}\right)^2\right)$ |

---



## Butterworth filter

---

$n$

$B(s)$

---

1

$$\left(1 + \frac{s}{\omega_c}\right)$$

2

$$\left(1 + \sqrt{2} \frac{s}{\omega_c} + \left(\frac{s}{\omega_c}\right)^2\right)$$

3

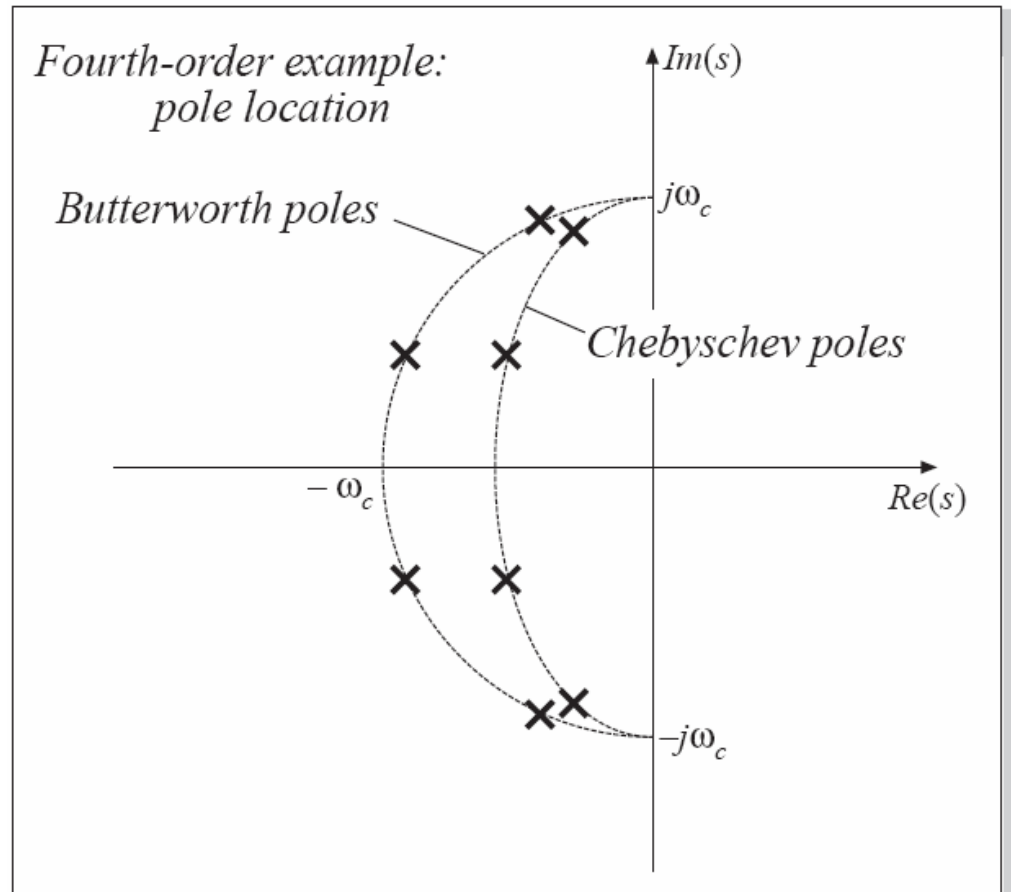
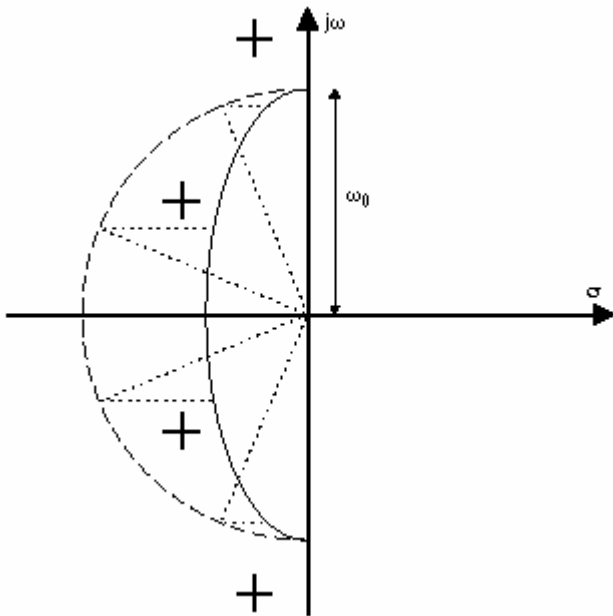
$$\left(1 + \frac{s}{\omega_c}\right)\left(1 + \frac{s}{\omega_c} + \left(\frac{s}{\omega_c}\right)^2\right)$$

4

$$\left(1 + 0.7654 \frac{s}{\omega_c} + \left(\frac{s}{\omega_c}\right)^2\right)\left(1 + 1.848 \frac{s}{\omega_c} + \left(\frac{s}{\omega_c}\right)^2\right)$$


---

Um filtro de Chebyshev pode ser obtido movendo-se cada pólo para mais próximo da linha central do eixo imaginário  $j$  de um filtro Butterworth na mesma proporção, de modo que os pólos se encontrem em uma elipse.

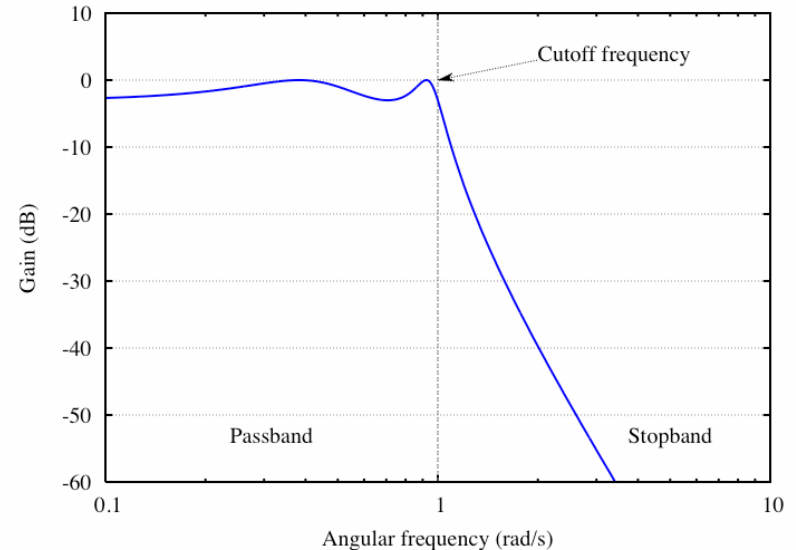




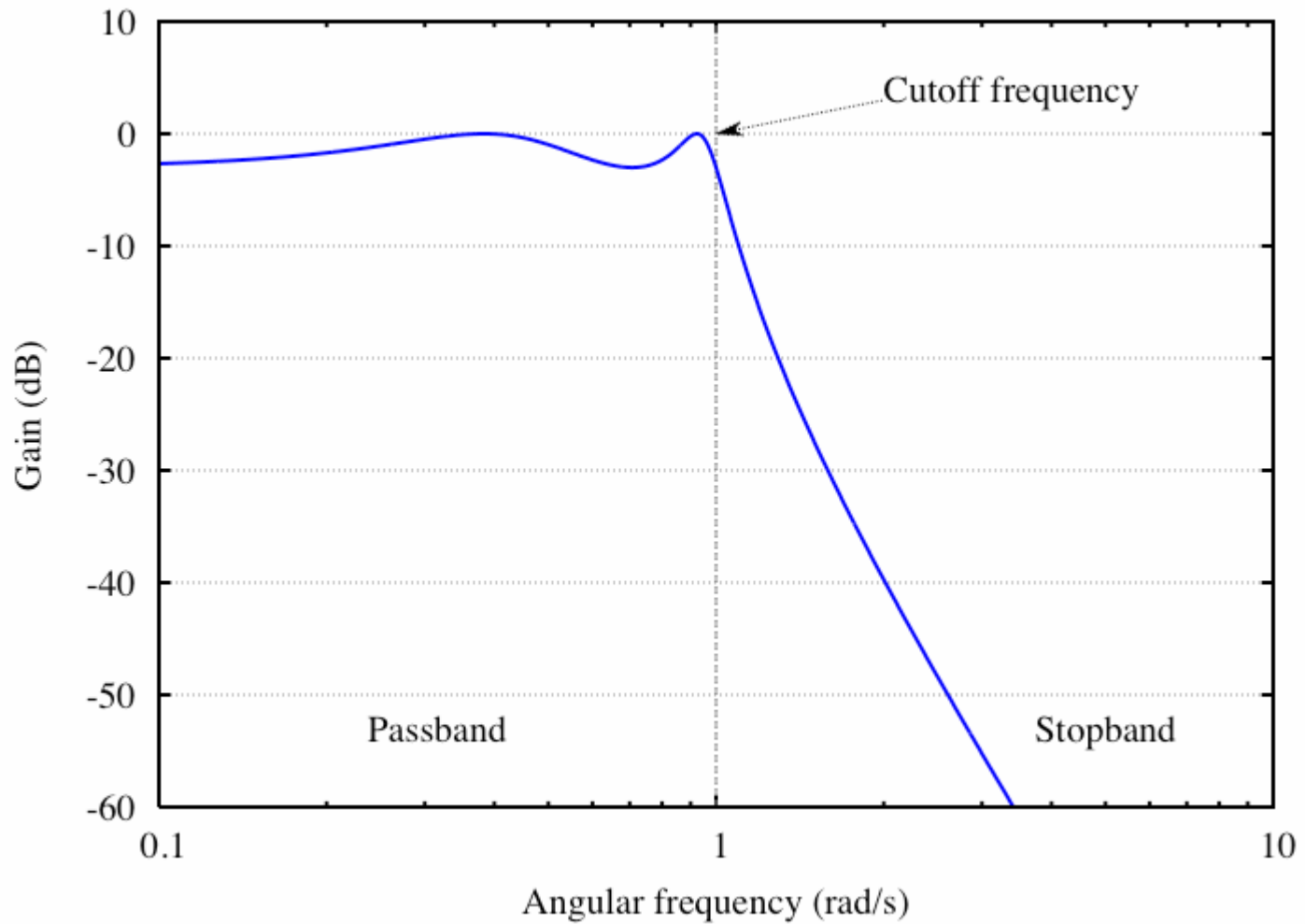
# *Filtro Chebyshev*

- Os *Filtros Chebyshev* são filtros analógicos ou digitais que possuem um aumento na atenuação mais íngreme e uma maior ondulação (ripple) na banda passante que os Filtros Butterworth.

$$|H(\omega)| = \frac{1}{\sqrt{1 + \epsilon^2 T_n^2 \left( \frac{\omega}{\omega_0} \right)}}$$



# *Filtros Chebyshev*



| N   | Fatores de Polinômios Normalizados   |
|---|--|
| <b>ONDULAÇÃO DE 0,5 dB (<math>\epsilon = 0.3493</math>)</b> |  |
| 1   | $s + 2.863$  |
| 2   | $s^2 + 1.425s + 1.516$   |
| 3   | $(s + 0.626)(s^2 + 0.626s + 1.142)$  |
| 4   | $(s^2 + 0.351s + 1.064)(s^2 + 0.845s + 0.356)$   |
| 5   | $(s + 0.362)(s^2 + 0.224s + 1.036)(s^2 + 0.586s + 0.477)$  |
| 6   | $(s^2 + 0.1554s + 1.024)(s^2 + 0.4142s + 0.5475)(s^2 + 0.5796s + 0.157)$                           |
| 7   | $(s + 0.2562)(s^2 + 0.1014s + 1.015)(s^2 + 0.3194s + 0.6657)(s^2 + 0.4616s + 0.2539)$              |
| 8   | $(s^2 + 0.0872s + 1.012)(s^2 + 0.2484s + 0.7413)(s^2 + 0.3718s + 0.3872)(s^2 + 0.4386s + 0.08805)$ |
| <b>ONDULAÇÃO DE 1.0 dB (<math>\epsilon = 0.5089</math>)</b> |  |
| 1   | $s + 1.965$  |
| 2   | $s^2 + 1.098s + 1.103$   |
| 3   | $(s + 0.494)(s^2 + 0.494s + 0.994)$  |
| 4   | $(s^2 + 0.279s + 0.987)(s^2 + 0.674s + 0.279)$   |
| 5   | $(s + 0.289)(s^2 + 0.179s + 0.988)(s^2 + 0.468s + 0.429)$  |
| 6   | $(s^2 + 0.1244s + 0.9907)(s^2 + 0.3398s + 0.5577)(s^2 + 0.4642s + 0.1247)$                         |
| 7   | $(s + 0.2054)(s^2 + 0.0914s + 0.9927)(s^2 + 0.2562s + 0.6535)(s^2 + 0.3702s + 0.2304)$             |
| 8   | $(s^2 + 0.07s + 0.9942)(s^2 + 0.1994s + 0.7236)(s^2 + 0.2994s + 0.3408)(s^2 + 0.3518s + 0.00702)$  |

# *Coefficientes do Filtro Chebyshev*

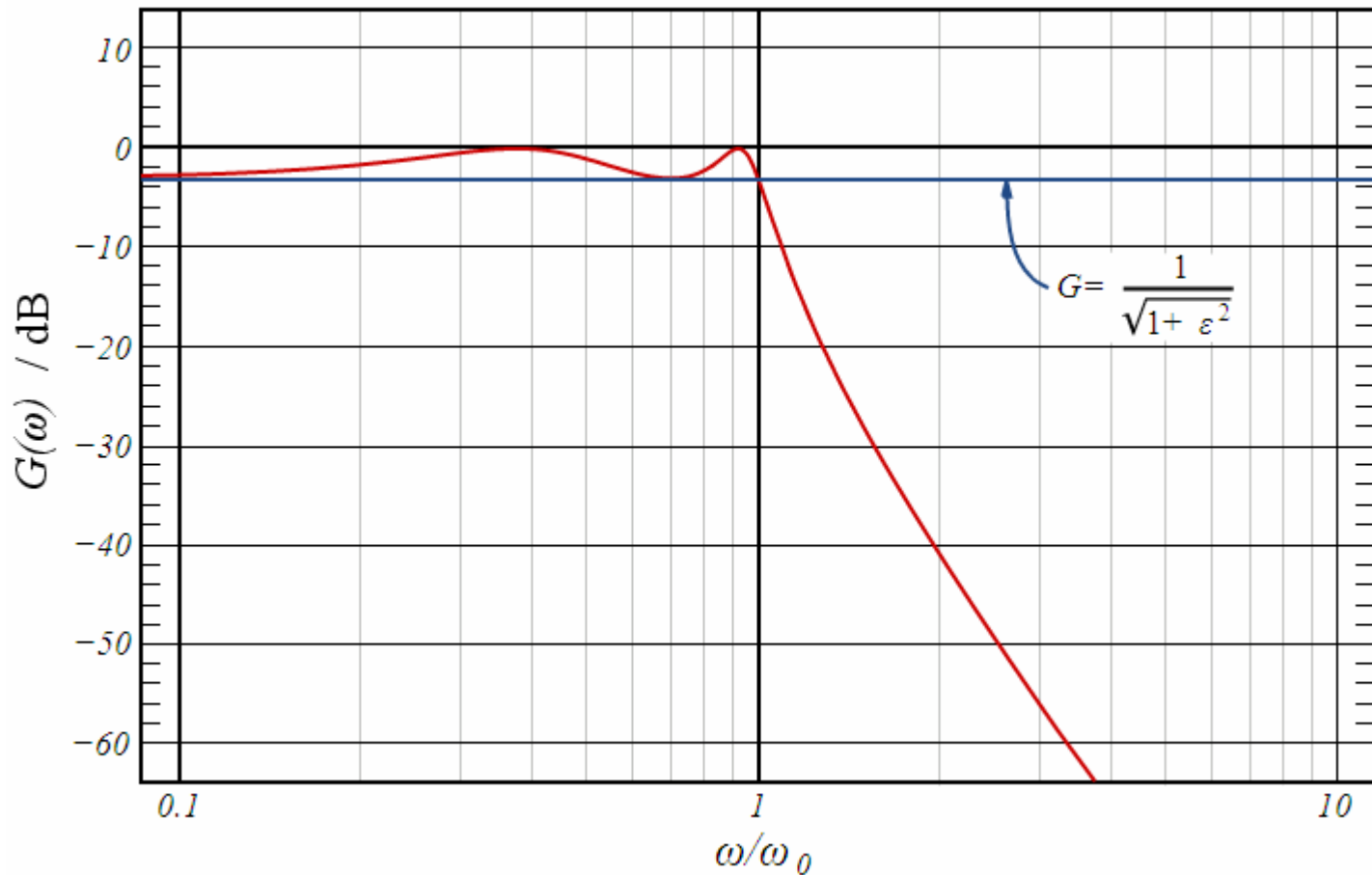
## **Chebyshev filter**

| <i>n</i> | <i>G(s)</i>  |
|----------|--|
| 1        | $\frac{1}{\left(1 + \frac{s}{\omega_c}\right)}$  |
| 2        | $\frac{1}{\sqrt{2}} \frac{1}{\left(1 + 0.7654 \left(\frac{s}{0.8409 \omega_c}\right) + \left(\frac{s}{0.8409 \omega_c}\right)^2\right)}$   |
| 3        | $\frac{1}{\left(1 + \left(\frac{s}{0.2980 \omega_c}\right)\right) \left(1 + 0.3254 \left(\frac{s}{0.9159 \omega_c}\right) + \left(\frac{s}{0.9159 \omega_c}\right)^2\right)}$  |
| 4        | $\frac{1}{\sqrt{2}} \frac{1}{\left(1 + 0.1789 \left(\frac{s}{0.9502 \omega_c}\right) + \left(\frac{s}{0.9502 \omega_c}\right)^2\right) \left(1 + 0.9276 \left(\frac{s}{0.4425 \omega_c}\right) + \left(\frac{s}{0.4425 \omega_c}\right)^2\right)}$ |

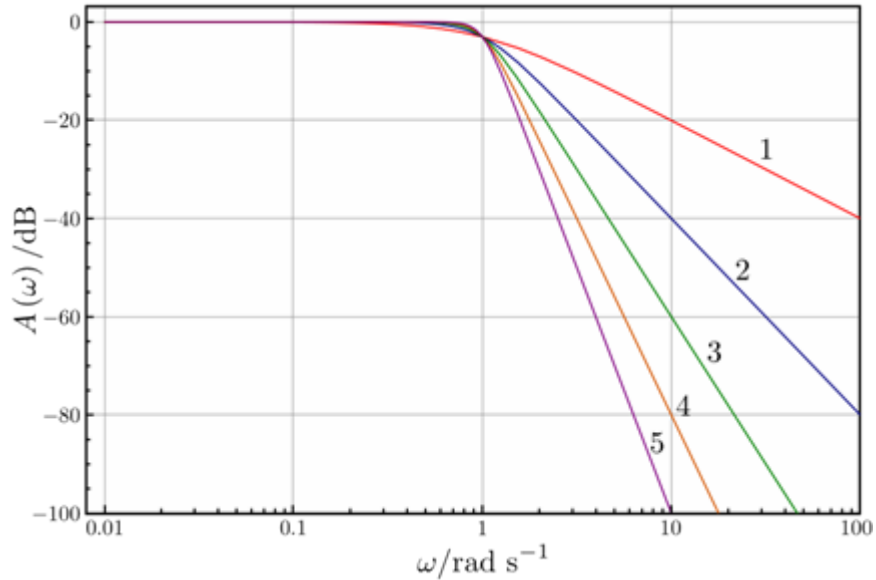
# *Função de Transferência*

## *Filtro Chebyshev*

$$|H_n(j\omega)| = \frac{1}{\sqrt{1 + \varepsilon^2 T_n^2\left(\frac{\omega}{\omega_0}\right)}}$$

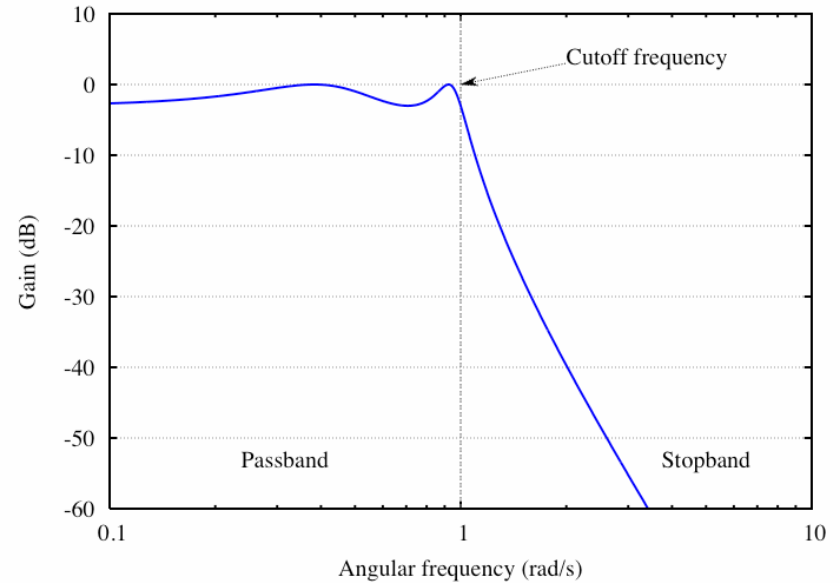


## Butterworth

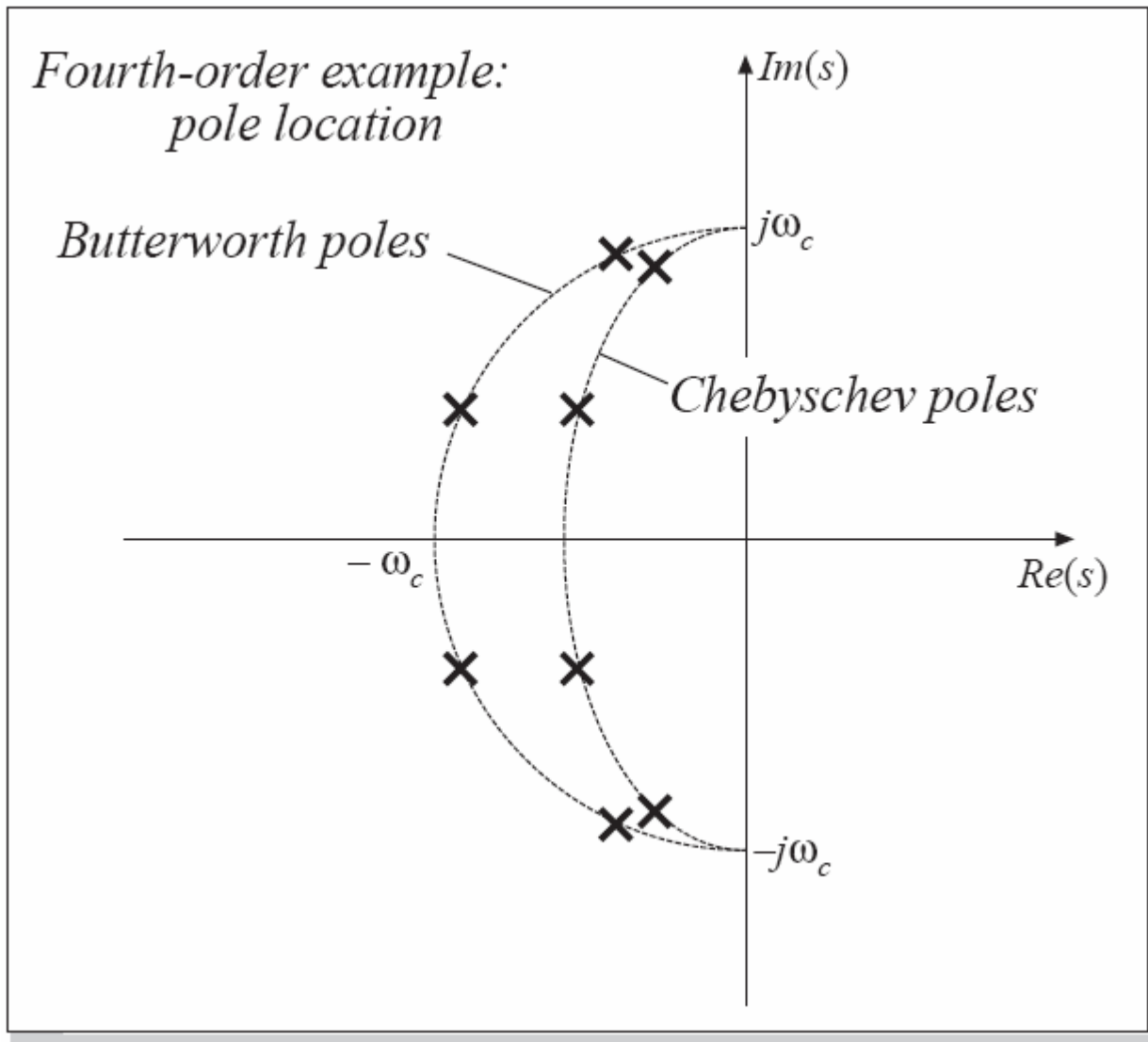


$$|H(\omega)| = \frac{1}{\sqrt{1 + (\omega/\omega_c)^{2n}}}$$

## Chebyshev



$$|H(\omega)| = \frac{1}{\sqrt{1 + \epsilon^2 T_n^2\left(\frac{\omega}{\omega_0}\right)}}$$



$$H(s) = \frac{a_o}{(s^2 + as + b)(s^2 + cs + d)}$$

## *Butterworth*

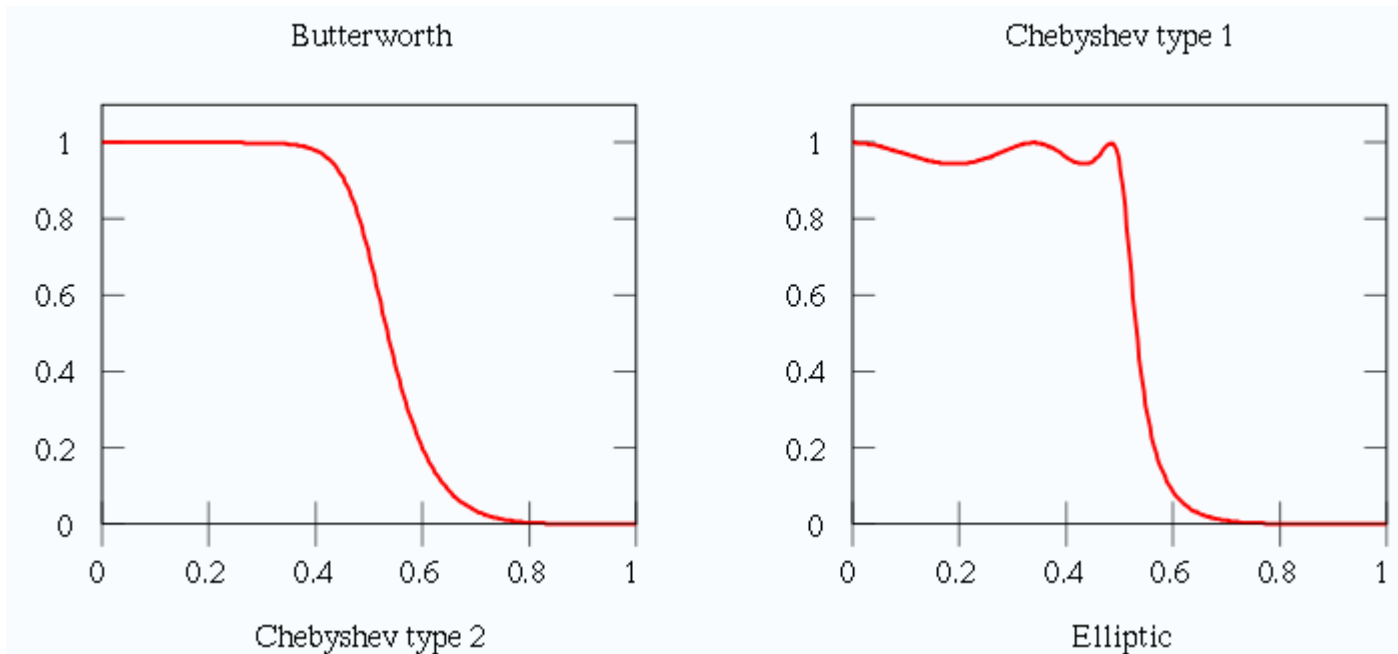
$$G(s) = \frac{1}{\left(1 + 0.7654\left(\frac{s}{\omega_c}\right) + \left(\frac{s}{\omega_c}\right)^2\right)\left(1 + 1.848\left(\frac{s}{\omega_c}\right) + \left(\frac{s}{\omega_c}\right)^2\right)}$$

## *Chebyshev*

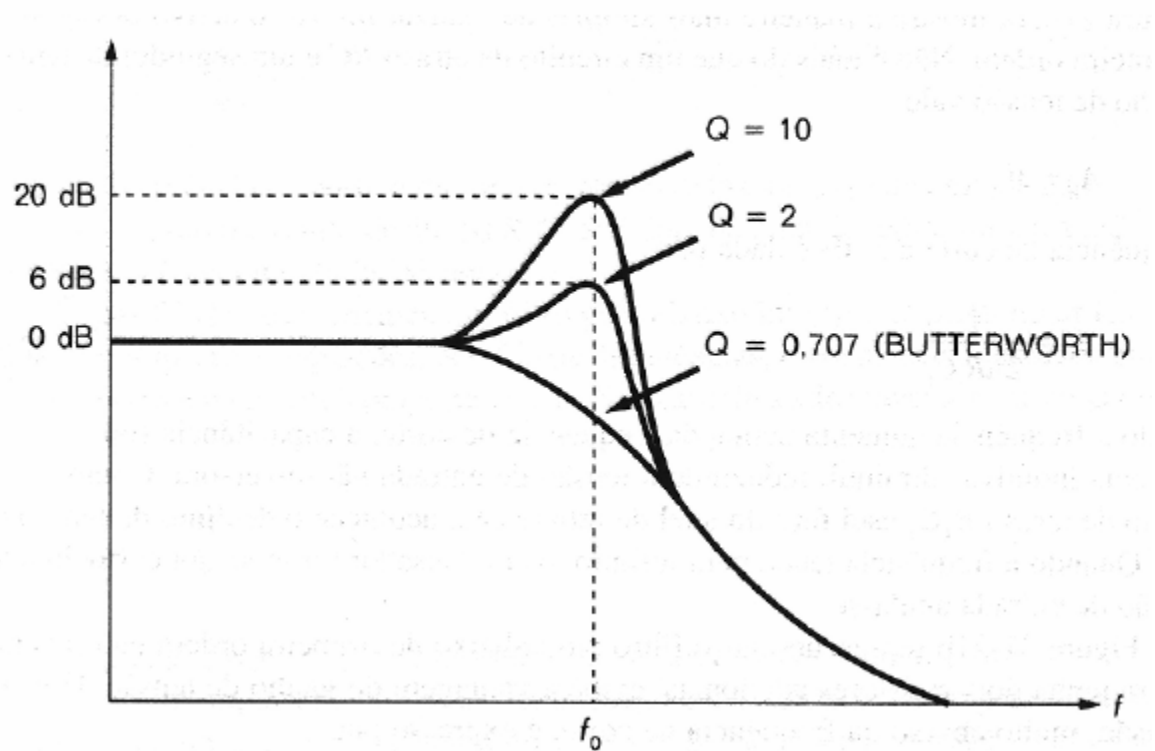
$$G(s) = \frac{1}{\sqrt{2}} \frac{1}{\left(1 + 0.1789\left(\frac{s}{0.9502\omega_c}\right) + \left(\frac{s}{0.9502\omega_c}\right)^2\right)\left(1 + 0.9276\left(\frac{s}{0.4425\omega_c}\right) + \left(\frac{s}{0.4425\omega_c}\right)^2\right)}$$

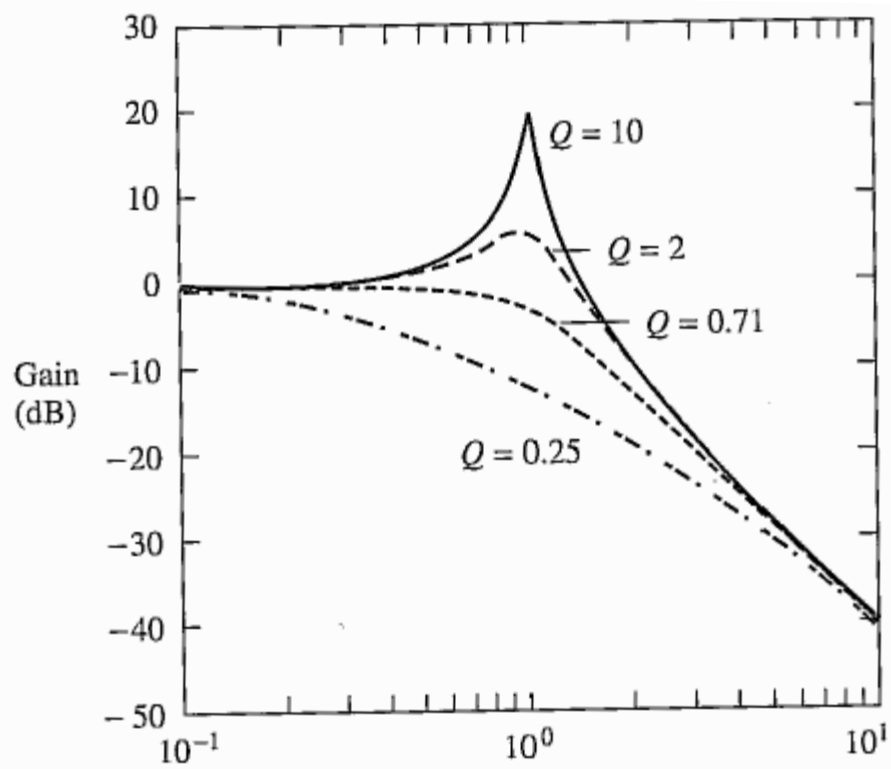


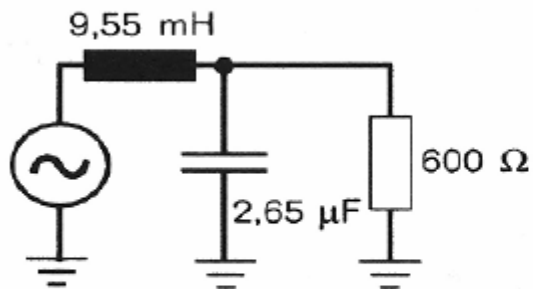
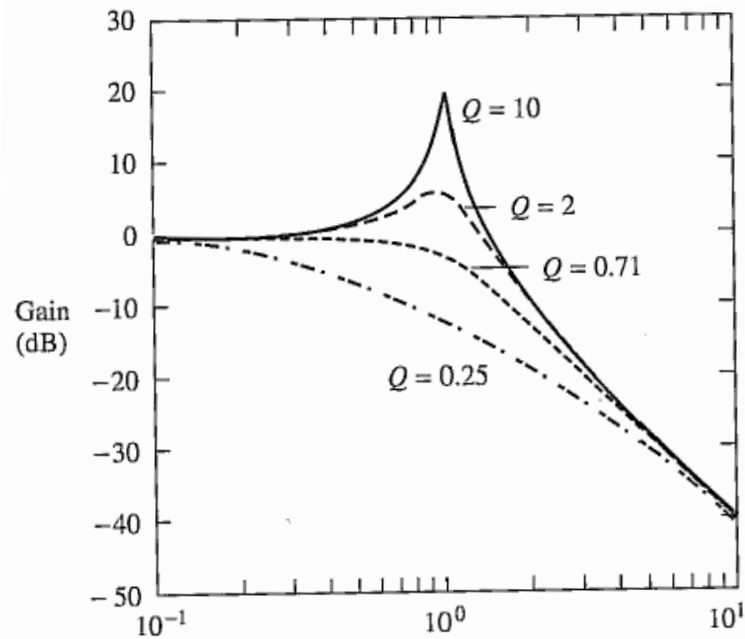
# *Comparação entre os tipos de aproximações para filtros*



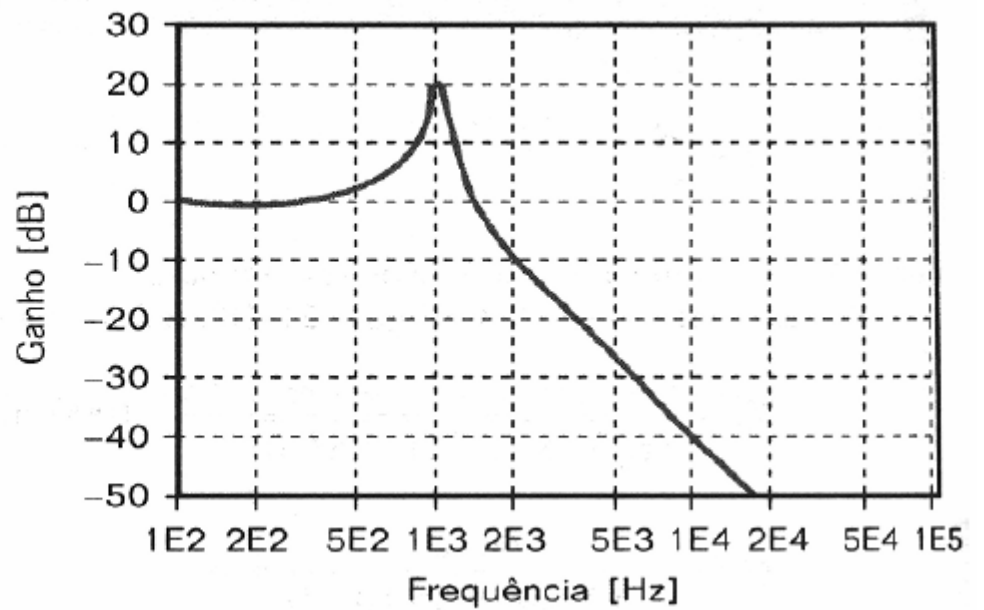
A figura seguinte resume o efeito de  $Q$  num filtro de 2ª ordem:



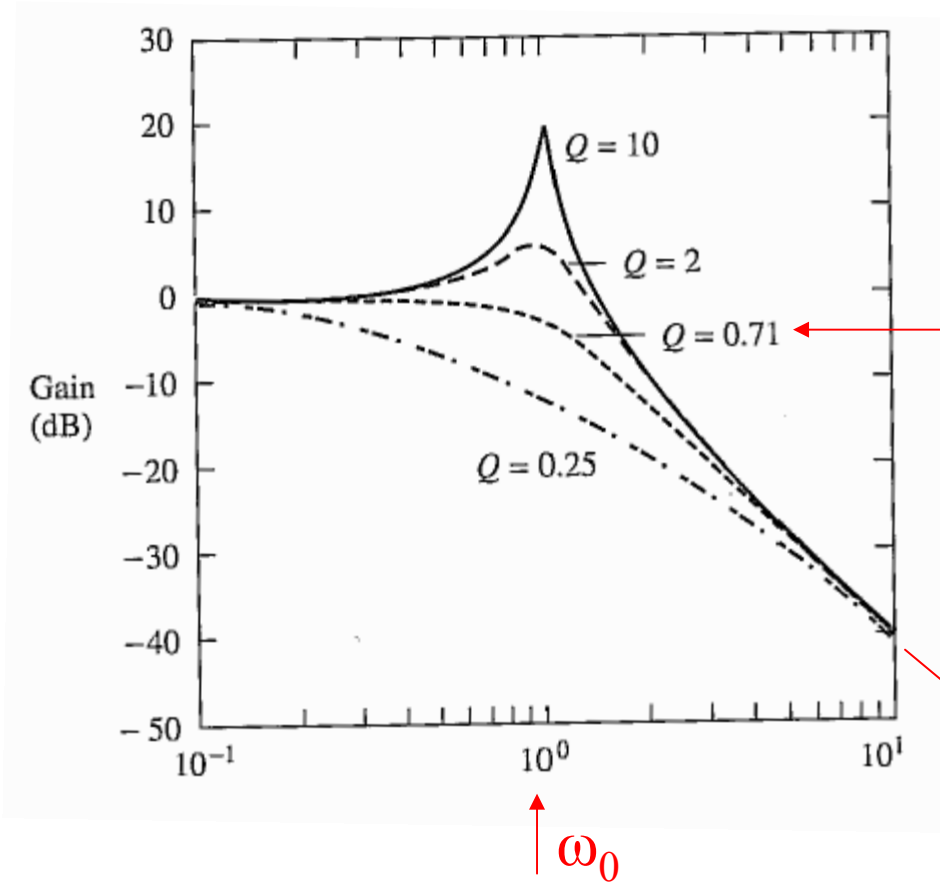




$f_0 = 1\text{kHz}$  ,  $Q = 10$



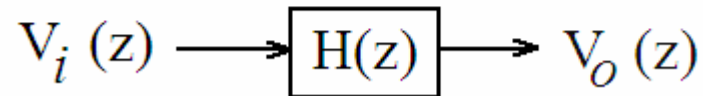
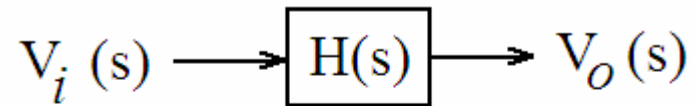
# Two-pole Low-pass Filter



$Q = 1/\sqrt{2} = 0.707$   
called “Butterworth” –  
RMS deviation from ideal  
is minimum

High frequency roll-off  
of 40db/decade since  
2<sup>nd</sup> order low-pass filter

# Filtro Analógico → Filtro Digital



# Filtro Analógico → Filtro Digital

$$V_i(s) \longrightarrow \boxed{H(s)} \longrightarrow V_o(s)$$

$$V_i(z) \longrightarrow \boxed{H(z)} \longrightarrow V_o(z)$$

$$\begin{array}{ccc} & \text{Transformação} & \\ & \text{Bilinear} & \\ H(s) & \xrightarrow{\hspace{10em}} & H(z) \\ & s \longrightarrow \frac{2z-1}{Tz+1} & \end{array}$$

# *Filtros a Capacitor Chaveado*

- **A técnica de filtros a capacitor chaveado é bastante utilizada no projeto de filtros em circuito integrado (CI)**

## *Vantagens*

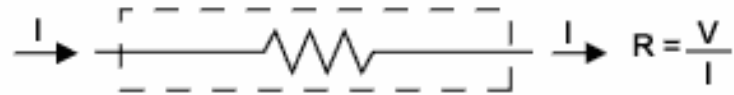
- *Frequência de corte ajustável pela frequência de chaveamento;*
- *Boa exatidão na frequência de corte (depende da relação entre capacitores que pode ser da ordem de 0,1%);*
- *Ocupa pequena área do CI (baixo custo).*



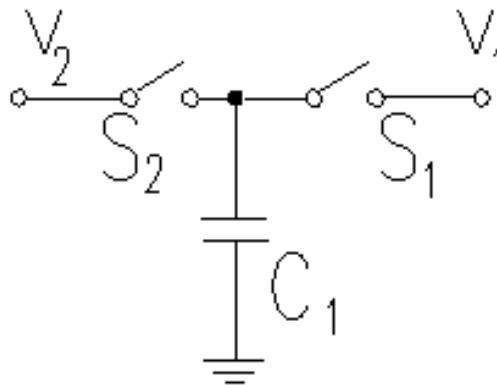
- *A técnica para produção de filtros com capacitores chaveados é baseada no princípio de que um capacitor (C), que periodicamente alterna entre dois pontos do circuito, é equivalente a uma resistência (R) conectando esses pontos do circuito.*
- **Para que o capacitor varie sua conexão são utilizadas chaves que alternam seu estado numa frequência do relógio  $f=1/T$ .**

# Capacitor Chaveado

## Funcionalidade – Circuito equivalente

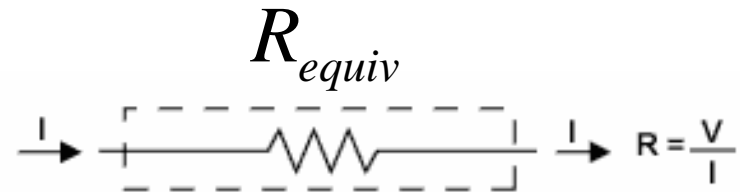
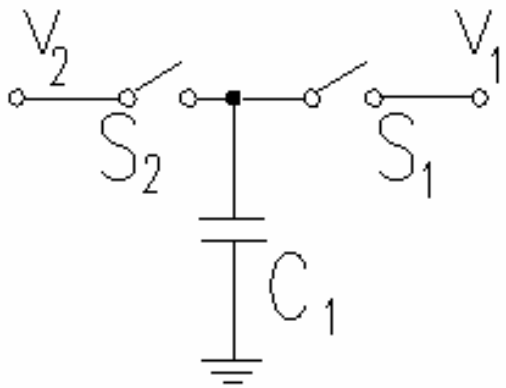


$$R = \frac{T}{C} = \frac{1}{f C}$$



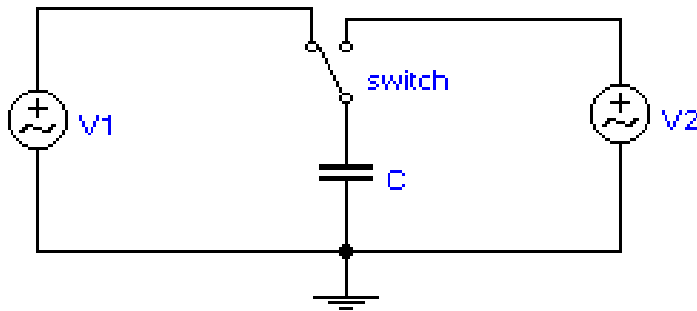
# *Ganho controlado por frequência*

## *Capacitor Chaveado*

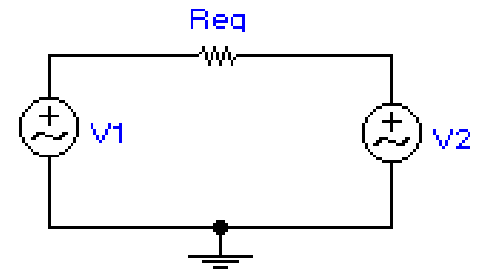


$$R_{equiv} = \frac{T}{C} = \frac{1}{fC}$$

- *Observe que a fonte nunca será conectada diretamente ao circuito, pois as chaves possuem funcionamento dual.*

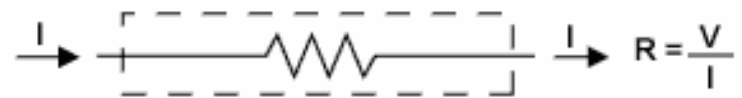
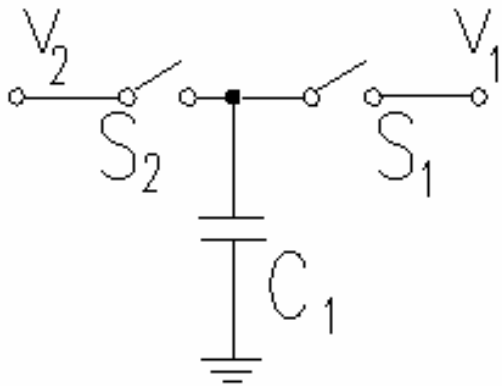


$$R = \frac{T}{C} = \frac{1}{f C}$$

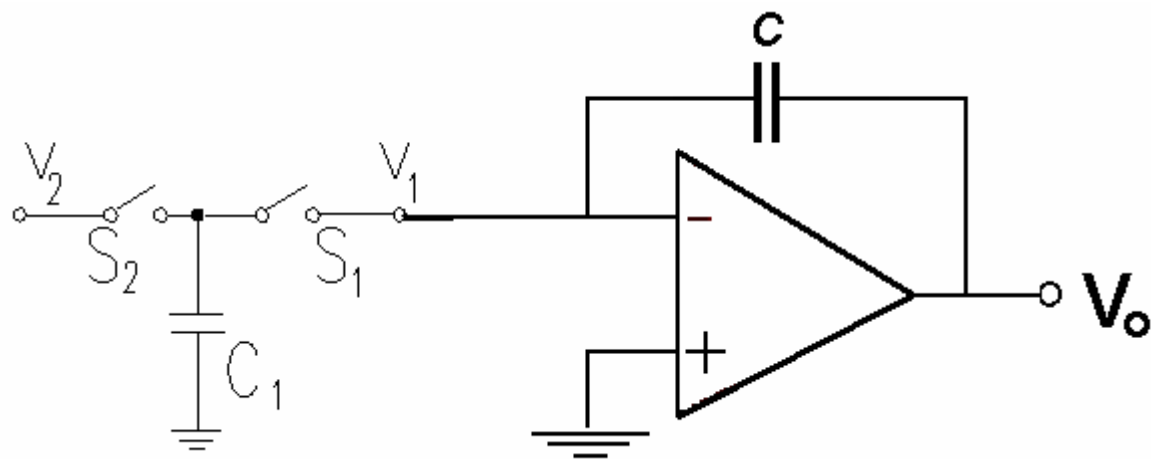
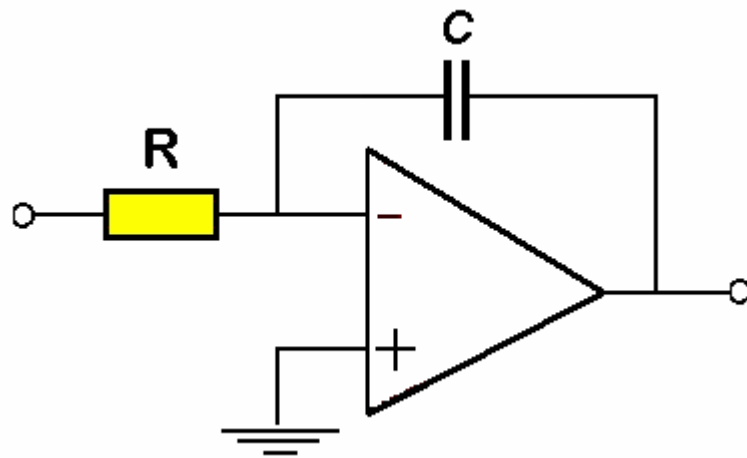


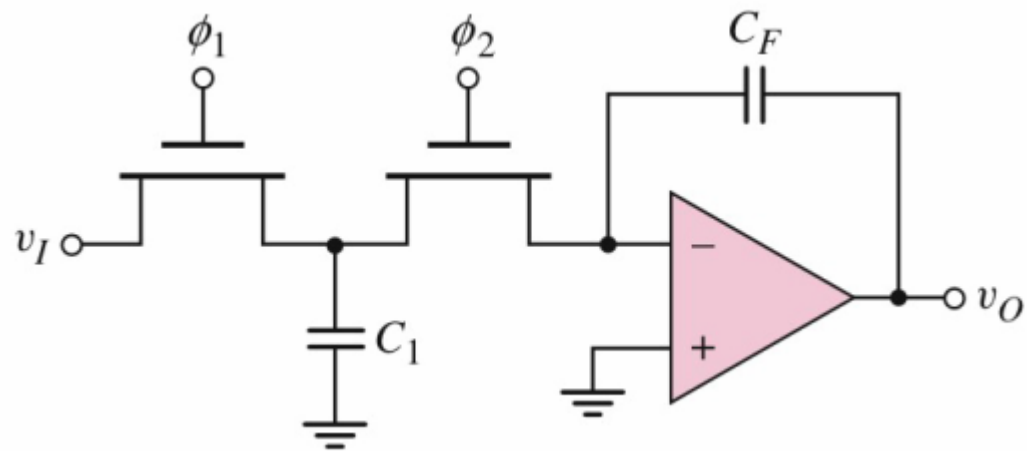
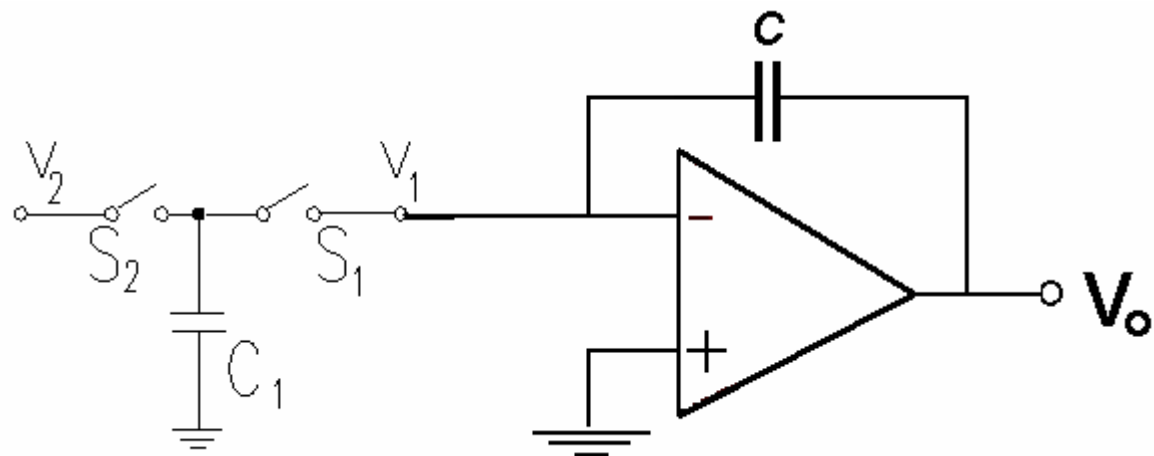
- *Estas geram uma transferência de cargas que resulta em uma corrente pulsante.*

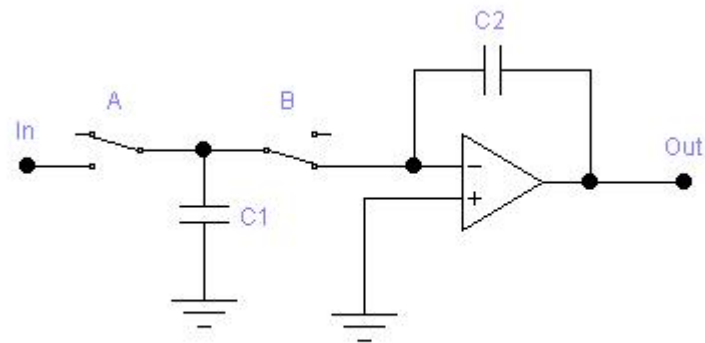
*A corrente média pode ser calculada e, se a frequência chaveada é bastante alta, esta corrente será equivalente à corrente direta no resistor.*



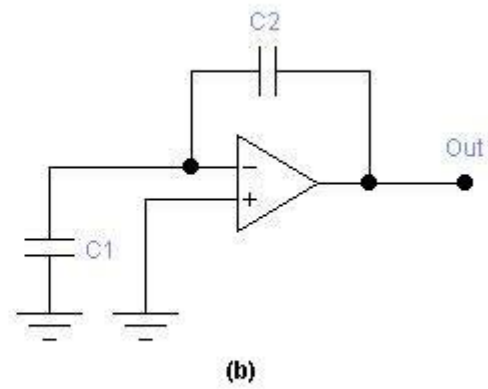
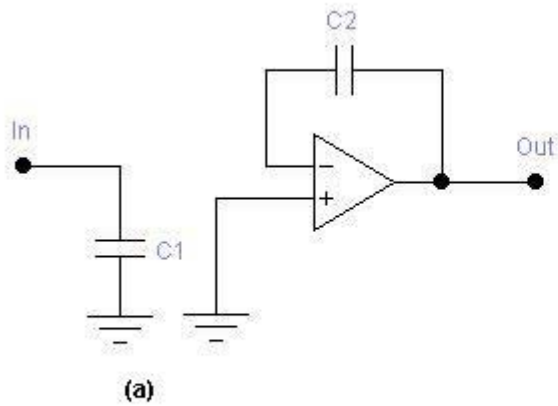
$$R = \frac{T}{C} = \frac{1}{f C}$$





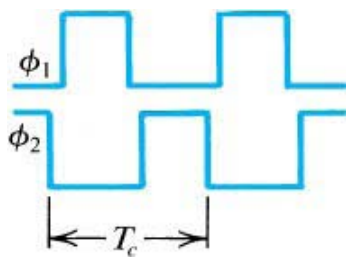
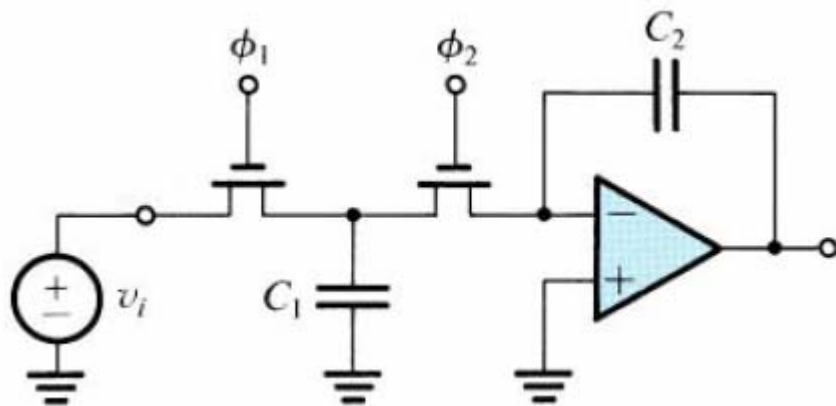
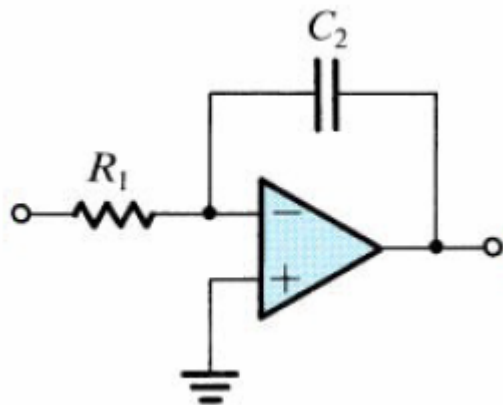


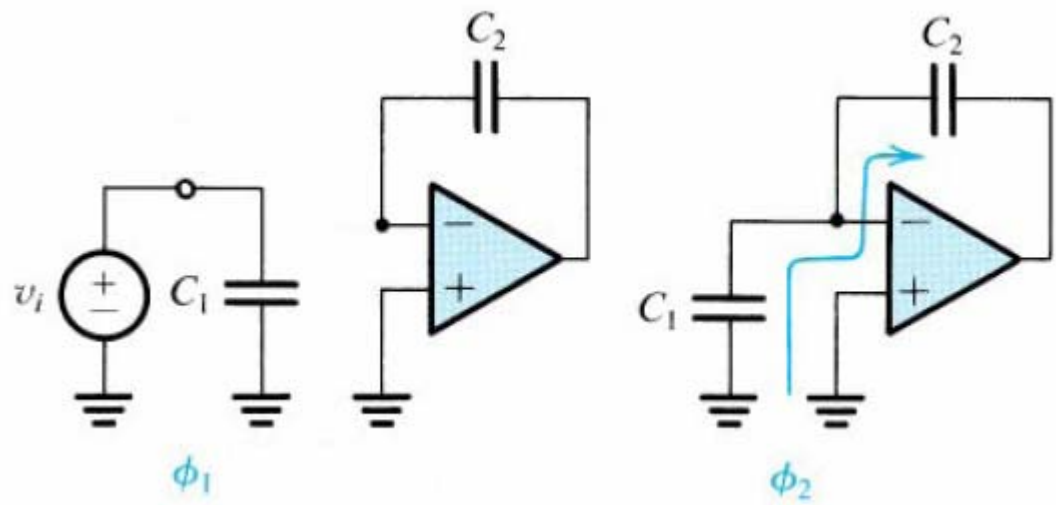
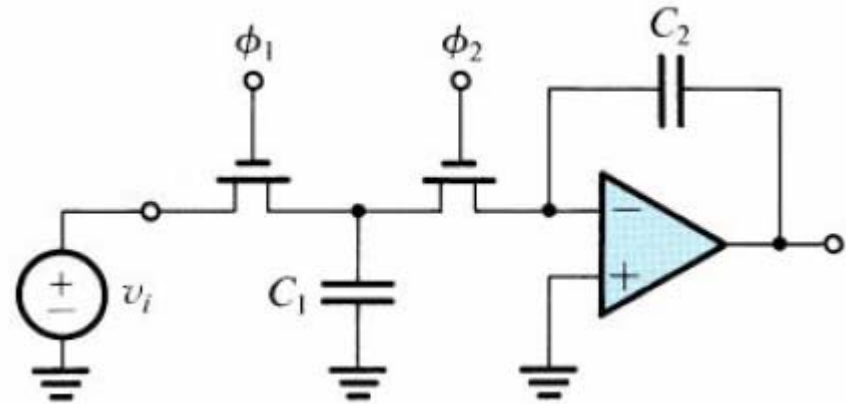
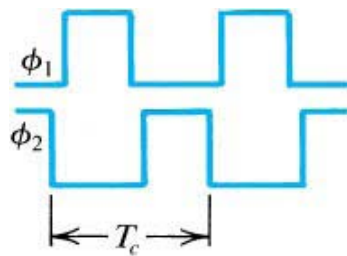
**Filtro com capacitores chaveados.**

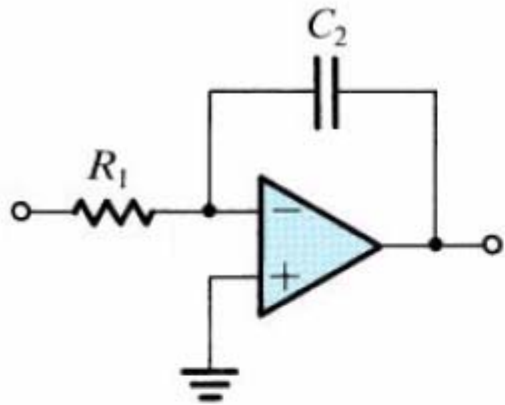


**Circuitos operando: (a) com C1 conectado a entrada, carregando o capacitor; e (b) com C1 conectado a C2, descarregando.**

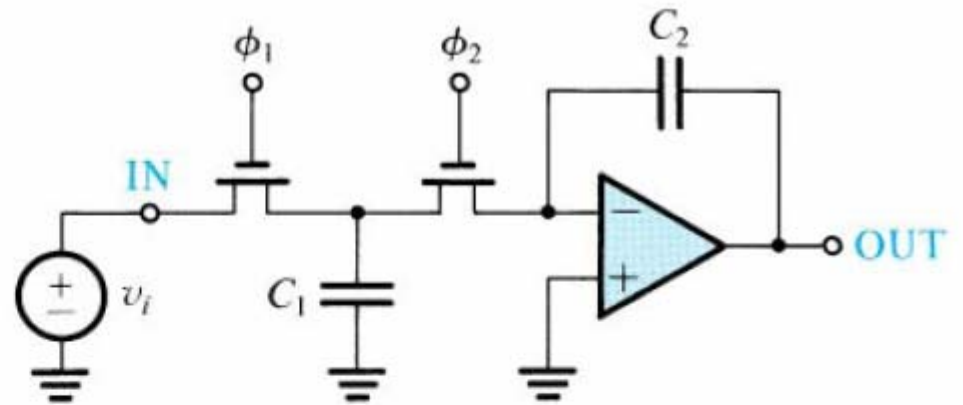




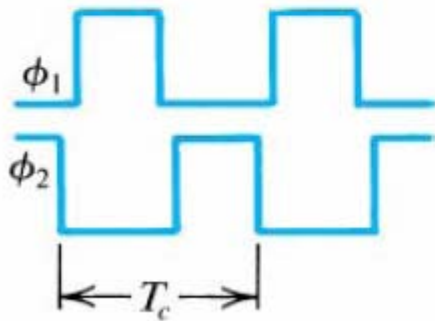




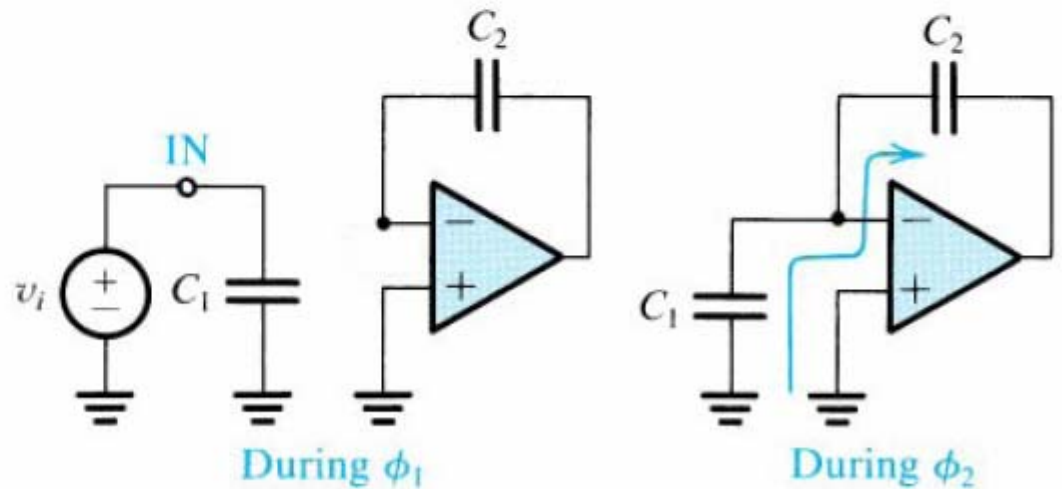
(a)



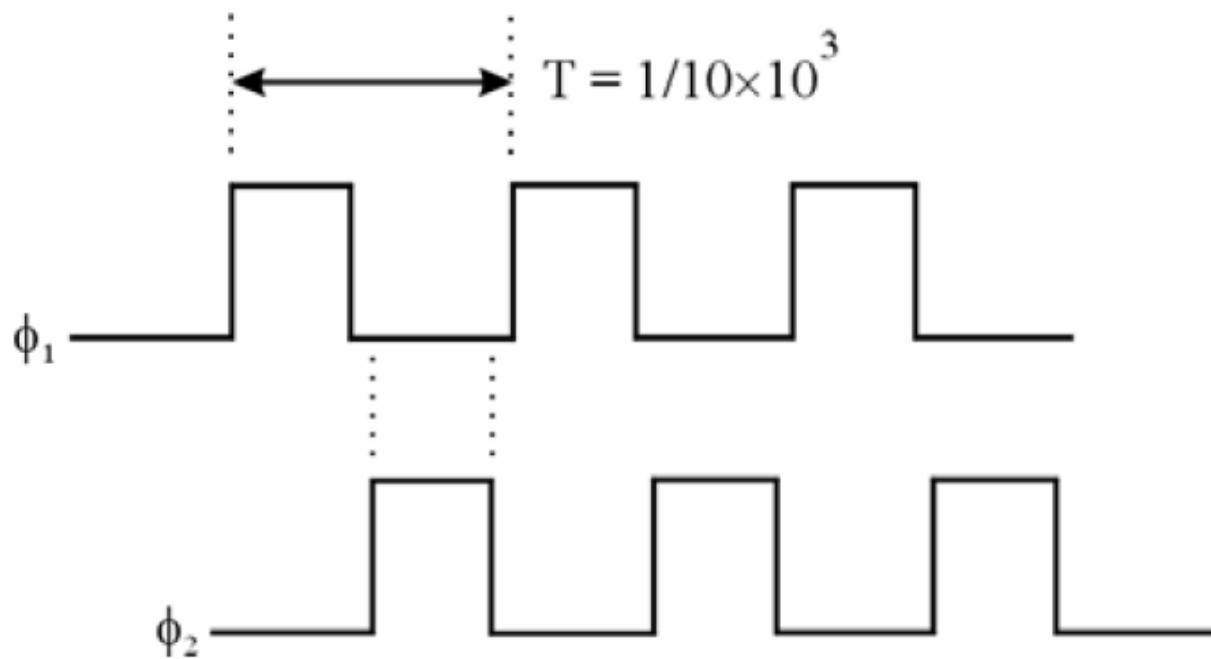
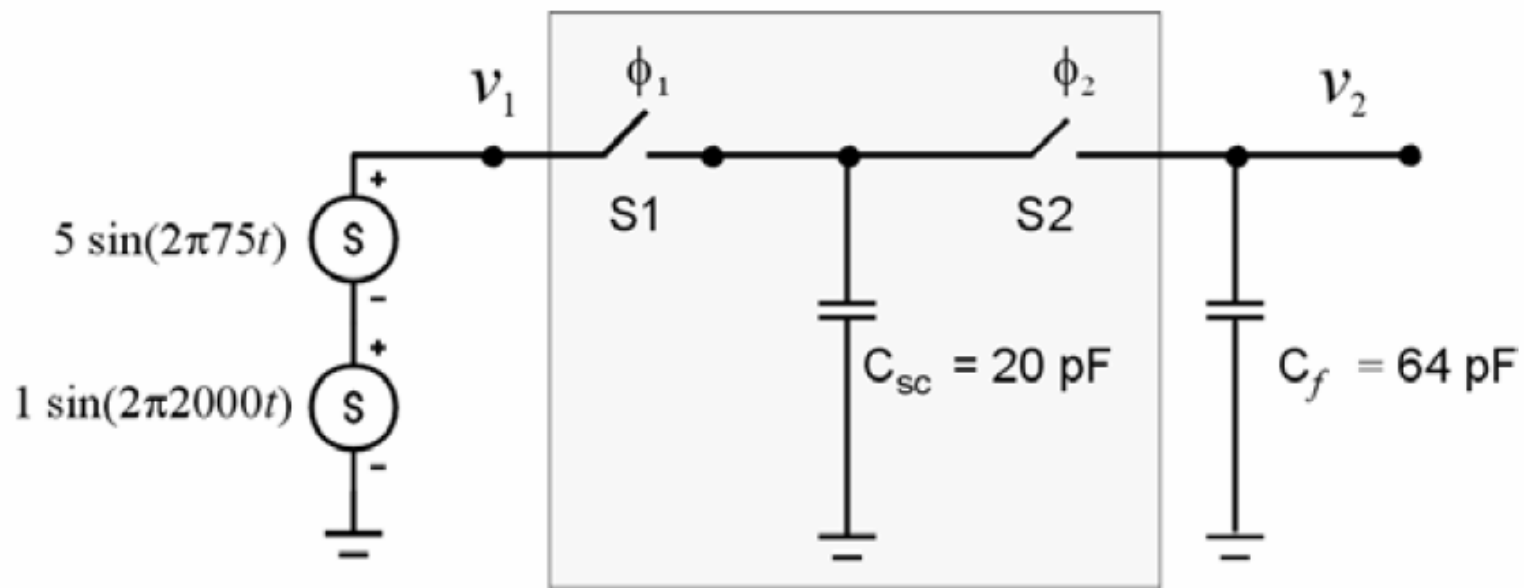
(b)

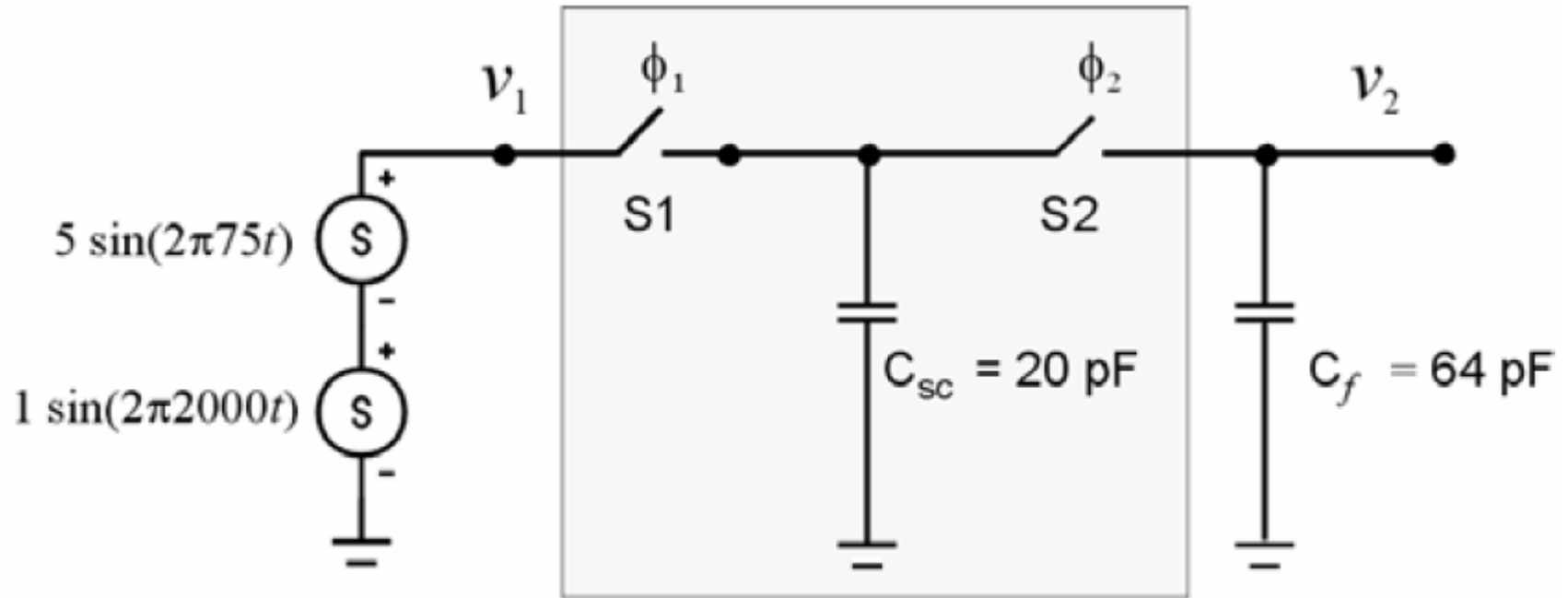


(c)

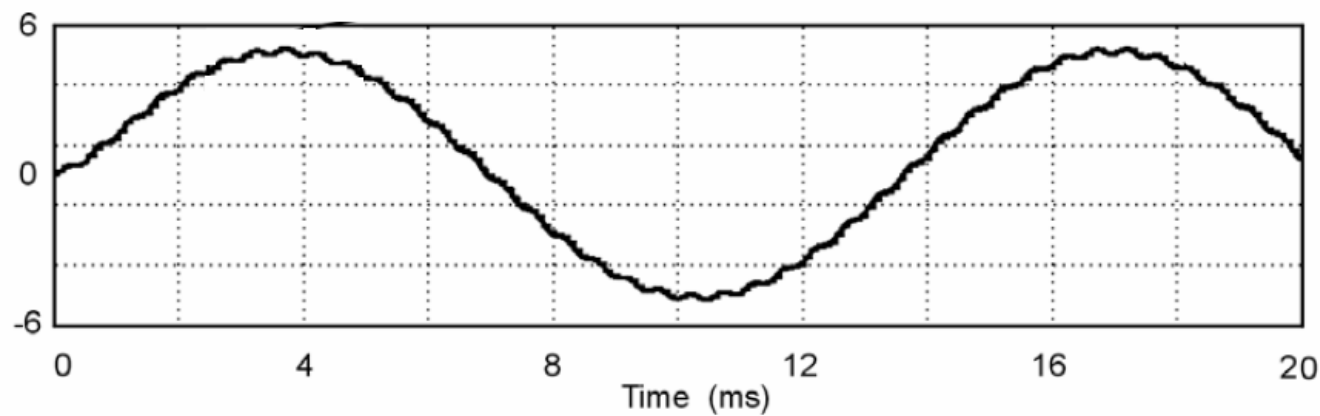
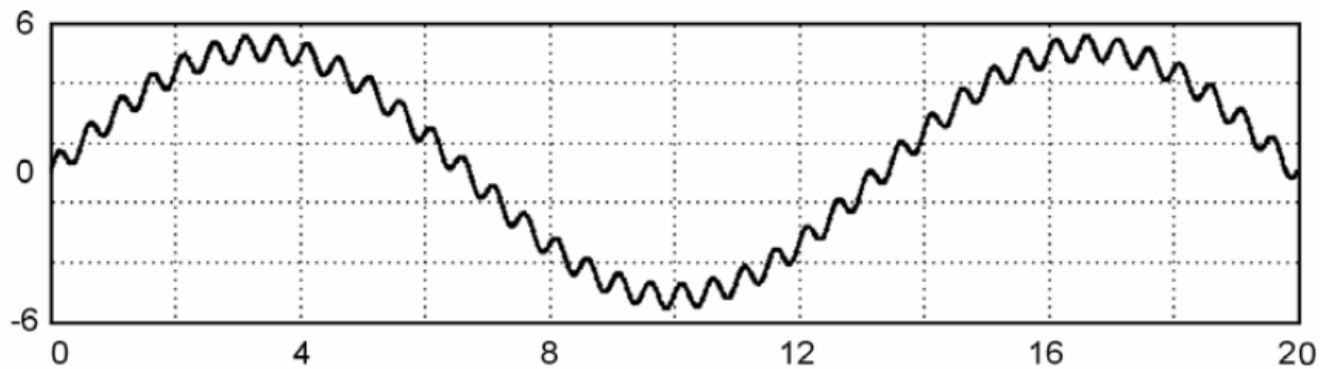
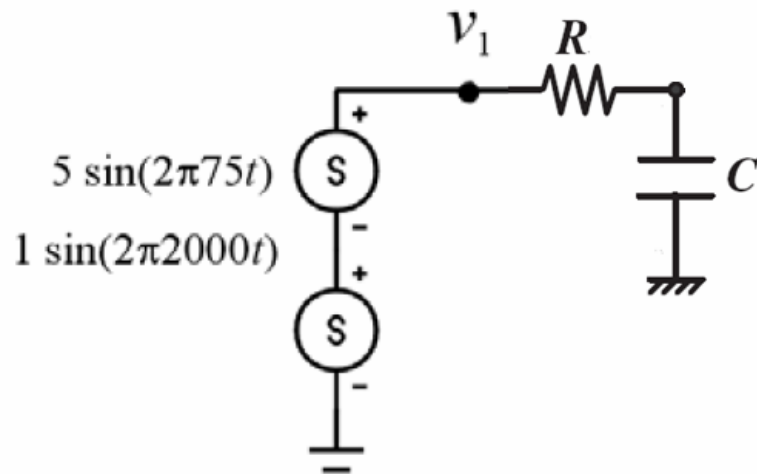


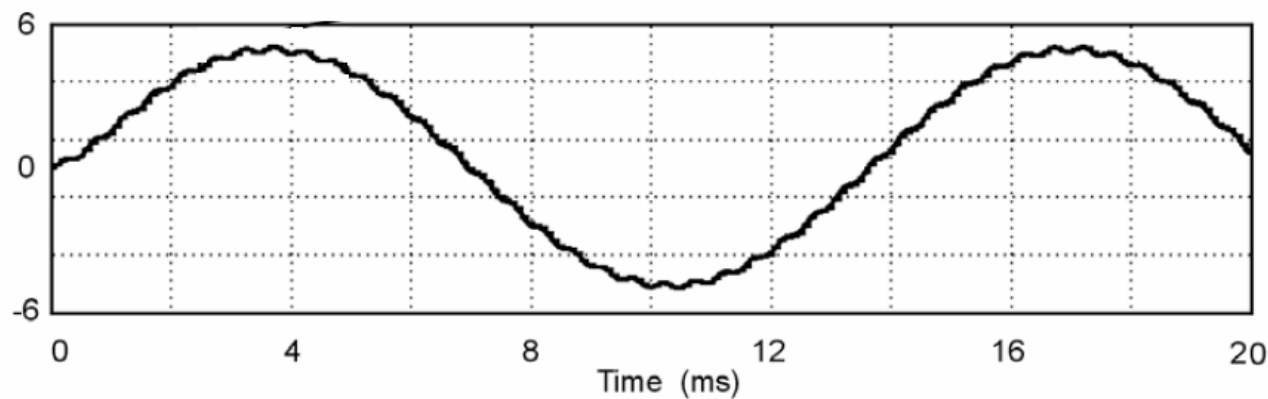
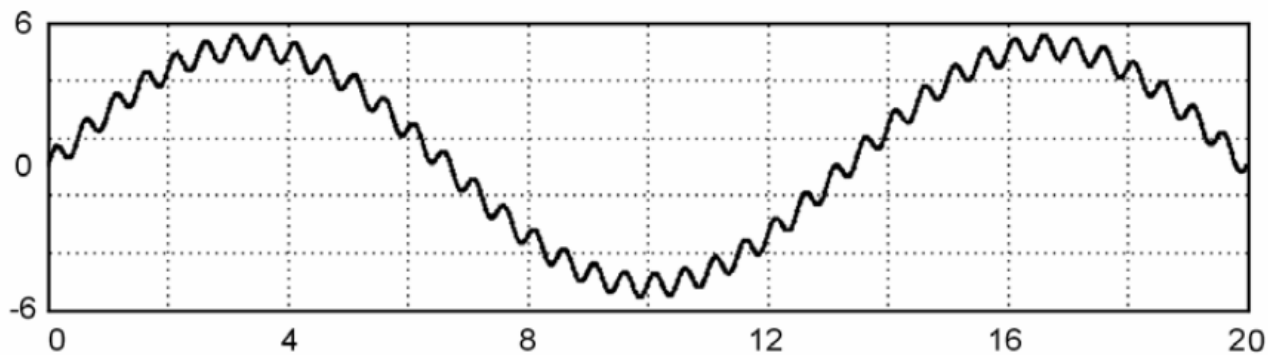
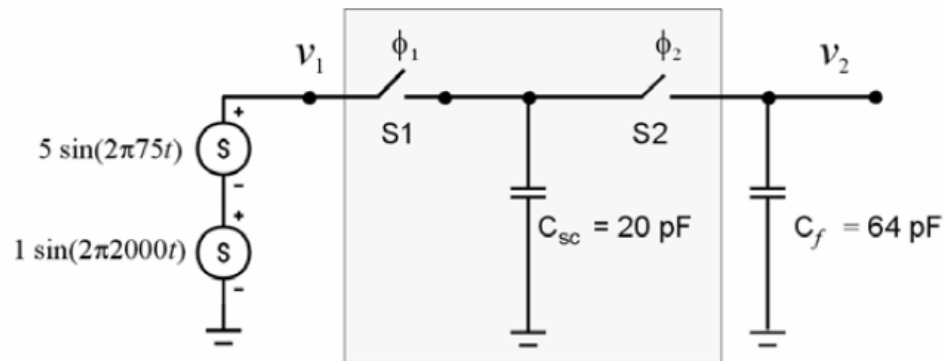
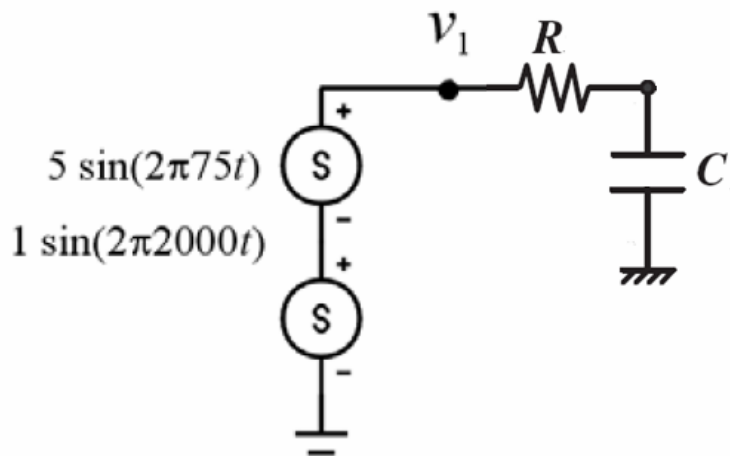
(d)

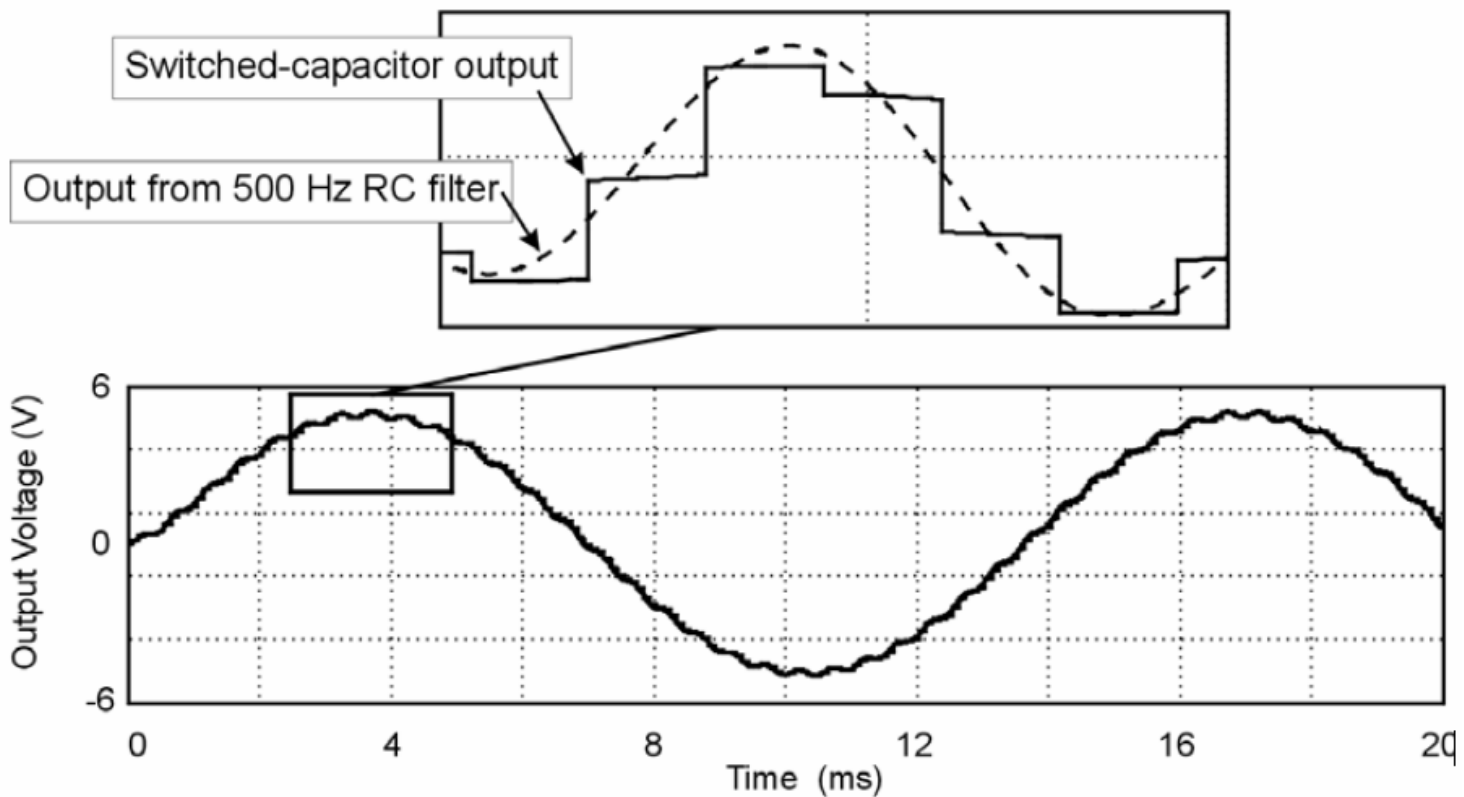
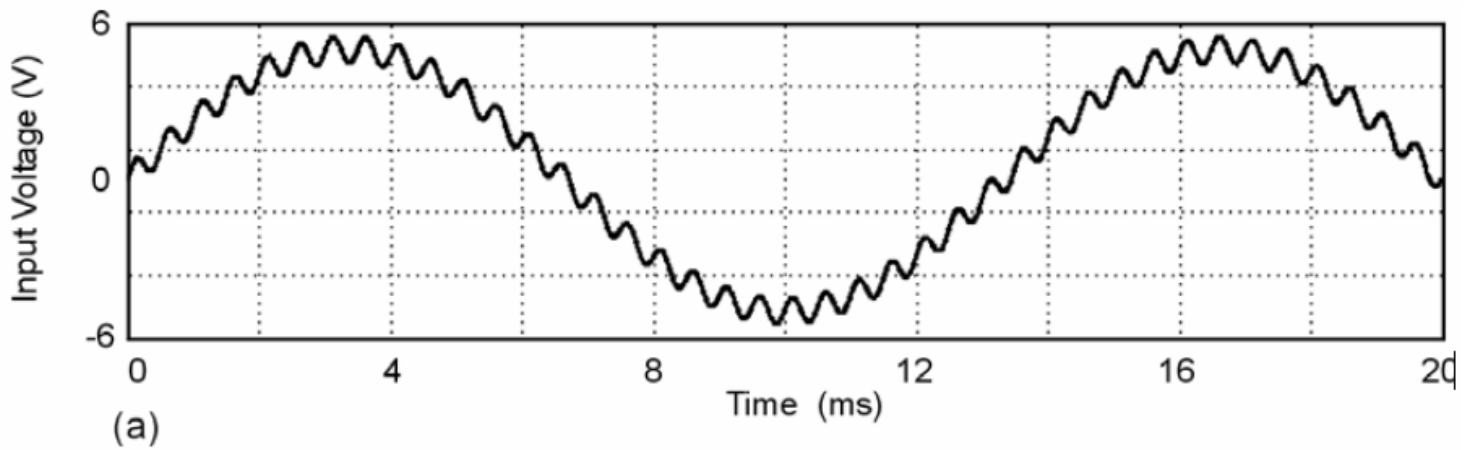




$$f_{3dB} = \frac{1}{2\pi R_{sc} C_f} = \frac{1}{2\pi \left( \frac{1}{f_{clk} C_{sc}} \right) C_f} = \frac{f_s C_{sc}}{2\pi C_f}$$



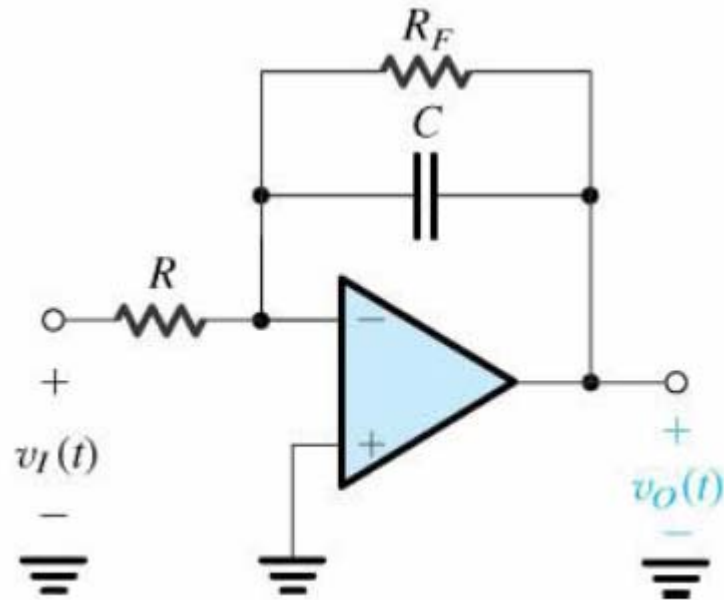






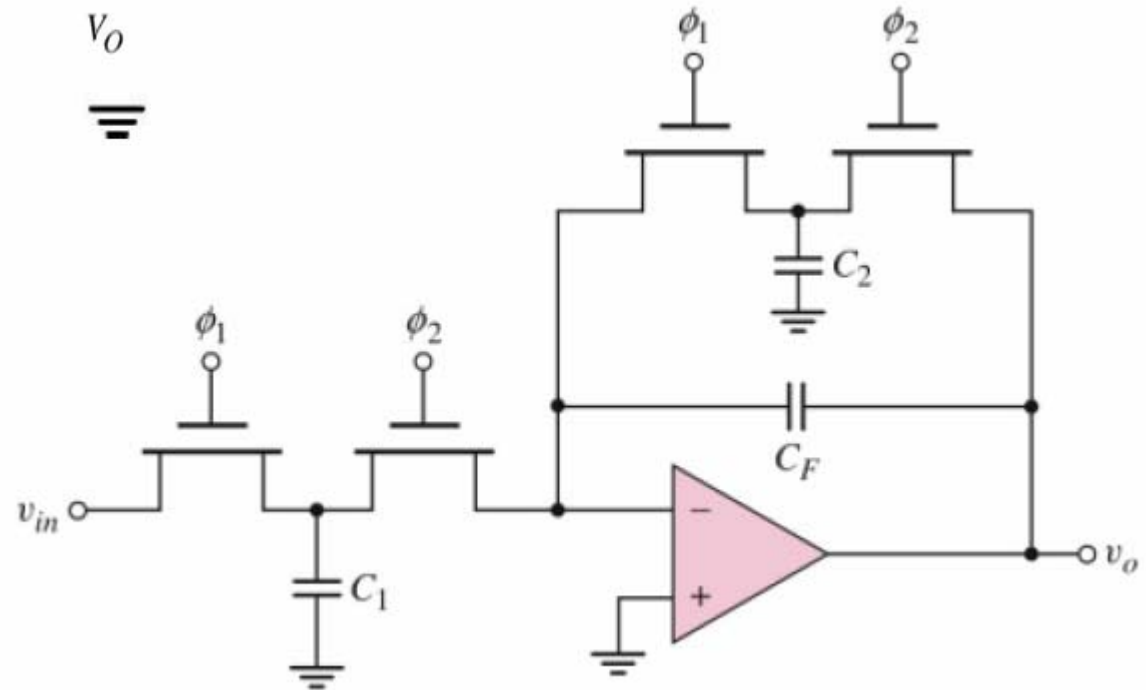
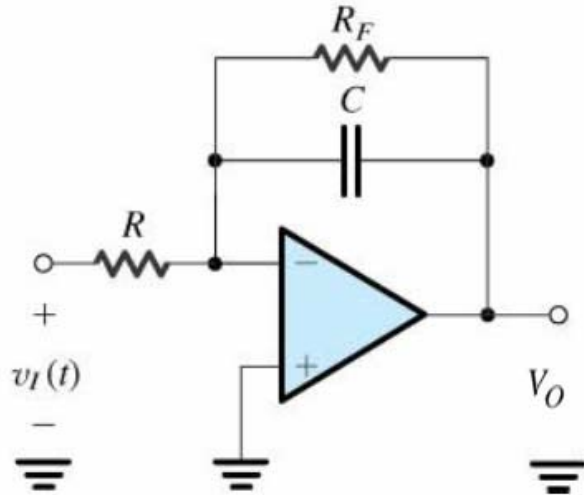
# *Filtro Passa Baixa*

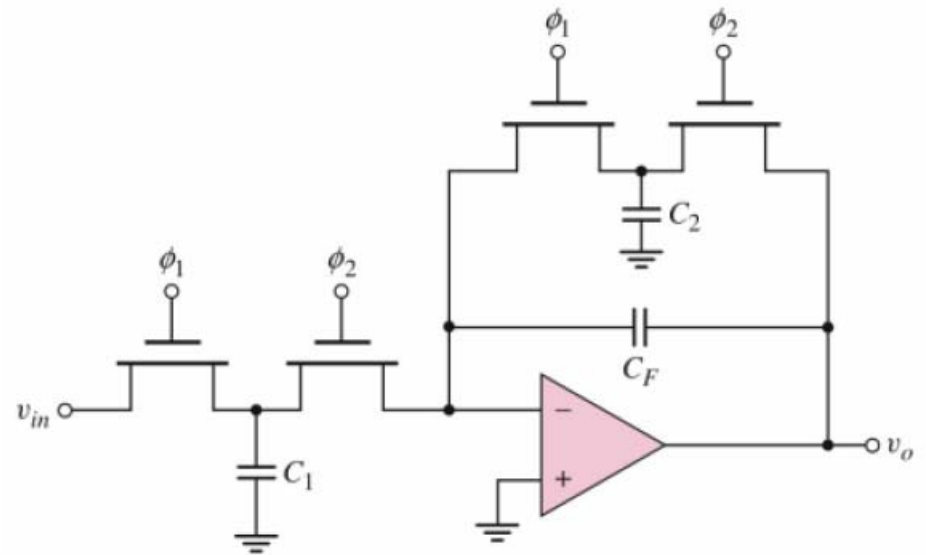
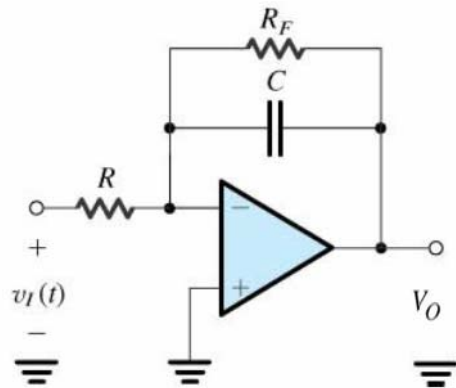
## *Capacitor Chaveado*



$$\frac{V_o(s)}{V_i(s)} = -\frac{R_F/R}{1 + sCR_F}$$

# Filtro Passa Baixa Capacitor Chaveado

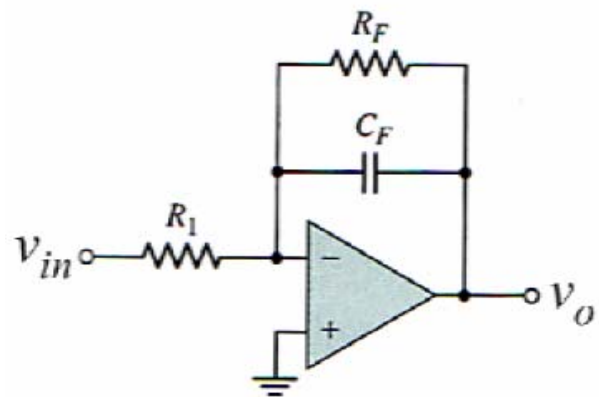




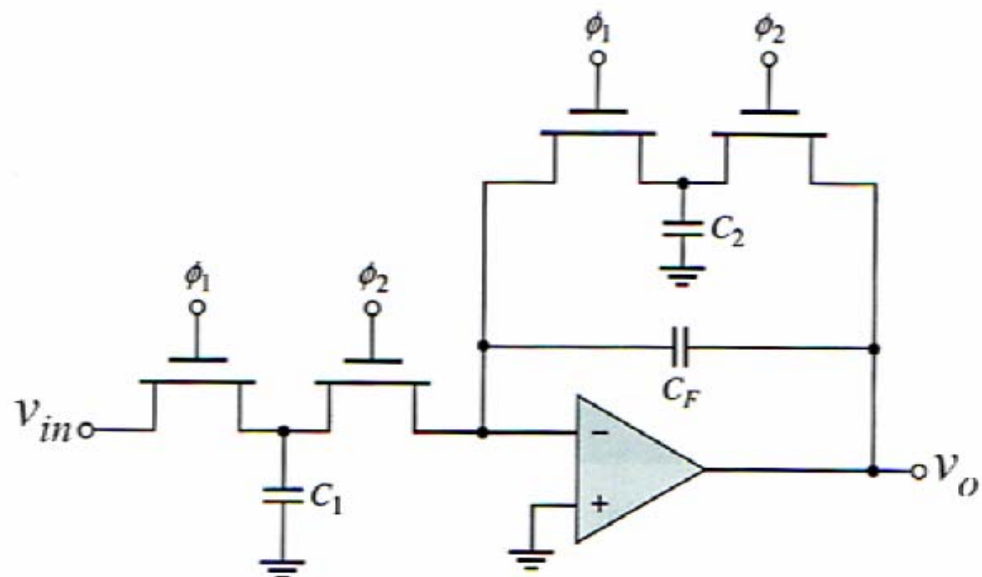
$$T(s) = \frac{v_o(s)}{v_{in}(s)} = -\frac{R_F}{R_1} \cdot \frac{1}{1 + sR_F C_F}$$

$$R_{1(sc)} = \frac{1}{f_{clk} C_1}, \quad R_{F(sc)} = \frac{1}{f_{clk} C_2}$$

$$T(s) = -\frac{1/f_{clk} C_2}{1/f_{clk} C_1} \cdot \frac{1}{1 + s[1/(f_{clk} C_2)]C_F} = -\frac{C_1}{C_2} \cdot \frac{1}{1 + \frac{sC_F}{f_{clk} C_2}}$$



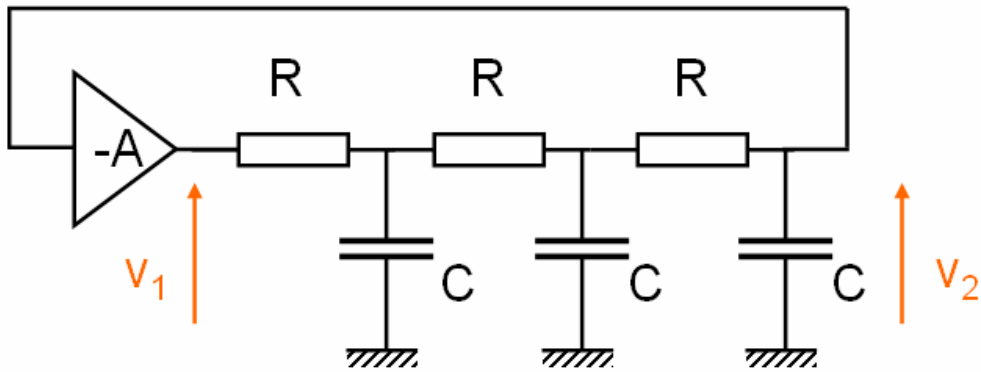
(i)



(ii)

$$T(s) = -\frac{C_1}{C_2} \cdot \frac{1}{1 + \frac{sC_F}{f_{clk}C_2}}$$

| <b>Part</b>              | <b>Manufacturer</b>               | <b>Comments</b>   |
|--------------------------|-----------------------------------|---|
| MF10                     | <a href="#">National</a>          | Two independent, universal filters. Each can be used to implement 1 <sup>st</sup> and 2 <sup>nd</sup> order filters. The filters can be cascaded to form 3 <sup>rd</sup> and 4 <sup>th</sup> order sections. Any of the classical filter (Butterworth, Bessel, Cauer and Chebyshev) can be implemented. |
| LMF100                   | <a href="#">National</a>          | Pin-compatible with MF10, but offers better performance.  |
| MF4                      | <a href="#">National</a>          | 4 <sup>th</sup> Order lowpass Butterworth filter. Has on-board clock that is set with an external resistor and capacitor or accepts an external clock.  |
| TLC04                    | <a href="#">Texas Instruments</a> | Texas Instruments' implementation of the MF4.   |
| MAX7480                  | <a href="#">Maxim</a>             | 8th Order, lowpass, Butterworth filter.   |
| MAX263/264<br>MAX267/268 | <a href="#">Maxim</a>             | Independent center frequency and Q selectable via input pins. Filter design software available.   |
| MAX291/292               | <a href="#">Maxim</a>             | 8 <sup>th</sup> Order lowpass filters with on-board or external clock. Frequency range 0.1 Hz—25 kHz.   |
| MAX295/296               | <a href="#">Maxim</a>             | Same as above, but frequency range is 0.1 Hz—50 kHz   |
| MAX260/261/262           | <a href="#">Maxim</a>             | Dual, universal, microprocessor-controllable filter with more features than most people will ever need. Filter design software available.   |



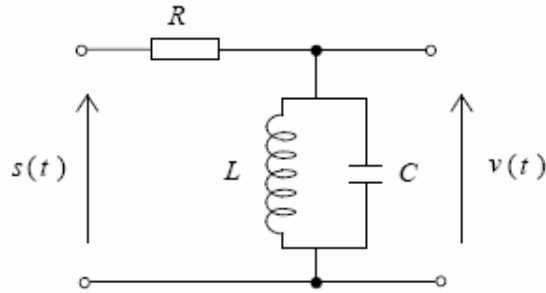
$$\frac{v_2}{v_1} = \frac{1}{1 + 6jRC\omega - 5R^2C^2\omega^2 - jR^3C^3\omega^3}$$

$$\Rightarrow \omega_0 = \frac{\sqrt{6}}{RC}$$

$$\Rightarrow A = 29$$

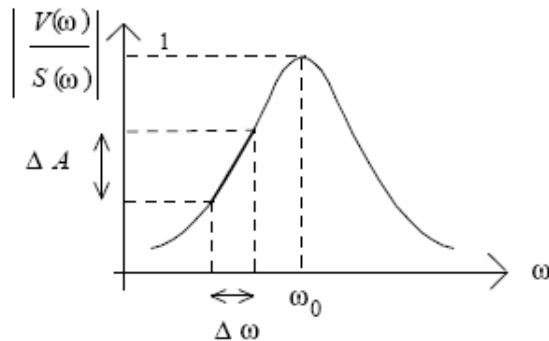
## 5.5. Démodulation de Fréquence

Ex. : Démodulateur à flancs



Soit :  $Z = Z(jL\omega) = \frac{jL\omega}{1 - LC\omega^2}$

$$Z = \frac{jL\omega}{1 - \frac{\omega^2}{\omega_0^2}} \quad \text{avec : } \omega_0 = \frac{1}{\sqrt{LC}}$$

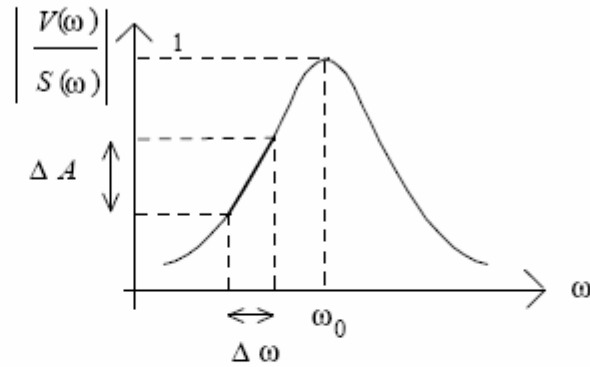
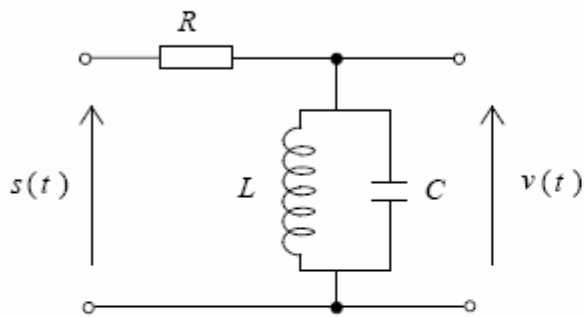


$$\frac{V(\omega)}{S(\omega)} = \frac{Z}{Z + R}$$

On transforme la variation de fréquence  $\Delta\omega$  en variation d'amplitude  $\Delta A \rightarrow$  On peut ensuite effectuer une démodulation d'amplitude (à PLL par exemple).

*Inconvénient :* la plage utilisable est faible si on veut une bonne linéarité (pas de déformation du signal).  
(il faut :  $\Delta A =$  fonction linéaire de  $\Delta\omega$  si on veut réaliser une transmission fidèle).

## Démodulation de Fréquence



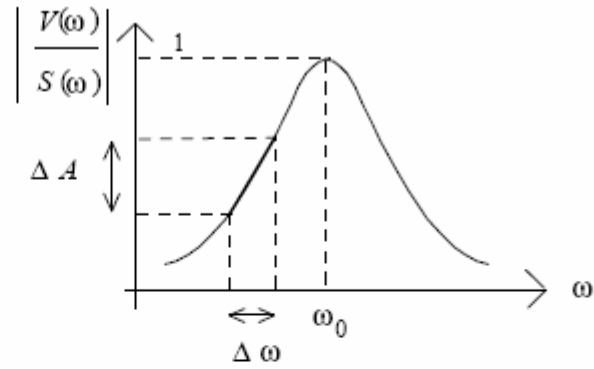
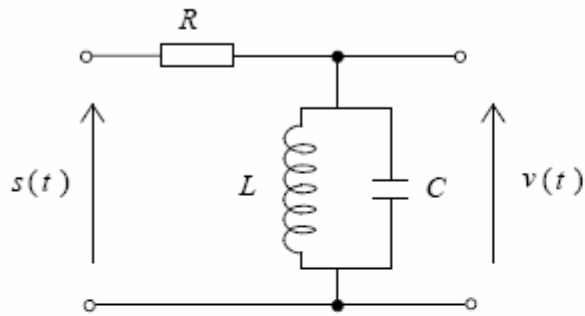
$$Z = Z(jL\omega) = \frac{jL\omega}{1 - LC\omega^2}$$

$$Z = \frac{jL\omega}{1 - \frac{\omega^2}{\omega_0^2}} \quad \text{avec :} \quad \omega_0 = \frac{1}{\sqrt{LC}}$$

$$\frac{V(\omega)}{S(\omega)} = \frac{Z}{Z + R}$$



# Demodulador FM



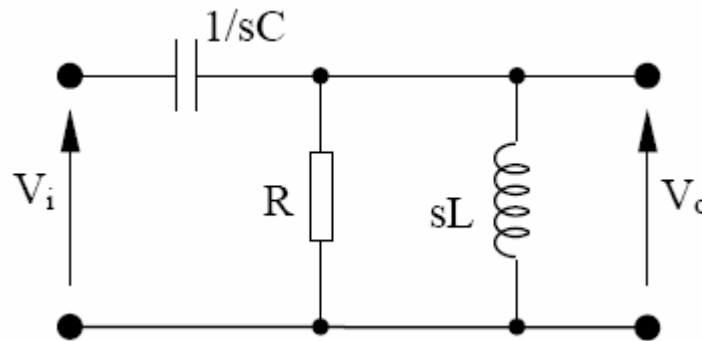
$$Z = Z(jL\omega) = \frac{jL\omega}{1 - LC\omega^2}$$

$$Z = \frac{jL\omega}{1 - \frac{\omega^2}{\omega_0^2}}$$

$$\omega_0 = \frac{1}{\sqrt{LC}}$$

$$\frac{V(\omega)}{S(\omega)} = \frac{Z}{Z + R}$$

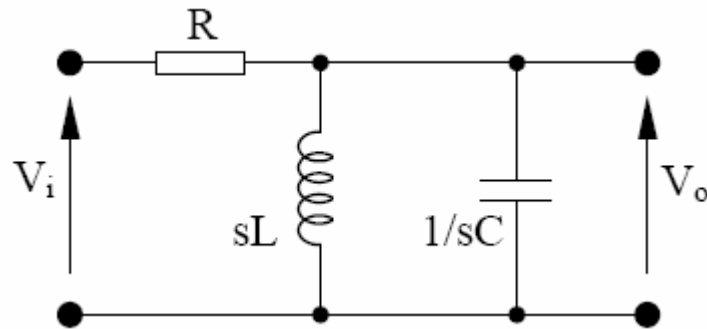
# Filtro Passa-alta



$$H(s) = \frac{\frac{sRL}{R + sL}}{\frac{1}{sC} + \frac{sRL}{R + sL}} = \frac{s^2}{s^2 + s(1/RC) + 1/LC}$$

$$H(s) = H_0 \frac{s^2}{s^2 + s \frac{\omega_0}{Q_0} + \omega_0^2}$$

# Filtro Passa-faixa

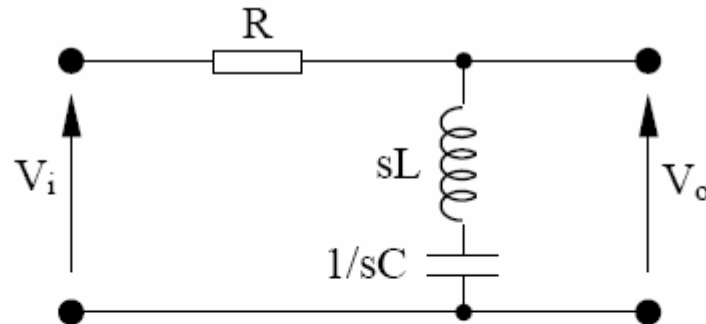


$$H(s) = \frac{\frac{sL}{s^2 LC + 1}}{R + \frac{sL}{s^2 LC + 1}} = \frac{s(1/RC)}{s^2 + s \frac{\sqrt{L}}{R\sqrt{C}\sqrt{LC}} + 1/LC} = \frac{s \frac{1}{R} \sqrt{L/C} \frac{1}{\sqrt{LC}}}{s^2 + s \frac{1}{R} \sqrt{L/C} + 1/LC}$$

$$\omega_0 = 1/\sqrt{LC} \quad e \quad \frac{1}{Q_0} = \alpha = \frac{1}{R} \sqrt{L/C}$$

$$H(s) = H_0 \frac{s(\omega_0 / Q_0)}{s^2 + s \frac{\omega_0}{Q_0} + \omega_0^2}$$

# Filtro Rejeita-faixa

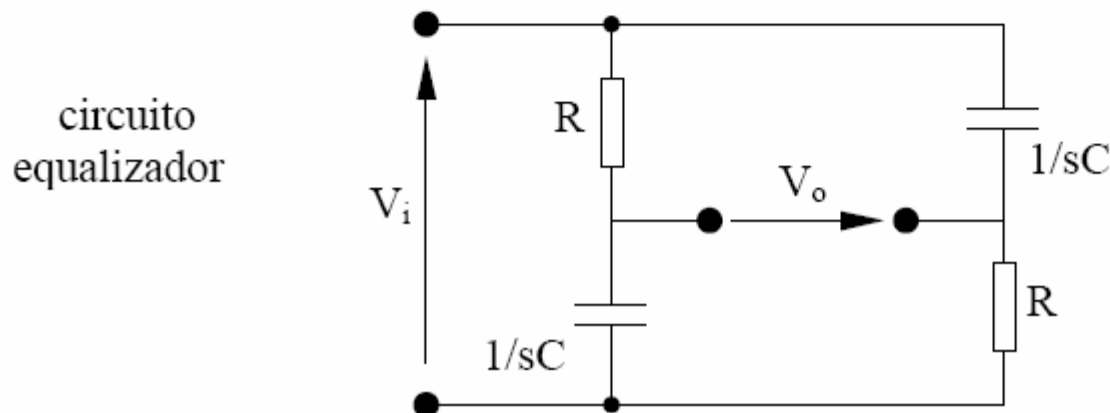
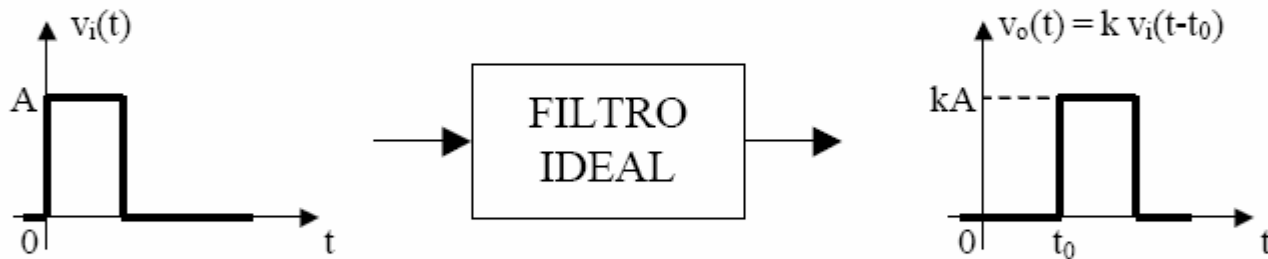


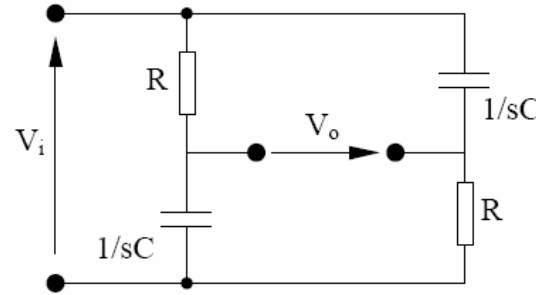
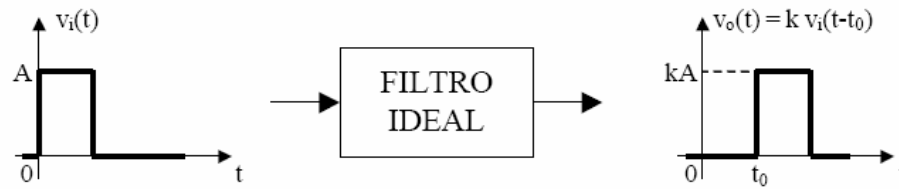
$$H(s) = \frac{\frac{s^2 LC + 1}{sC}}{R + \frac{s^2 LC + 1}{sC}} = \frac{s^2 LC + 1}{s^2 LC + sRC + 1} = \frac{s^2 + 1/LC}{s^2 + sR\sqrt{C/L} \frac{1}{\sqrt{LC}} + 1/LC}$$

$$H(s) = H_0 \frac{s^2 + \omega_0^2}{s^2 + s \frac{\omega_0}{Q_0} + \omega_0^2}$$

## Exemplo de equalizadores de fase

Em geral os filtros não apresentam resposta de fase linear com frequência. Nas aplicações de transmissão de voz ou música isto não tem importância, pois o ouvido humano não é sensível à variação de fase. Entretanto, na transmissão de pulsos, é desejável que todos os harmônicos tenham o mesmo tempo de atraso, a fim de que haja uma perfeita recomposição harmônica. No domínio do tempo, um filtro ideal é aquele que fornece uma saída proporcional à entrada, admitindo apenas um atraso de  $T$  segundos. O equalizador de fase tem a função de tornar o atraso total (filtro + equalizador) igual para todos os harmônicos.





Equalizador de fase

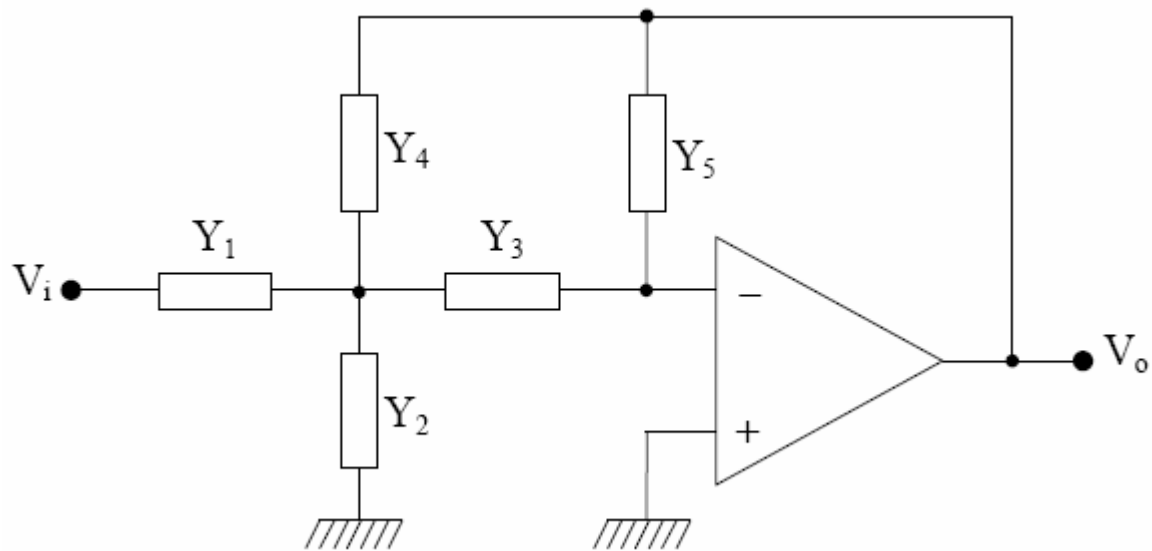
$$H(s) = \frac{V_o}{V_i} = \frac{R}{R + 1/sC} - \frac{1/sC}{R + 1/sC} = \frac{s - 1/RC}{s + 1/RC}$$

Fazendo  $\omega_0 = 1/RC$ , tem-se  $H(s) = \frac{s - \omega_0}{s + \omega_0}$ .

Em regime permanente senoidal, com  $s = j\omega$ , vem

$$H(j\omega) = \frac{j\omega - \omega_0}{j\omega + \omega_0} \quad \therefore \quad |H(j\omega)| = \sqrt{\frac{\omega^2 + \omega_0^2}{\omega^2 + \omega_0^2}} = 1 \quad (\text{em qualquer freq\u00eancia})$$

Daí vem o nome de *filtro passa-tudo* dado ao equalizador de fase. O circuito introduz apenas defasagens adicionais, em freq\u00eancias apropriadas, compensando as defasagens introduzidas pelo filtro.



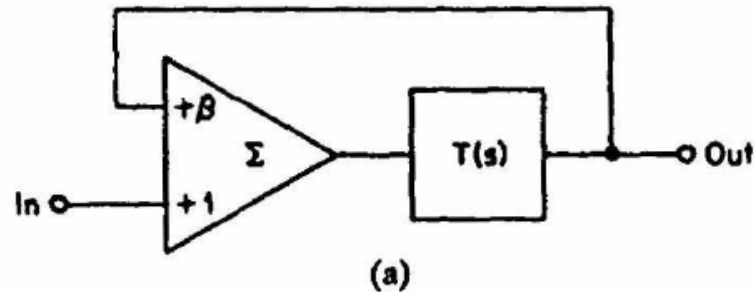
$$H(s) = \frac{-H_0 \omega_0^2}{s^2 + s\alpha \omega_0 + \omega_0^2} \equiv -\frac{Y_1 Y_3}{Y_5(Y_1 + Y_2 + Y_3 + Y_4) + Y_3 Y_4}$$

Butterworth de 2ª ordem é  $H(s) = \frac{-1}{s^2 + s\alpha + 1}$

$$H(s) = \frac{-1}{s^2 + s\sqrt{2} + 1} \equiv -\frac{Y_1 Y_3}{Y_5(Y_1 + Y_2 + Y_3 + Y_4) + Y_3 Y_4}$$

$$H(s) = \frac{-1/R_1 R_3}{s^2 C_2 C_5 + s C_5 (1/R_1 + 1/R_3 + 1/R_4) + 1/R_3 R_4}$$

## Q Multiplier Active Bandpass Filters



If  $T(s)$  in circuit A corresponds to a bandpass transfer function of:

$$T(s) = \frac{\frac{\omega_r}{Q} s}{s^2 + \frac{\omega_r}{Q} s + \omega_r^2}$$

The overall circuit transfer function becomes:

$$\frac{\text{Out}}{\text{In}} = \frac{\frac{\omega_r}{Q} s}{s^2 + \frac{\omega_r}{Q} s + \omega_r^2} \cdot \frac{Q}{1-\beta}$$

The middle term of the denominator has been modified so the circuit Q is given by  $Q/(1-\beta)$  where  $0 < \beta < 1$ . The Q can then be increased by the factor  $1/(1-\beta)$ . Note that the circuit gain is increased by the same factor.



A simple implementation of this circuit is shown in figure B.

The design equations are:

First calculate  $\beta$  from  $\beta = 1 - \frac{Q_r}{Q_{eff}}$

where  $Q_{eff}$  is the overall circuit Q and  $Q_r$  is the design Q of the bandpass section.

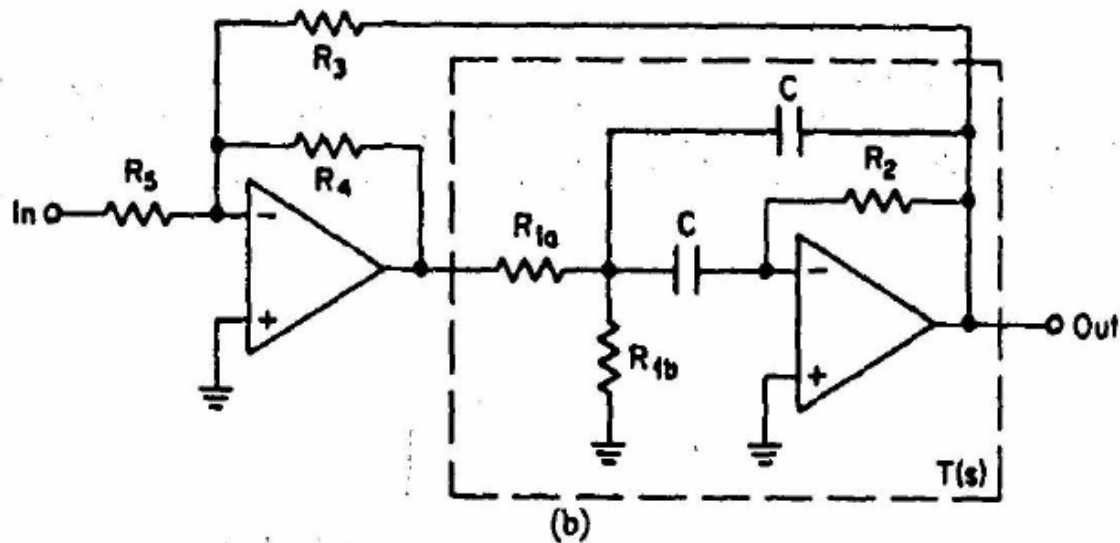
The component values can be computed from:

$$R_3 = \frac{R}{\beta} \qquad R_2 = \frac{Q_r}{\pi f_r C} \qquad R_{1b} = \frac{R_{1a}}{2Q_r^2 - 1}$$

$$R_4 = R \qquad R_{1a} = \frac{R_2}{2}$$



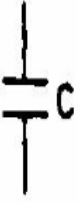
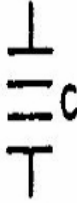

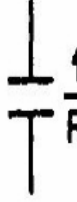
$$R_5 = \frac{R}{(1 - \beta)A_r}$$

Where R and C can be conveniently chosen.

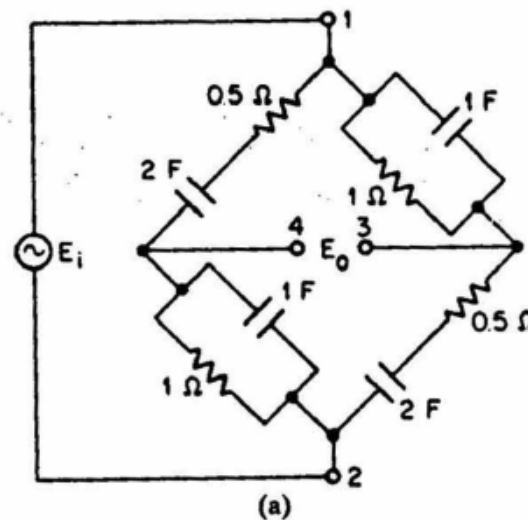


A função de transferência  $H(s)$  de um circuito permanece inalterada se todas as impedâncias forem multiplicadas ou divididas por um mesmo fator.

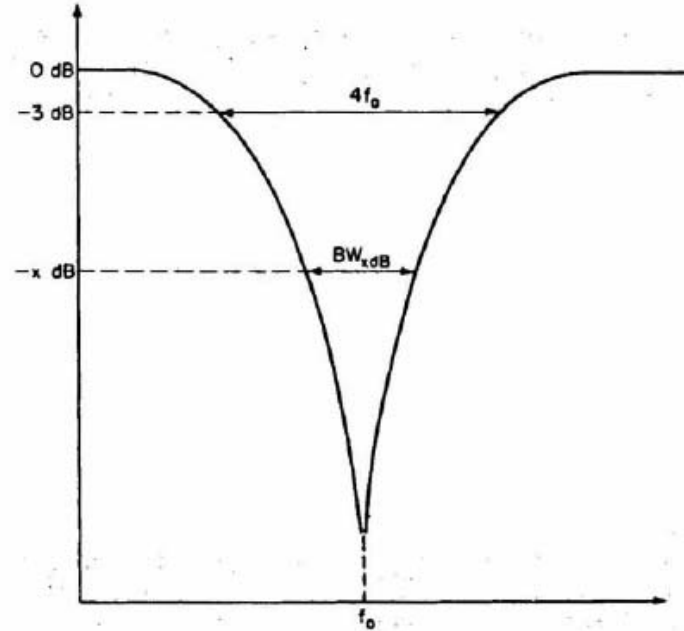
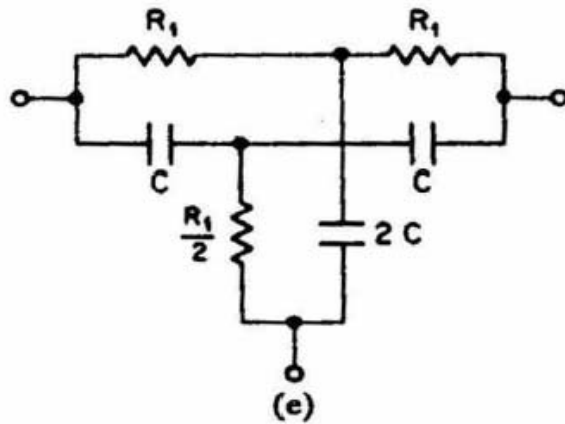
### The 1/S Transformation

| Element   | Impedance      | Transformed Element   | Transformed Impedance |
|---|----------------|---|-----------------------|
|    | $sL$           |    | $L$                   |
|   | $\frac{1}{sC}$ |   | $\frac{1}{s^2C}$      |
|  | $R$            |  | $\frac{R}{s}$         |

## High-Q Notch Filters



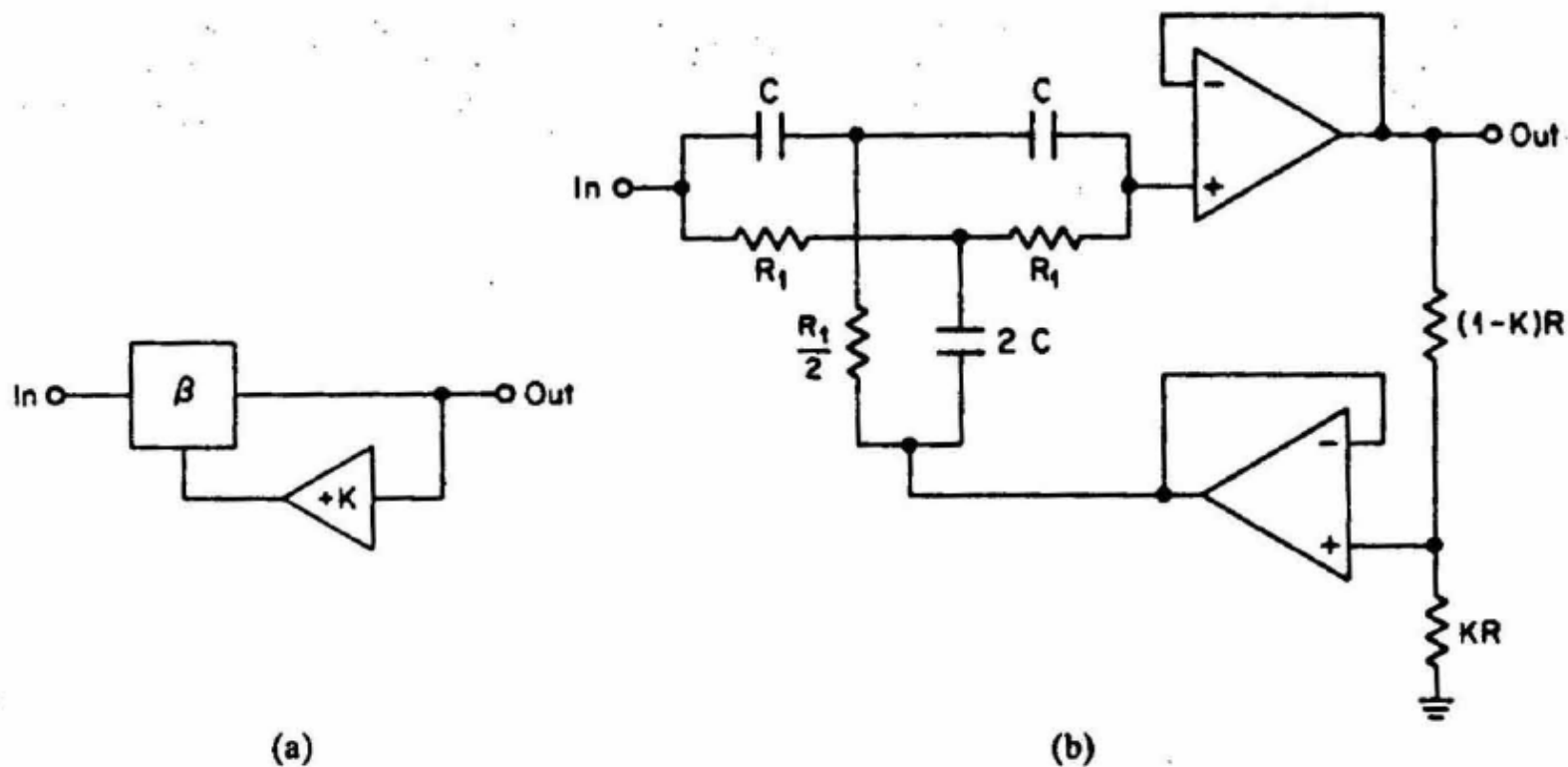
This circuit is in the form of a bridge where a signal is applied across terminals' 1 and 2 the output is measured across terminals' 3 and 4. At  $\omega=1$  all branches have equal impedances of  $0.707 \angle -45^\circ$  so a null occurs across the output.



To calculate values for this circuit pick a convenient value for C. Then

$$R_1 = \frac{1}{2\pi f_0 C}$$

The Twin-T has a Q ( $f_0/BW_{3dB}$ ) of only  $1/4$  which is far from selective.



Circuit A above illustrates bootstrapping a network  $\beta$  with a factor  $K$ . If  $\beta$  is a twin-T the resulting  $Q$  becomes:

$$Q = \frac{1}{4(1-K)}$$

If we select a positive  $K < 1$ , and sufficiently close to 1, the circuit  $Q$  can be dramatically increased. The resulting circuit is shown in figure B.